Topics in Safety, Risk, Reliability and Quality

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Introduction to Optimization Analysis in Hydrosystem Engineering

Springer

Topics in Safety, Risk, Reliability and Quality

Volume 25

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ISSN 1566-0443 ISSN 2215-0285 (electronic)
ISBN 978-3-319-04399-9 ISBN 978-3-319-04400-2 (eBook)
DOI 10.1007/978-3-319-04400-2
Springer Cham Heidelberg New York Dordrecht London

Library of Congress Control Number: 2014930225

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

To my lovely wife, Mina

Preface

Population increase and socioeconomic rise of developing countries in many parts of the world has escalated the water demand for various uses including agriculture, municipal, recreational, and industrial demands. The increased demands in the past few decades have put severe stresses on the available water resources across the world particularly in arid and semi-arid regions. Hence, optimal management of water resources is a crucial issue and it is imperative to adopt realistic policies to ensure that water is used more efficiently in various sectors. In this book, we present the latest tools and methods to assist the students, researchers, engineers, and water managers to properly conceptualize and formulate the resource allocation problems, and deal appropriately with the complexity of constraints in water demand and available supplies. Although existing references supply total information relevant to the optimization analysis in water resources engineering, providing a book for undergraduate and graduate students and newcomers to this field is a requirement. In other words, what is needed is to present concepts more simply on the basics of optimization theories, get directly to the principal points, and apply simple examples in preparation for the use of more advanced texts.

In this book, the basics of linear and nonlinear optimization analysis for both single and multiobjective problems in conjunction with several examples with various levels of complexity in different fields of water resources engineering are presented. The main advantages of the current book rather than existing publications briefly are:

1. The authors' idea is to use simple examples and solve them step by step as the best way to introduce the materials in the book, and also to provide useful information to better understand the implementation of theoretical concepts. Hence, each chapter of the book contains some examples related to the basic principles of linear and nonlinear optimization analysis for both single and multiobjective problems (Chaps. 2–4).
2. As EXCEL, LINGO, and MATLAB are three of the well-known computer programs used today in optimization analysis, the process of solving optimization problems using those programs are presented in details as an alternative. This characteristic teaches the application of the noted computer programs in optimization analysis and makes analyzing, organizing, interpreting, and presenting results quick and easy (Chap. 5).

3. Real case studies are important resources for students to apply theoretical formulas, and computer programs to analyze real events. Hence, three real case studies as a valuable source for students, practitioners, and researchers are presented in the last chapters of book to show how the optimization concepts and theoretical formulas are used in analyzing real world problems (Chaps. 6–8). The case studies in brief are;

   - Reservoir Optimization and Simulation Modeling,
   - Reservoir Operation Management by Optimization and Stochastic Simulation, and
   - Water supply optimization in central Florida (simulation-optimization using integrated surface and groundwater modeling to allocate groundwater pumping that is protective of the natural ecosystem while meeting water supply demands of over two million people using a mix of surface water, groundwater, and desalinated water).

4. Finally, complete lists of most optimization studies on hydrosystem engineering (1963–2013) are presented in the Appendix of the book in table format. These tables include authors' names, dates of study, and a brief description of their work. With the help of these tables, readers can easily find all previous studies related to hydrosystem optimization analyses that may of particular interest to them.

To sum up, the main purpose of this book is to serve as a guide for conducting and incorporating optimization analyses in water resource planning processes. This book's main theme is to improve the understanding of the quantity and quality of information we have, and the importance of information we do not have, for the take only out purpose of improving decision making. The principal audiences of this book are undergraduate and graduate students of water engineering and all new researchers who are interested in academic research associate with optimization analysis as well as practitioners in the field of water resources management. Furthermore, this book can be used as reference for teaching in various fields of water engineering including: hydrology, hydraulic, water resources analysis, water quality analysis, etc. This book is also a useful reference for practicing engineers/professionals as well as students and individual researchers. They can apply optimization analysis as a useful tool to make best informed decisions when designing for unaccounted loads.

Acknowledgments

I would like to gratefully and sincerely thank Dr. AliReza Aghasi for his help, guidance, and expertise on this book. The authors also want to acknowledge Dr. Zia Hosseinipour not only for his contributions to this book, but also for the outstanding mentorship that he has provided me. I correspondingly would like to gratefully acknowledge the contribution of Drs. Nisai Wanakule and Alison Adams from Tampa Bay Water, Florida, on this book. In addition, I appreciate all Dr. Ozeair Abessi help during writing this book.

Contents

Chapter 1
Importance of Optimization Analysis

Abstract This introductory chapter introduces the challenges that the water professionals will encounter in water management and allocation of water supply, factors that impact the water availability, water-energy-climate change relationships, water transfer and the role of various stakeholders, and finally policy decisions deriving the investments needs for planning and maintaining water supply systems.

1.1 Introduction

The world's readily available fresh water resources are becoming increasingly scarce due to higher demands by municipal, industrial, recreational, and agricultural sectors mostly because of population increase and higher standards of living in many areas, but also in part due to changes in land use and global climate change as a result of rapid development. In fact, the nexus between water and energy use seems to be a real issue that needs the attention of decision makers at all levels of governments and international organizations. The water energy nexus and related stresses do not subscribe to jurisdictional and political boundaries recognized nationally or internationally, and hence requires multi-organizational/ stakeholders solutions. Effective management of natural and water resources is becoming one of the most important challenges of our era to resolve, for maintaining and/or improving the living standards we enjoy in the developed and developing counties. In addition, the relations between energy, water, food, and environmental issues must be considered carefully in the development of water management plans and ultimately towards the goal of Integrated Regional Water Management (IRWM) Plan which is closely tied to sound watershed planning. The realities of water management include a limit to the availability of water whether local or imported supplies. This places a greater emphasis on innovative local/ regional projects that are multi-faceted and multi-purpose considering a holistic approach and consensus from various stakeholders. The water professionals must

E. Goodarzi et al., *Introduction to Optimization Analysis in Hydrosystem Engineering*, Topics in Safety, Risk, Reliability and Quality 25, DOI: 10.1007/978-3-319-04400-2_1,

focus on many aspects when performing initial studies as a building block for water management plan developments, those include:

1. Future water supply availability/reliability,
2. Direct and indirect municipal/irrigation water reuse,
3. Water quality through salinity management and desalting opportunities,
4. Water projects and tie-into climate change adaptation,
5. Future funding needs and revenue streams,
6. What are the pros and cons of various funding options? As well as other site specific consideration.

To overcome the stresses on natural resources and in particular the fresh water supply sources multi-purpose and multi-objective water/natural resources management is taking root across the world. These days we see professionals in various fields of environmental and water resources engineering as well as allied disciplines of economics and social sciences collaborate to develop water resources management solutions that meet the urban, agriculture, industrial, habitat, environmental, recreational, and ecosystems requirements with constraints and priorities that must be consensus based by the stakeholders.

Water is a valuable resource everywhere in the world even areas that have seemingly plenty of precipitation. For example, although southeastern USA and Great Lakes states in Midwest receive above average rainfall with respect to other regions of the world, these areas still have water management challenges because of extreme events, water quality issues and management of non-point source pollution. The world is facing water supply challenges that will test the technical and managerial skills of trained professionals and the expertise of water scientists to the fullest extent in the next two–three decades. The effects of climate change, extreme floods and the economic and structural damages by frequent floods, pollution from urban and agricultural run-off, require collaboration from multitude of engineering and scientific experts as well as other stakeholders. In short, often there are competing interests in managing and protecting this vital resource in every region of the world. One area of utmost importance is the key component of how stakeholders consider water issues and make appropriate policy decisions or rank different priorities during water shortage and other emergencies.

Water management requires input from a multidisciplinary team from hydrologists to ecologists and other experts. To assess the availability of water for various uses under different conditions experts often develop water management models for a proposed project. These models often look at all sources available considering economics, water quality, specific use, and socioeconomic issues. For example, specialists may be looking at the hydrogeology of the area (ground and surface water interaction) in high water table conditions like much of south and central Florida and how exploitation of a water source impacts the other and the surrounding ecosystem; the economic aspects of water use and the impact of water use on the environment is of great interests to social scientists and ecologists. For instance, the potential relation between the ecosystem value and economic benefits of water use has been studied by ecological economists over the last decades.

Climate scientists also are looking at the effects of climate change and variability on water availability and scarcity, while behavioral scientists are examining people's biases and beliefs and the effects on the policy and decision making process.

1.2 Challenges Facing Water Management and Policy Professionals

Water resources management presents a variety of challenges, and growing world population make certain demands on the existing water resources across the world. Industry and industrial waste management cause other impacts, while agricultural water use bring about a variety of challenges from meeting water demand during droughts to soil water logging, salinization to nutrient and pesticide migration to groundwater aquifers and surface waters. Economic development and vitality is quite simply dependent on water availability at a reasonable price.

Water resources are among the most important factors which could be affected by climate changes and recent global warming. In addition, increasing water use in turn can increase the negative impacts of climate changes on ecosystems and local hydro-climate. With most developments the environment typically gets short-changed, that is why we need to look at ecosystem sustainability as part of the equation. Engineers need to work closely with economists, information technologists, and ecologists for information on the economic value of ecosystem services and the impacts of water use on ecosystems. Resource management professionals want to figure out how we can support both ecosystem protection and economic development with the limited amount of available water. This requires managing the water supplies using schemes that can take into account various objectives and constraints with given priorities, this is called "optimization". The water management system that uses optimization is amenable to an adaptive management approach, based on various scenarios, which the study team can analyze and provide the results to the stakeholders for informed decision making. For example in a given area the scenarios may assume significant sea level rise and its impact on groundwater availability and quality degradation, rainfall and temperature changes over land, and a range of population and economic growth rates, and economic trade off among various uses.

Various stakeholders as well as scientists/engineers participate in the study helping the team in the course of developing appropriate plans for water management to find out public support on data and latest technological tools. The water resources professionals job is not only to solve the problem of water scarcity in every region, say for example in south Florida or southern California, rather to use a regional example as a case study to see how multiple stakeholders can cope with complex issues and move towards more sustainable water use on a consensus based approach that optimizes the use of available supplies simply because there are very limited additional sources often at much higher costs. Another area that requires input from water professionals as well as social scientist, economists and well informed stakeholders is considering reclaimed water as an available resource that can be used for various uses including municipal supply.

1.3 Local, Regional, and International Competition for Water and Ensuing Conflicts

In 2010 United Nations (U.N.) General Assembly declared 2013 as the International Year of Water Cooperation (IYWC). The U.N. is aware about the competition over the existing finite fresh water resources in the world. Current and past water conflicts and disputes have included confrontations between countries in the Middle East (Israel and Jordan, Israel and Lebanon, Turkey and Iraq, Palestine and Israel, etc.), in Southwest Asia (India and Pakistan, India and Bangladesh), in Africa (Egypt, Sudan and Ethiopia), and in South America (Bolivia, Peru and Chile), among many other places. Even within countries there are sometimes conflicts among different regions over water allocations of trans-boundary water resources and inter-basin transfers; for example in the USA, states of Alabama, Georgia and Florida have been fighting for decades over *Apalachicola-Chattahoochee-Flint River System* (ACF) and tributaries' inflow that end up in the Gulf of Mexico; and Colorado River transfer to California, Nevada and Arizona has been a source of hot debates for the past few decades; in central west Iran, water transfer from Zayandehrud in Isfahan to Yazd and from Karoon Basin to Zayandehrud Basin have caused local protests and in some cases physical conflicts. Another recent example is the damming of tributaries of Lake Urmia in northwest Iran which has caused the lake to shrink significantly posing irreversible ecological damage in the region and a lively national debate on dam building and irrigation water use (Fig. 1.1).

In the case of ACF, water allocation and establishment of Minimum Flows and Levels (MFLs) required for aquatic and ecologic health of downstream habitats especially during low flows and droughts was a source of conflict that took decades of negotiations and law suits and eventually an act of US Congress to develop an agreement among tri-states which was hard to accept by the parties, but their best options was to agree to share the limited resource rather than devoting time and money to endless conflict resolution (Fig. 1.2).

Conflicts like these have shown that the water professional/managers need not only to be well versed in the science and engineering aspect, but also need knowledge of applicable laws, regulations, negotiation skills and applied optimization principles in order to formulate feasible options that can be looked at as win–win for all parties involved. So, the upcoming conflicts will be extremely dependent on the human ability to deal with the water demand challenges; if we are able to increase water use efficiency and productivity such that we can free up

Fig. 1.1 Lake Urmia, the third largest salt water lake on earth at **a** 2003, and **b** 2010 (Payvand 2011)

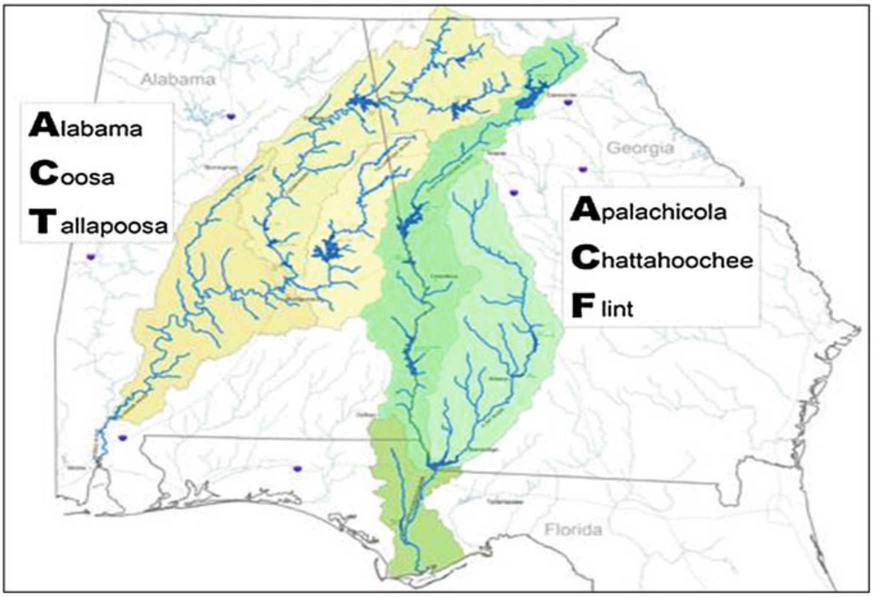

Fig. 1.2 Map of the ACF river basin watershed in the southeast USA showing the Apalachicola river and its two main tributaries, the Chattahoochee river and Flint river (Atlanta Regional Commission 2010)

water resources for protecting our environment, thereby ensuring the sustainability of the supply, and allowing for new users and uses, it will be easier to cooperate. If we cannot manage the available water demands, water management will become difficult like a zero-sum exercise, and so there would be permanent challenges on the available water resources.

According to the recent figures, nearly 800 million people in the world live without safe water, that is, roughly 15 % of the world population. Some 2.5 billion others live without access to sanitation, about 40 % of the world's population (United Nation Water 2013). These figures portray a grim scenario for political and social stability in many areas of the world for the foreseeable future: ethnic conflicts, regional tensions, political instability and mass migrations would prevail without immediate actions by governments and international organizations.

In the not too distant future, many countries will certainly face water related problems including shortages, water quality issues, epidemics due to contaminated water, or floods, and these problem increase the risk of instability and regional tensions (Global Water Security: Intelligence Community Assessment 2012). In this report these issues are connected to a world where the population is growing fast and the demand for freshwater is growing even faster. Therefore, close coordination and cooperation between various sectors in governments, NGOs, international organizations is of fundamental importance if we are to successfully share and manage our most precious resource (fresh water), for which we need reliable and defensible information to make sound and consensus based decisions.

To address these issues there is a need to cooperate with players outside the water sector, to foster collaboration between the various decision-making entities, between the private, public and civic sectors as well as between actors who work in water research, public policy and public relations. That is, only through sound and forward-looking consensus based partnerships a water wise world may be achieved. Because of population growth and pressures on water and natural resources within and among nations, sound and fair resource management is a clear imperative; water professionals will have no time to waste to come up with a solution to natural resources management in general, and water resources in particular.

One area of great need is the optimal management and operation of existing reservoirs and water allocation which are critical issues in sustainable water resource management due to increasing water demand by various sectors. Multiplicity of stockholders with different objectives and especially water utilities make reservoir operation and other sources of available water a complicated problem with a variety of constraints, and at times conflicting objectives to be met. In such cases, the conflict resolution models can be efficiently used to determine the optimal water allocation scheme considering the utility and relative authority of different stakeholders. Water resources planning and management is a combined process of sharing water that very often involves specific difficulties and complex decisions on resolving conflicts among decision makers, water users and stakeholders. Because of limitations on the quantity and quality of water resources, the optimal operation of reservoirs in a watershed is very important for providing a secure water supply from a system's point of view, Karamouz et al. (2003) discuss this issue in details.

1.4 Urban and Agricultural Water Supply Challenges

Worldwide approximately more than 70 % of available freshwater is used for irrigation of crops to supply food and fiber for the growing populations. Although great strides have been made in irrigation efficiency in a number of developed and developing countries to reduce water use in the agricultural sector, there is still progress to be made into save water and reduce environmental impacts. The remaining 30 % is used for other uses including municipal water supply. For a new framework for urban water systems to become fully established, industrialized countries should serve as a model for future water supply management in developing countries. Sweeping changes will be needed in the ways that engineers and managers of urban water systems approach the planning, design, and operation of urban water infrastructure. For this change to take hold, it will be necessary to embrace not only new technologies but also innovative management strategies that can create more resilient, economically sustainable water systems that will better serve society's present and future needs. Public acceptance, particularly for new technologies and unfamiliar practices (e.g., grey water recycling, reclaimed water use), will require more effective communication about the processes and their established safety.

For the purpose of understanding current and future development needs, the elements of the new framework for urban water systems may be subdivided into four themes: increasing water availability through improved system efficiency, demand management, desalination to augment water supply, stormwater harvesting, and water reuse/reclaimed water. Broadening treatment options by developing technologies that lead to more resilient systems, linking water quality to its intended use and incorporating managed natural systems into urban water infrastructure; considering wastewater as a resource through energy and nutrient recovery; and establishing an enabling environment by explicitly addressing institutional and management challenges related to a need to account for non-monetary benefits, manage tradeoffs among alternatives and more effectively engaging stakeholders and public at large. Let's look at each of these themes in more details.

1. *Increasing Water Availability*: In the past, water supply issues were frequently solved by building huge structures such as dams or water distribution networks for storage, transfer and conveyance of water. Conservation has been another way to save water in the residential, agricultural, and industrial sectors over the last decades. For example, Singapore applied a water conversation tax to reach 11 % reduction in average monthly water consumption between 1995 and 2004 to improve water availability (Tortajada 2006). These efforts are definitely helping to meet increasing demands and should be encouraged.
2. *Demand Management*: Many water utilities in the USA and other countries have provided incentives to households to replace older home water fixtures with newer ones that use less water (e.g., low use shower heads, toilets using less water per flush, etc.). While water use efficiency will continue to serve as

an important component of urban water supply in the coming decades, there are signs that it will eventually become less attractive, as the least expensive water conserving appliances and industrial process modifications are implemented. Using water losses strategies is one of the best practical approaches to increase the efficiency of urban water systems and better manage demands. For example, about 14 and 40 % of treated water in the U.S. and developing countries respectively, is lost to leaks (Grant et al. 2012). A common occurrence in many old cities of the world is pipe breakage which is not only the cause of significant water loss, but it is also the cause of collateral damage like flooding. In this case, the modern management techniques can be used to save water through effective leak detection and in a more cost-effective manner.

3. *Desalination*: Although many challenges remain with regard to environmental impacts and large initial capital costs, desalination is now considered a viable option for urban water supply, particularly in situations where either climate change or short-term events (e.g., catastrophic floods) compromise water quantity and quality. The acceptability of seawater desalination has come about principally because of the reduction in power consumption of the reverse osmosis stage due to improved membrane design and implementation of energy recovery technologies. Many cities in different regions of the world rely partially or fully on desalinated sea water or brackish groundwater (e.g., Arab nations of the Persian Gulf, Tampa Bay Region of Florida, Australia's driest capital city of Perth, San Diego Region in southern California, among many others. The examples cited, receive up to half of their water supply by desalination plants. Many other municipalities are also planning for desalination plants as supplementary source during droughts and other emergency situations (i.e., disruptions in regional transmission facilities). One are of concern is the potential increase of greenhouse gas emissions associated with operation of the desalination plants, but that could be offset by energy from renewable sources (wind farms and solar panels).
4. *Stormwater Harvesting*: Based on this scheme the runoff could be captured and recharged into the aquifers or stormwater and reuse for non-drinkable usages by combination of urban runoff and flood control management. This underutilized water source can be used to supply some parts of water requirements of cities. A known example in the course of stormwater harvesting is the Los Angeles County Department of Public Works which runs 27 spreading basins that recharge about 150 million m^3 of surface water runoff in one year (Los Angeles County Department of Public Work 2012). Similar approaches are implemented in other areas of southwest USA as well as arid countries in the Middle East (central and southeastern Iran). Although some of the recharged runoff consists of dry weather flows from rivers that receive wastewater effluent, the majority of the recharged water is associated with wet weather flows. Other areas in southern California are also pursuing efforts to further enhance the recharge of stormwater as part of a strategy for coping with possible decreases in imported water sources (Ventura County). One caveat in this scheme is that more research is needed to assess the water quality implications

of this practice, and, when necessary, integrate passive treatment processes into recharge systems. Even, in high rainfall regions like southeast USA aquifer storage and recovery (ASR) has been studied and in some cases implemented in the last two decades for water supply augmentation.

5. *Water Reuse/Wastewater Reclamation*: This option is becoming more and more attractive and even imperative in areas with limited water supply of their own that relay on imported water and/or desalinated water. Options for reuse include using highly treated wastewater (tertiary treated) for irrigating urban landscape (parks, gulf courses, and lawns), in some cases for agricultural uses (greenbelts around cities in the arid and semi arid areas of the world, or tree farming), use in industrial operations when applicable such as fertilizer manufacturing and steel mill cooling requirements, and in the case of advanced treatment for municipal water supply (Orange County, California has constructed and currently operates the most advance treatment plant which is more like a refinery than a wastewater treatment plant) for municipal use by injecting the reclaimed water into the underground aquifer to maintain a seawater intrusion barrier and feeding percolation basins used to augment their imported source via groundwater recharge, in effect an ASR with reclaimed water. This plant is an acclaimed state-of-the-art in wastewater treatment/reclamation and attracts visitors from all over the world (www.ocwd.com or www.gwrsystem.com). Orange County Water District (OCWD) operates the plant in a series of steps. After wastewater is treated at the Orange County Sanitation District, it flows to the Groundwater Replenishment System (GWRS) where it undergoes a state-of-the-art purification process involving microfiltration, reverse osmosis, ultraviolet light and lastly treatment with hydrogen peroxide. The product water is near-distilled-quality. About 70 million gallons (265,000 m^3) per day of the GWRS water are used to both pumped into injection wells to create a seawater intrusion barrier to protect freshwater aquifers, and transfer to the percolation basins in Anaheim where the GWRS water naturally filters through sand and gravel to the deep aquifers of the groundwater basin; about half the total is used for each operation (Groundwater Replenishment System 2003).

As seen various approaches are being used to meet water supply requirements of municipalities. However, water allocation rules may lose their validity in terms of supplying reliable water to current and future water demands due to changing hydrologic and socio-economic conditions, as well as, changes in land use and development requirements in a given region. Therefore, water sector decision makers are very interested in knowing when and how they must update the water allocation rules, especially the withdrawal ratios from reservoirs, river and groundwater, to fulfill the current water demands while minimizing the costs associated with fulfillment of unmet water demands; in short, how to factor in uncertainty. Hydrologic and socio-economic uncertainties are the most influential parameters in water supply and demand management, which in turn affect water allocation rules for long-term planning. Hydrologic uncertainty may be included explicitly in water supply management models for minimizing the long-term operational cost or maximizing the water supply reliability in drought conditions. Integrated water supply and demand simulation is also useful for achievable improvements in water supply reliability by a combined supply–demand management strategy like the scheme used by Tampa Bay Water (Chap. 8). However, the challenge is that integrated holistic models put serious restrictions on the predictive accuracy and the size of the problem to be solved. In addition to long-term impacts, uncertainty and integrated modeling for water allocation, inclusion of operational and performance objectives in terms of equity, reliability and social acceptability are the required criteria for assessment of dependable water allocation systems (Dinar et al. 2005; Joshi and Gupta 2010). Water allocation rules need to be derived and classified based on hydrologic and socio-economic conditions in the basin, for example Normal Operation Policy (NOP) and Emergency Operation Policy (EOP) that are triggered based on a drought index value for reservoir operation rules (Eum et al. 2011). Engineers dealing with hydro-systems mostly focus on optimum operational analysis that reveals how much improvement is achievable by changing the attitude towards water system operation in terms of preferences or the degree of integration between different disciplines influencing the project. Within this framework, different constraints on optimal operation (i.e., spatial interactions, uncertainties, long-term impacts and performance indices) can be incorporated for analysis and classification of different water allocation rules.

Optimization application in conjunctive use of surface water-groundwater-desalinized water, and managing downstream water quality, as well as, aquatic and ecosystem needs related to Minimum Flows and Levels (MFL) and power production is an area of much interest in face of uncertain hydrologic conditions. Many examples of these applications are cited in the literature, for example, Tampa Bay Region of Florida in the USA (Chap. 8 in this volume), south-eastern USA like Tennessee Valley Authority (TVA) and Zayandehrud and Karoon Rivers in Iran, Euphrates and Tigris in Turkey-Syria-Iraq, as well as others. Water allocation using integrated Water Quality-Quantity modeling has been applied to study water resource issues and conjunctive use by Javier et al. (2010), Zhang et al. (2010), Qin et al. (2009), and Wang et al. (2008) to use water for beneficial consumptions while making sure ecosystem and aquatic needs are met while meeting local and national regulations.

Water Management on a regional and national basis is even more complex and requires a multi-purpose and multi-disciplinary approach. For example, let's consider an inter-basin water transfer that will be crucial for economic survival of the receiving basin but will also affect the donor basin in many aspects (Zayandehrud and Karoon Rivers in Iran). For such analysis an Integrated Stochastic Dynamic Programming (ISDP) approach seems appropriate, but what are the factors and parameters that need to be considered and optimized? Let's contrast an ISDP for reservoir operation with an inter-basin water transfer. In the former, the water storage and inflow are state variables and the release from the reservoir is the decision variable. In the latter, the operation of both reservoir in the donor and GW in the receiving basin would be state variable in addition to many other parameters in order to arrive at an optimum water transfer policy. Many parameters may have to be considered in an ISDP such as: net benefits to the water users in both basins, water demands of both basins, characteristics of the reservoir in the donor and the aquifer in the receiver basin, pumping costs for the receiver, etc. An example of a potential ISDP application is the operation of the three Gorges Dam in China that needs to incorporate all these factors as well as many others.

1.5 Global Climate Change Impacts on Integrated Regional Water Management

Recently, many proposed and enacted Integrated Regional Water Management Planning (IRWMP) have required that the plans must include an evaluation of the adaptability to Climate Change and its impact on the water management systems in the region. Given the currently predicted effects of Climate Change on water resources, IRWM Plans are to address adapting to changes in the various characteristics of runoff, storage, and recharge such as their timing, intensity, amount, and quality (RMC 2013). For example, areas of southern California that receive water from the Sacramento-San Joaquin River Delta, the area within the San Francisco Bay Delta, and areas served by coastal aquifers will have to consider the effects of sea level rise on water supply conditions and identify suitable adaptation measures. Decisions about adapting water management schemes, as well as, mitigating Climate Change through reductions in Green House Gases (GHG) emissions, should take into account the risks to the region of no action alternative. A key factor in assessing the effects of Climate Change and adapting to those changes as it relates to water supply is the use of adaptive management. IRWM plans need to contain policies and procedures that promote adaptive management. As more effects of Climate Change manifest and new tools are developed, new information becomes available that the Regional Water Management officials must adjust for in their IRWM plans accordingly. However, tools to properly assess the risk of any one effect of Climate Change on a region are currently not well developed, and the abilities of different regions to use such tools vary considerably. The challenge is how to account for this impact in an IRWM?

In addition to responding to the effects of Climate Change, IRWM plans can also help mitigate Climate Change by reducing energy consumption, especially the energy embedded in water use, and ultimately help decreasing GHG emissions. Water management in developed countries results in the consumption of significant amounts of energy and the accompanying production of GHG emissions, especially where water must be pumped for long distances; from the underground aquifers; or over significant elevations. As an example, according to California Energy Commission November, 2005 CEC-700-2005-011 California's Water—Energy Relationship Final Staff Report, 19 % of the electricity and 30 % of the non-power plant natural gas of the State's energy consumption are spent on water-related activities.

Fig. 1.3 The Cornalvo dam, Spain—1,500 years young—a well maintained dam can be operated for many years for sustainable water supply. The Cornalvo dam in Spain was built by the Romans almost 1,500 years ago and it is still fully functional (WikiPedia 2013)

The close connection between water resource management and energy is an important consideration to meet national and international GHG emission reduction goals. All aspects of water resources management have an impact on GHG emissions, including the development and use of water for habitat management and recreation, domestic, municipal, industrial, and agricultural supply and hydroelectric power production and flood control. Therefore, water professionals need to be also cognizant of water-energy nexus and its impact on GHGs as a driver of Global Climate Change (Fig. 1.3).

1.6 The Incentive for This Book

The above introductory sections enumerated the challenges that water professional face in designing, building, and managing water supply infrastructures of all kinds (canals, reservoirs, desalinization plants, wastewater reclamation, etc.). The main driver behind all this is to provide more water supply while reducing costs both in building the systems and maintaining them through their design life (sustainability). Therefore, students in this field as well as practitioners need to become well versed in optimal use of resources both to build and maintain the facilities, as well as managing the output from the facilities (source of water supply for all users). This applied optimization book is designed to fill the gap that currently exists for all parties interested in learning how to use the optimization principles to solve their water management problems. There are many text books in the field of operations research and engineering systems, as well as water management that discuss optimization theory and its application areas, but they lack clear and so, concise example problems that a beginner student needs to easily relate to the theories discussed here. This book is different in the sense that after introduction of various methods and concepts, example problems that are readily understandable are introduced and solved step-by-step using commonly available software to help the reader connect the theory with applications. Various example problems that are commonly encountered by water professionals are introduced after the presentation of each topic and then solved to elaborate the application of the method in the real world. This introductory chapter introduces the challenges that the water professionals will encounter in water management and allocation of water supply, factors that impact the water availability, water-energy-climate change relationships, water transfer and the role of various stakeholders, and finally policy decisions deriving the investments needs for planning and maintaining water supply systems.

Chapter 2 introduces the reader to the concept of Linear Optimization and its applicability to solve classes of problems that are amenable to this method. Chapter 2 covers the basic concepts of linear optimization analysis and its applications in water resources engineering. In this chapter, the graphical and simplex solution methods for solving linear optimization problem are discussed and then illustrated step by step with some example problems. In addition, the applications of simplex method in solving water distribution network plus one- and two- dimensional confined aquifer optimization problems using the Solver tool in Excel are presented. The graphical and simplex methods are discussed in detail with example problems in urban water management and aquifer pumping optimization.

Chapter 3 introduces the reader to nonlinear programming and the conceptual framework for nonlinear optimization, as well as its applications in water resources engineering. In addition, different nonlinear optimization methods including one-dimensional optimization techniques, unconstrained and constrained optimization methods in conjunction with a number of example problems to indicate why and how nonlinearities arise in a wide range of water resources optimization problems. Random search method, Newton method, Univariate Method, and Steepest Descent Method are then discussed with example problems in each section to help readers connect the concepts with applications.

In Chap. 4, the reader is introduced to Multi-objective Optimization (MOO) and the fact that most real world problems are multi-objective in nature and require consideration of several minimizing or maximizing objective functions to be optimized simultaneously. For multi-objective problems, a set of optimal solutions instead of a single solution needs to be determined. This chapter provides the idea behind multi-objective optimization in water resources engineering projects in today’s complex world of water supply-water demand management and lays the foundation for using readily available software to address and solve these multi-faceted problems. This chapter also provides a number of applicable and commonly encountered example problems and their step-by-step solutions to help the reader better understand the concept and applied side of multi-objective optimization. In these examples, Weighted Method and its application to different aspect of water supply and environmental management issues are demonstrated.

Chapter 5 introduces the reader to LINGO and MATLAB softwares and their adaptability to solve all classes of optimization problems. These well-known applications in science and engineering field can be readily used to formulate, express, and solve optimization models. In this chapter, the process of solving both single and multi-objective optimization problems using these programs is presented in details. Furthermore, a number of useful examples are provided and solved step-by-step to help readers better understand the application of LINGO and MATLAB in solving linear and nonlinear optimization problems. The remaining chapters of this book discuss case studies applying optimization schemes to address real world water supply-water demand issues.

Chapter 6 presents a combination of optimization (LINGO) and simulation (HEC-ReSim) models to determine monthly operating rules for the Zayandehrud Reservoir system in central west Iran. A single-objective framework is used to optimize and determined outflow-storage of the reservoir and simulate the system behavior over 47 years. The results show that optimizing the operation of Zayandehrud reservoir could increase its storage by 88.9 % as well as increase the reliability index of regulated water for all downstream demands more than 10 %.

Chapter 7 presents Stochastic Simulation principles and its application for optimized reservoir operation and water supply management. Managing optimal use of available water supplies resources is a vital issue in many parts of the world especially in arid and semi-arid regions. It is imperative to adopt realistic policies to ensure that not only water is used efficiently in various sectors, but it is also allocated effectively for best use during droughts and/or high demand periods. This chapter presents an optimization analysis to determine monthly operating rules for the Doroudzan Reservoir located in southern Iran. Different strategies under limited water availability conditions are analyzed by running an optimization model based on observed and synthetic inflow data, and the performance indicators of each strategy are presented. Each strategy includes a minimum requirement release in the optimization process that results in a specific operational policy. In this example, LINGO is applied to determine optimum operational parameters and synthetic inflows are generated using the Monte Carlo simulation method. The results demonstrated that the applied methods could effectively optimize the current operational policy of an existing reservoir in a single-objective framework.

Chapter 8 presents a comprehensive application of real world simulation–optimization scheme to manage multi-wellfield, reservoir, and desalinated water supply in central west Florida. Tampa Bay Water is Florida's largest wholesale water supplier, serving more than 2.3 million people with annual average daily demand ranging from 220 to 262 million gallons (mgd). The water supply agency mission has been to develop, store, and supply water for municipal purposes in a manner to reduce adverse environmental impacts due to excessive groundwater withdrawals from concentrated areas. Conflict between meeting water demands and preventing harm to surrounding wetlands and lake systems was intensified in the early 1990’s, making it difficult to manage wellfields effectively. In 1996, the predecessor Agency was mandated by Florida Legislature to develop regional water supply solutions and a comprehensive answer to the water supply needs of the Tampa Bay area in a manner that is protective of the environment (groundwater cutback) and meets the long term water demand of the region.

To relieve ecosystem stress and develop an environmentally sustainable water supply system, Tampa Bay Water implemented an Optimized Regional Operation Plan (OROP) that incorporated additional water supply sources (surface water and desalinated water). The Optimized Regional Operations Plan (OROP), includes a region wide integrated ground water-surface water simulation–optimization model that is used to schedule well pumpage among eleven wellfields with an objective of maximizing surficial aquifer water levels which are correlated closely to wetland and lake water levels in the surrounding areas of wellfields. This scheme also envisioned a mitigation plan which provides rehydration for hydrologically-stressed wetlands and lakes. As additional water supply sources came on-line, the OROP model and the mitigation plan were used to ensure that water production would not result in unacceptable adverse environmental impacts, and that historical impacts from groundwater production were gradually addressed.

The OROP was designed to minimize production impacts to wetlands and lakes by rotating among sources in response to target levels set in surficial aquifer monitoring wells. These target levels were determined by statistical correlations between minimum levels established for wetlands and lakes and surficial aquifer water levels (the applied concept of Minimum Flows and Levels, MFLs). The establishment of minimum wetland and lake levels was based on regulatory criteria that relate environmental health to indicators of historical wetland and lake levels (known as historical normal pool).

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Chapter 2
Linear Optimization

Abstract The purpose of this chapter is to cover the basic concept of linear optimization analysis and its applications in water resources engineering. The graphical and simplex solution methods for solving linear optimization problems are illustrated step by step. In addition, the applications of simplex method in solving water distribution network and one and two dimensional confined aquifer optimization problems using the Solver tool in Excel are presented.

2.1 Linear Programming

A linear optimization problem can be defined as solving an optimization problem in which the objective function and all associated constraints are linear. The linear optimization is also known as linear programming (LP) and it can be defined as the process of minimizing or maximizing a linear function to find the optimum value (minimum or maximum) for linear and non-negative constraints. The term *programming* here implies the way of planning and organizing (formulation) to find the optimal solutions, and it is different from its meaning in coding and computer programming. In general, this method is a relatively simple technique to find realistic solutions for a wide range of optimization problems and includes three essential elements listed below:

1. Identify decision variables: decision variables are the unknown variables of the problem statement that need to be determined to solve the problem. Defining decision variables precisely is a fundamental step in formulating a linear optimization model.
2. Obtain the objective function: in this step we need to define the objective of desired problem statement which shows the main goal of the decision-maker. Afterward, the relations between decision variables and the objective should be accurately determined. It should be noted that the function cannot include any nonlinear component such as exponential, products, or division of variables,

E. Goodarzi et al., *Introduction to Optimization Analysis in Hydrosystem Engineering*, Topics in Safety, Risk, Reliability and Quality 25, DOI: 10.1007/978-3-319-04400-2_2,

and variables under a root sign. All variables only must be added or subtracted in a linear fashion.

3. Determine the constraints: constraints explain the requirements that desired problem shall meet, and it can be in the forms of either equalities (=) or inequalities (≤, ≥).

The general form of linear programming model can be written as:

$$\min \mathrm{f}(x_1, x_2, \ldots, x_n) = c_1x_1 + c_2x_2 + \cdots + c_nx_n \tag{2.1}$$

Subject to the following constraints:

$$\begin{array}{c} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \cdot \\ \cdot \\ \cdot \\ a_{k1}x_1 + a_{k2}x_2 + \cdots + a_{kn}x_n = b_k \\ x_i \geq 0 \quad \text{for} \quad i = 1, 2, \ldots, n \end{array} \tag{2.2}$$

where, x_i are decision variables and a, b, c are known constants. The above equations also can be presented in the matrix form as:

$$\min \mathrm{f}(X) = c^T X \tag{2.3}$$

in which,

$$c = \begin{bmatrix} c_1 \\ c_2 \\ \cdot \\ \cdot \\ c_n \end{bmatrix} \quad \text{and} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ x_n \end{bmatrix}$$

Subject to the following constraints:

$$aX = b \tag{2.4}$$

$$X \geq 0 \tag{2.5}$$

where,

$$a = \begin{bmatrix} a_{11} & a_{12} & \cdot & \cdot & a_{1n} \\ a_{21} & a_{22} & \cdot & \cdot & a_{2n} \\ \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & & \cdot \\ a_{k1} & a_{k2} & \cdot & \cdot & a_{kn} \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} b_1 \\ b_2 \\ \cdot \\ \cdot \\ b_n \end{bmatrix}$$

It is important to note that maximization of any objective function is equivalent to negative minimization of that function. Hence, a maximization problem can be simply converted to a minimization problem in any linear programming.

$$\max \mathrm{f}(\mathrm{X}) = \min \; -\mathrm{f}(\mathrm{X}) \tag{2.6}$$

In addition, the constraints sometimes are presented in the form of inequalities, while, they can be simply presented in the form of equalities by adding or subtracting slack variables (s) as:

$$\begin{array}{cc} \text{inequality constraint} & a_{k1}x_1 + a_{k2}x_2 + \cdots + a_{kn}x_n \leq b_k \\ \text{equality constraint} & a_{k1}x_1 + a_{k2}x_2 + \cdots + a_{kn}x_n + s = b_k \end{array} \tag{2.7}$$

or

$$\begin{array}{cc} \text{inequality constraint} & a_{k1}x_1 + a_{k2}x_2 + \cdots + a_{kn}x_n \geq b_k \\ \text{equality constraint} & a_{k1}x_1 + a_{k2}x_2 + \cdots + a_{kn}x_n - s = b_k \end{array} \tag{2.8}$$

The solution to an optimization problem can be categorized into four types as: (1) a unique optimal solution, (2) an infeasible solution, (3) an unbounded solution, and (4) multiple solutions. The unique optimal solution is obtained when all of the constraints are satisfied and the minimum or maximum values of the objective function are precisely determined. In this case, the set of all feasible solutions is called the feasible region. On the other hand if we cannot find any solution that satisfies the desired constraints, the problem is called infeasible. Therefore, the feasible region is empty and there is no optimal solution since there is no solution in this condition. The unbounded solution in LP problems happens when the objective value is feasible while its value increases or decreases indefinitely and approaches to negative or positive infinity. And finally, there are multiple solutions, if more than one solution can be found for the desired optimization problem. In this case, all values of objective function are equaled and can be considered as optimum value.

In the following sections, the graphical and simplex methods for linear programming are presented with a few examples for each technique.

2.2 Graphical Method

Graphical methods can be applied to solve linear optimization problems involving two decision variables and a limited numbers of constraints. As the graphical methods are visual approach, they can increase our understanding from the basics of linear programming and the steps to find the optimal value in an optimization problem. To be more familiar with the concept of these methods, a simple example is presented in the following section.

Example 2.1 Maximize the function f(x)

$$f(x) = 15x_1 + 18x_2$$

Subject to the following constraints:

$$2x_1 + 3x_2 \leq 35$$
$$4x_1 + 2x_2 \leq 50$$
$$x_1 \geq 0 \quad \text{and} \quad x_2 \geq 0$$

Solution: The first step is drawing the constraints to find the feasible region. In this case, we need to replace the inequality sign of each constrain with the equality sign as follow:

$$2x_1 + 3x_2 = 35$$
$$4x_1 + 2x_2 = 50$$

Now, assume $x_1 = 0$ and solve for x_2 from first constraint equation and repeat the process again by assuming $x_2 = 0$ and solving for x_1. For the first constraint, we have:

$$\text{If } x_1 = 0 \rightarrow x_2 = 11.66$$
$$\text{If } x_2 = 0 \rightarrow x_1 = 17.5$$

And, for the second constraint equation:

$$\text{If } x_1 = 0 \rightarrow x_2 = 25$$
$$\text{If } x_2 = 0 \rightarrow x_1 = 12.5$$

Now, draw a line to connect the points (0, 11.66) to (17.5, 0) for the first constraint and (0, 25) to (12.5, 0) for the second constraint, as shown in the Fig. 2.1.

The shaded area in Fig. 2.1 shows the feasible region and all points that are in this domain satisfy both constraints of the model. The best points in the feasible region that maximize the function f(x) would be the optimal solution. For the sake of analysis, the feasible solution is redrawn as shown in Fig. 2.2. The intersection point between the two constraints can be calculated by solving the following two equations with two unknowns simultaneously as:

$$\begin{cases} 2x_1 + 3x_2 = 35 \\ 4x_1 + 2x_2 = 50 \end{cases} \Rightarrow (x_1, x_2) = (10, 5)$$

Therefore, we need to draw a straight line from points (0, 11.66) to (10, 5), and (10, 5) to (12.5, 0).

To find the optimal solution in the feasible area, we assume a value for the objective function, for example f(x) = 270, and draw a line to see it is inside the

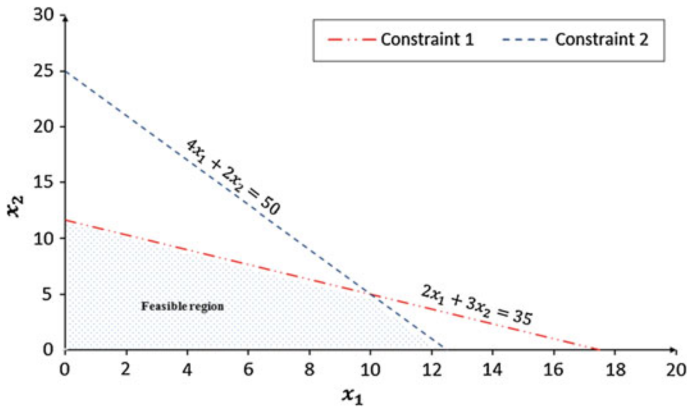


Fig. 2.1 The feasible solution domain for Example 2.1

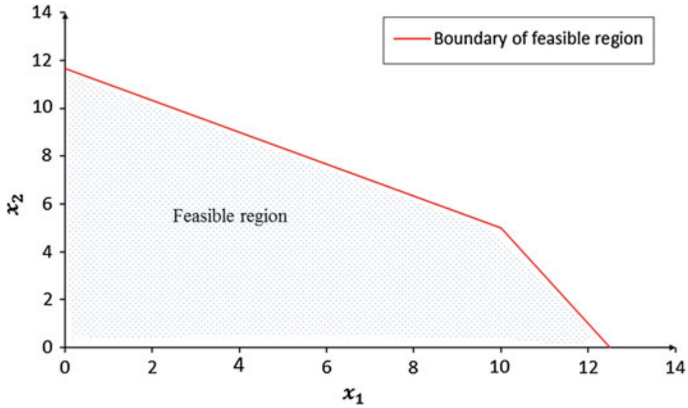


Fig. 2.2 The feasible region of the problem

feasible region for $15x_1 + 18x_2 = 270$ or not (Fig. 2.3). In this case, the line does not intersect the solution domain, therefore we know that the value for the objective function must be less than 270. It is important to note that all points with different values of x_1 and x_2 on this line have the same value of 270. We can consider other values for the objective function and draw new lines with lower values for f(x). The optimal point which is the maximum value will happen at the intersection of the last possible point in the feasible region and the associated objective line. In this problem, the maximum value occurs when f(x) = 240 since it is the last line that intersects the boundary of feasible solution domain (constraints). As noted above, the values of x_1 and x_2 can simply be calculated by solving the constraints equations simultaneously and the results will be $x_1 = 10$ and $x_2 = 5$.

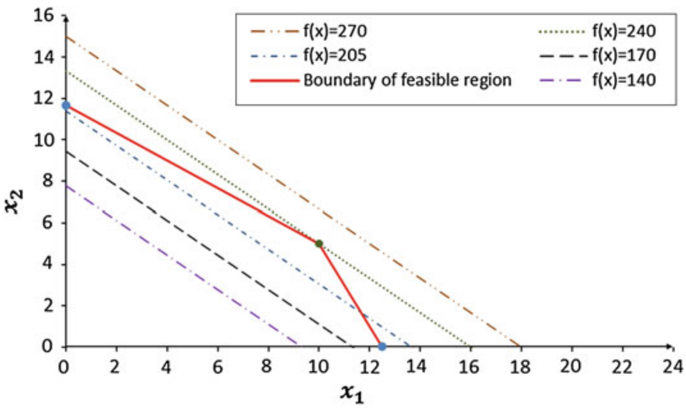


Fig. 2.3 The procedure of finding maximum of f(x)

This problem also can be solved as minimization problem by changing the sign of function f(x) into −f(x) and write it as following:

$$\min \;\; g(x) = -f(x) = -(15x_1 + 18x_2)$$

In this case we need to find the line that intersects the boundary of constraints to determine the minimum for the function g(x). Based on Fig. 2.4, the line g(x) = −240 is the last intersecting point, and so, it can be considered as the optimal value of the function g(x). As described above, the values of x_1 and x_2 can be simply calculated by solving the constraint equations simultaneously as $x_1 = 10$ and $x_2 = 5$. If the minimum value of g(x) or −f(x) is multiplied by a negative sign, it will result in the maximum for the function f(x).

2.3 Simplex Method

As noted above, the graphical method can only be used for LP problems with one or two decision variables, while many real LP problems involve more than two decision variables and so, we need to apply other optimization techniques to find the optimal solution. The simplex method is a well-known mathematical technique for solving LP models by constructing an acceptable solution domain and improving it step by step until the best solution is found and the optimum value is reached. The necessary steps in simplex method to find the optimal solution are presented in the following example.

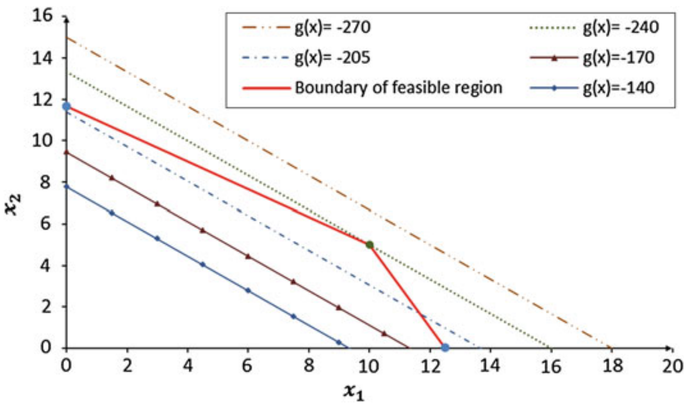


Fig. 2.4 The procedure of finding minimum of g(x)

Example 2.2 Maximize the objective function f(x) using the simplex method.

$$\max \ \mathrm{f}(x) = 2x_1 + 3x_2$$

Subject to

$$x_1 + x_2 \leq 27$$
$$2x_1 + 5x_2 \leq 90$$
$$-x_1 + x_2 \leq 11$$
$$x_1 \geq 0 \quad \text{and} \quad x_2 \geq 0$$

Solution: The following steps illustrate the whole process of solving a LP model using the simplex method.

Step 1: Convert desired LP problem into a standard form.

To convert a LP model to its standard form, all inequality constraints should be presented in the equality forms by considering the following conditions:

1. Adding a non-negative *slack* variable s_i for the constraints in the form of:

$$a_{i,1}x_1 + a_{i,2}x_2 + \cdots + a_{i,m}x_m \leq b_i$$

Hence, the constraint can be written as:

$$a_{i,1}x_1 + a_{i,2}x_2 + \cdots a + a_{i,m}x_m + s_i = b_i$$

2. Subtracting a non-negative *surplus* variable s_i for the constraints in the form of:

$$a_{i,1}x_1 + a_{i,2}x_2 + \cdots + a_{i,m}x_m \geq b_i$$

Therefore, the constraint can be written as:

$$a_{i,1}x_1 + a_{i,2}x_2 + \cdots + a_{i,m}x_m - s_i = b_i$$

Thus, transforming equalities into the standard form based on the above conditions can be written as:

$$\begin{cases} x_1 + x_2 + s_1 = 27 \\ 2x_1 + 5x_2 + s_2 = 90 \\ -x_1 + x_2 + s_3 = 11 \\ x_1 \geq 0 \quad \text{and} \quad x_2 \geq 0 \end{cases}$$

It is important to note that non-negative constraints remain in inequality forms of $\geq$ or $\leq$. Figure 2.5 shows the feasible region and the corresponding constraints for example 2.2.

Step 2: Determine the basic and non-basic variables.

By considering f(x) as objective function and putting it along with the constraints, we will get the following system of linear equations:

$$Z - 2x_1 - 3x_2 = 0$$

$$\begin{cases} x_1 + x_2 + s_1 = 27 \\ 2x_1 + 5x_2 + s_2 = 90 \\ -x_1 + x_2 + s_3 = 11 \\ x_1 \geq 0 \quad \text{and} \quad x_2 \geq 0 \end{cases}$$

The variables x_1 and x_2 are considered as *non-basic* variables and the other slack variables (s_1, s_2, and s_3) denoted as *basic* variables. In other words, the variables that only appear in one equation are basic and the other ones which are repeated in objective function and other equations are non-basic variables.

Step 3: Obtain entering and leaving variables.

The LP model in this problem includes five unknown variables ($n = 5$) x_1, x_2, s_1, s_2, and s_3 and three equations ($m = 3$) which are the constraints of the problem. As the numbers of unknown variables are more than equations ($n > m$), we will assume that the two non-basic variables are equal zero in order to find a basic solution for desired problem. It is important to note that the possible solution for LP model can be obtained if $n - m$ non-basic variables exist at the Zero level. By setting $x_1 = x_2 = 0$, we have: $f(x) = 0$, $s_1 = 27$, $s_2 = 90$, $s_3 = 11$.

Now the question is: can we still increase the objective function, or should this answer be considered as the optimal solution? By looking at the objective function equation, it can be seen that increasing x_1 or x_2 results in an increase in the values of f(x). Because both variables x_1 and x_2 have negative coefficients -2 and -3 respectively (or positive coefficients 2 and 3 in the original form as $f(x) = 2x_1 + 3x_2$), we still can increase the value of f(x) by setting higher values

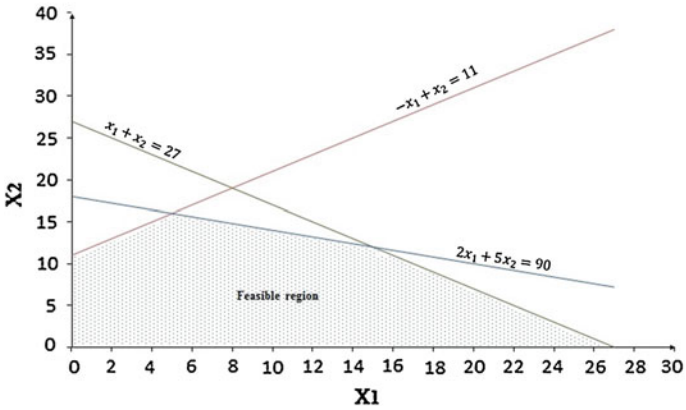


Fig. 2.5 The feasible region and corresponding constraints of f(x)

for non-basic variables. On the other hand, if all coefficients of the objective function are nonnegative, it can be concluded that the current basic solution, f(x) = 0 that is obtained by setting $x_1 = x_2 = 0$, is the optimum solution.

In this problem, the coefficients of both non-basic variable are negative, and hence, still it is possible to increase one of the variables x_1, or x_2 from zero to a higher value to increase the value of f(x). By increasing one of the non-basic variables from zero to a higher value, one of the basic variables should be pushed down to zero in order to maintain a feasible solution with $n - m$ non-basic variables. The selected non-basic variable which are going to be a basic variable, is called the *entering variable* and the basic variable that is changed to a non-basic variable is named *leaving variable*. The largest negative coefficient in the function f(x) or the largest positive coefficient in the original f(x) function can be selected as the entering variable in the maximization problems. The main reason for choosing a variable with the most negative coefficient is its potential to increase the objective function as much as possible. It is important to note that there is an inverse way to choose the entering variable for a minimization problem. In other words, the variable with largest positive coefficient of f(x) is selected as the entering variable. However, in the simplex method it is easier to convert minimization problems into a maximization problem and then find the optimum value of the desired objective function. The maximized function can be found by seeking the minimum of the negative of the same function by changing f(x) to −f(x).

Based on discussions above, the entering variable candidates in this problem are x_1 and x_2. As x_2 has the largest negative coefficient in $f(x) - 2x_1 - 3x_2 = 0$, it is selected as the entering variable. Once the *entering* variable is chosen, we need to determine one of the basic variables as *leaving* variable. The problem in the equality form can be written as:

$$\begin{cases} R_1 \rightarrow f(x) - 2x_1 - 3x_2 = 0 \\ R_2 \rightarrow x_1 + x_2 + s_1 = 27 \\ R_3 \rightarrow 2x_1 + 5x_2 + s_2 = 90 \\ R_4 \rightarrow -x_1 + x_2 + s_3 = 11 \end{cases}$$

where, R_i means the ith row of the equality system. To solve this LP model, the initial tableau corresponding to the equations can be represented as following:

The leaving variable can be selected by calculating the ratio of right side of equations (last column of table) to the non-negative coefficients of selected entering variable in the rows linked to the basic variables (here R_2, R_3, R_4). The entering variable and its coefficients are **bold** in Table 2.1. Finally, the leaving variable will be the current basic variable associated with the row with minimum ratio. In this problem the ratios are calculated as:

$$R_2 \rightarrow \frac{27}{1} = 27$$
$$R_3 \rightarrow \frac{90}{5} = 18$$
$$R_4 \rightarrow \frac{11}{1} = 11$$

The minimum value among the rations (27, 18, 11) is 11 and so, the current basic variable associated with this ratio which is s_3 is selected as the leaving variable and consequently become a non-basic variable.

Step 4: Determine pivot equation.

In this step, the row associated with the minimum ratio is selected as *pivot equation* and the coefficient of entering variable in the pivoting row is the *pivot element*. In this example, the pivot equation is R_4 and the pivot element is 1. To make a new simplex table, both side of pivot equation should be divided by pivot element to have a unit value for pivot element, and then, add or subtract multiples of the pivot equation to or from the other rows (here R_1, R_2 and R_3) in order to eliminate the selected entering variable (here x_2) from them. It is important to note that this method is called Gauss-Jordan elimination method. The equality equations after applying Gauss-Jordan elimination method will be changed as follows:

$$\begin{cases} R_1 + 3R_4 \rightarrow f(x) - 5x_1 + 3s_3 = 33 \\ R_2 - R_4 \rightarrow 2x_1 + s_1 - s_3 = 16 \\ R_3 - 5R_4 \rightarrow 7x_1 + s_2 - 5s_3 = 35 \\ R_4 \rightarrow -x_1 + x_2 + s_3 = 11 \end{cases}$$

The new coefficients of all basic and non-basic variables based on the simplex method are shown in the Table 2.2.

Table 2.1 Simplex tableau based on the coefficients of basic and non-basic variables

Equations	All variables →	f(x)	x_1	$\boldsymbol{x_2}$	s_1	s_2	s_3	Right side of equations
R_1	f(x)	1	−2	−3	0	0	0	0
	Basic variables							
R_2	s_1	0	1	**1**	1	0	0	27
R_3	s_2	0	2	**5**	0	1	0	90
R_4	s_3	0	−1	**1**	0	0	1	11

Table 2.2 Simplex tableau based on the new coefficients of basic and non-basic variables

Equations	All variables →	f(x)	$\boldsymbol{x_1}$	x_2	s_1	s_2	s_3	Right side of equations
$R_1 + 3R_4$	*f*(*x*)	1	−5	0	0	0	+3	33
	Basic variables							
R_2–R_4	s_1	0	**2**	0	1	0	−1	16
R_3–$5R_4$	s_2	0	**7**	0	0	1	−5	35
R_4	x_2	0	**−1**	1	0	0	1	11

Step 5: Find the optimal solution.

Once the new basic and non-basic variables are determined and the new simplex table is generated, we need to examine if the computed value for f(x) is an optimal solution or not. Based on Table 2.2, there is a non-basic variable (i.e., x_1) in the first row of the tableau with negative coefficient −5 that has potential to improve the value of f(x). In other words, for this maximization problem by setting higher values for non-basic variable x_1, the value of f(x) will be increased. Hence, the non-basic variable x_1 (it is **bold** in the Table 2.2) is considered as the entering variable and by using the same procedure described above the leaving variable should be selected. The ratios based on the new simplex table are:

$$R_2 - R_4 \rightarrow \frac{16}{2} = 8$$

$$R_3 - 5R_4 \rightarrow \frac{35}{7} = 5$$

As the leaving variable is chosen by dividing the ratio of right side of equations to the non-negative coefficients of entering variable in the constraint rows, the coefficient −1 should not be considered. The minimum of values (8, 5) is 5, and so, the current basic variable associated with this ratio, which is s_2, is chosen as the leaving variable. The pivot equation and pivot element here are $R_3 - 5R_4$ and 7, respectively. Afterward, both side of pivot equation is divided by pivot element to have a unit value for pivot element, and then, add or subtract multiples of the pivot equation to or from the other rows as $R_1 + 3R_4$, $R_2 - R_4$ and R_4 to eliminate the selected entering variable x_1 from them. The elimination procedure is:

$$\begin{cases} R'_1 = (R_1 + 3R_4) + 5R'_3 & \rightarrow \quad Z + \frac{5}{7}s_2 - \frac{4}{7}s_3 = 58 \\ R'_2 = (R_2 - R_4) - 2R'_3 & \rightarrow \quad s_1 - \frac{2}{7}s_2 + \frac{3}{7}s_3 = 6 \\ R'_3 = \frac{1}{7}(R_3 - 5R_4) & \rightarrow \quad x_1 + \frac{1}{7}s_2 - \frac{5}{7}s_3 = 5 \\ R'_4 = R_4 + R'_3 & \rightarrow \quad x_2 + \frac{1}{7}s_2 + \frac{2}{7}s_3 = 16 \end{cases}$$

The new coefficients of all new basic and non-basic variables are shown in the Table 2.3.

As it can be seen from this new tableaux, the value of the objective function is increased from 33 to 58. Now the question is "is this the optimal solution?" To find the appropriate answer for this question, we need to examine the coefficients of variables in the first row (R'_1) and find the negative coefficient. If all coefficients are nonnegative, the optimization process is done and so, the final value of 58 will be the maximum for f(x). But, there still is a coefficient with negative value in the row of objective function of Table 2.3. Therefore, it can be concluded that s_3 is the entering variable and the leaving variable should be determined in this step. The ratios based on the previous simplex tableau are:

$$R'_2 \rightarrow \frac{6}{3/7} = 14$$

$$R'_4 \rightarrow \frac{16}{2/7} = 56$$

The minimum value of (14, 56) is 14 and so, the current basic variable associated with this ratio, which is s_1 is chosen as the leaving variable. The pivot equation and pivot element here are R'_2 and 3/7 respectively. Now we will try to eliminate the selected entering variable s_1 from the equation system as follow:

$$\begin{cases} R''_1 = R'_1 + \frac{4}{7}R''_2 & \rightarrow \quad Z + \frac{4}{3}s_1 + \frac{1}{3}s_2 = 66 \\ R''_2 = R'_2 & \rightarrow \quad \frac{7}{3}s_1 - \frac{2}{3}s_2 + s_3 = 14 \\ R''_3 = R'_3 + \frac{5}{7}R''_2 & \rightarrow \quad x_1 + \frac{5}{3}s_1 - \frac{1}{3}s_2 = 15 \\ R''_4 = R'_4 - \frac{2}{7}R''_2 & \rightarrow \quad x_2 - \frac{2}{3}s_1 + \frac{1}{3}s_2 = 12 \end{cases}$$

The calculated coefficients for all new basic and non-basic variables are presented in the Table 2.4. As it can be seen from the objective coefficients in the row of R''_1, all coefficients are nonnegative and hence, no non-basic variable to increase the value of the objective function. Therefore, the current solution $x_1 = 15$ and $x_2 = 12$ are the optimum solution, and the maximum value of Z is 66.

The LP problems with many constraints and objective functions also can be solved quickly by applying powerful software like Excel. Excel contains a powerful tool, called Solver, to find the optimal solution of linear programming using the simplex method. This tool can be found on the Data tab of the Excel worksheet in which the opening window looks like Fig. 2.6. As seen from this figure, the first section of Solver Parameters window is *Set Objective*. In this part, we need to address the cell reference or name for the objective cell which contains a formula. Afterward, the value of the objective cell should be determined as: Max, Min, or a Value (the objective cell to be a certain value). The third part is *By Changing Variable Cells*

Table 2.3 New coefficients of basic and non-basic variables based on the simplex method

Equations	All variables →	Z	x_1	x_2	s_1	s_2	s_3	Right side of equations
R_1'	Z	1	0	0	0	$\frac{5}{7}$	$-\frac{4}{7}$	58
	Basic variables							
R_2'	s_1	0	0	0	1	$-\frac{2}{7}$	$\frac{3}{7}$	6
R_3'	x_1	0	1	0	0	$\frac{1}{7}$	$-\frac{5}{7}$	5
R_4'	x_2	0	0	1	0	$\frac{1}{7}$	$\frac{2}{7}$	16

Table 2.4 New coefficients of basic and non-basic variables based on the simplex method

Equations	All variables →	f(x)	x_1	x_2	s_1	s_2	s_3	Right side of equations
R_1''	f(x)	1	0	0	$\frac{4}{3}$	$\frac{1}{3}$	0	66
	Basic variables							
R_2''	s_3	0	0	0	$\frac{7}{3}$	$-\frac{2}{3}$	1	14
R_3''	x_1	0	1	0	$\frac{5}{3}$	$-\frac{1}{3}$	0	15
R_4''	x_2	0	0	1	$-\frac{2}{3}$	$\frac{1}{3}$	0	12

that is used for choosing the decision variable cell ranges. It is important to note that the decision variables must be related to the objective cell. To enter all constraints of the problem, the *Subject to the Constraints* box should be applied. In this case, click the *Add* option and in the *Cell Reference* box, enter the cell reference of constraints. Excel provides the following three different techniques to solve an optimization problem: (1) Simplex LP, (2) GRG Nonlinear, and (3) Evolutionary approaches. The application of Excel in solving LP problems are illustrated in the following examples. The following example is a linear problem, and the Simplex LP method is selected to find the optimum value of profit (R). It should be noted that Solver is not a very robust tool for non-linear and complex optimization problems, but it is useful to find solution of simple problems.

Example 2.3 The proper monitoring of earth dams and safety evaluation of the large structure under operational conditions require using a number of instruments such as piezometers to monitor the earthen embankment pore water pressures for potential engineering improvement (filter systems, cut off walls, sheet piles, low permeability apron, etc.) to prevent failure. The Caspian Company has two production lines and produces two types of piezometers called "P_1" and "P_2". The P_1 production line has a capacity of 25 piezometers per day, whereas the daily capacity of the P_2 line is only 35 piezometers. The labors requirement to produce P_1 and P_2 are 2 man-hours and 3 man-hours, respectively. The maximum capacity of Caspian Company is 140 labor hours per day to produce two types of piezometers. Determine the daily production, if the profit for the P_1 piezometer is \$20 and for the P_2 piezometer is \$25.

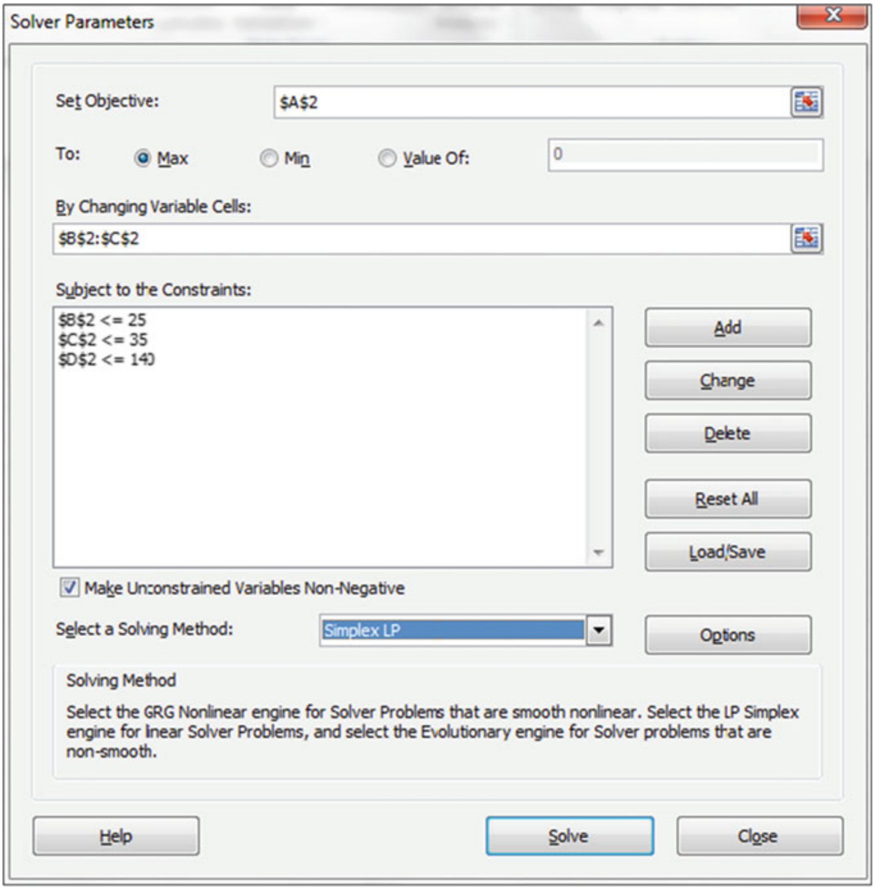


Fig. 2.6 The Solver parameters in Excel 2010

Solution:

1. The first step is to determine the objective and associated constraints of example problem. It is obvious that the objective function is maximizing the profit $R(\$)$ subject to the following constraints:

   (a) The number of P_1 piezometers (n_{p1}) produced each day are less than or equal to 25,
   (b) The number of P_2 piezometers (n_{p2}) produced each day are less than or equal to 35,
   (c) The total number of labor hours is a linear function of 2 and 3 man-hours for production of piezometers, that is: $2n_{p1} + 3n_{p2} \leq 140$.

And the profit function in this example can be written as follow:

$$R(\$) = \left(n_{p_1} \times 20\$\right) + \left(n_{p_2} \times 25\$\right)$$

Then, the exact statement of this problem or the objective function is:

$$\max R(\$) = \left(n_{p_1} \times 20\$\right) + \left(n_{p_2} \times 25\$\right)$$

Subject to:

$$n_{p_1} \leq 25; \quad n_{p_2} \leq 35; \quad 2n_{p_1} + 3n_{p_2} \leq 140$$

Figure 2.7 shows the governing conditions in the Caspian Company to produce the piezometers. Based on the constraints, the feasible production combinations are the points in the shaded area of Fig. 2.7 and we need to find a point in this area that makes the highest profit. To solve this problem, different values for n_{p1} are considered, then associated n_{p2} are calculated, and finally the target point considering all constraints is found. Based on the presented results in Table 2.5, the acceptable ranges that meet all constraints of this problem are from 30 to 34, for n_{p2} and from 19 to 25 for n_{p1} which are shown in a gray color. According to this range, the highest profit for n_{p2} and n_{p1} are equal to 30 and 25, respectively.

The first column of Table 2.5 includes various numbers of P_2 piezometers (n_{p2}), while the second column (n_{p1}) is calculated based on the relationship between two variables P_1 and P_2. According to the results, if the Caspian Company produces 25 P_1 type and 30 P_2 type piezometers per day, the highest profit will occur.

In summary, the procedure for finding optimal solution using the Excel Solver can be explained as:

1. Set Objective: set the profit as target value that should be maximized,
2. By Changing Variables Cells: consider n_{p1} and n_{p2} as decision variables,
3. Subject to the Constraints: determine the constraints as: $n_{p1} \leq 25$, $n_{p2} \leq 35$, and $2n_{p1} + 3n_{p2} \leq 140$.

The achieved results using the Solver tool are the same as the computed results in the previous section. It is important to note that the existence of solution is only dependent on the defined constraints of the desired problem and it is not a function of objective function.

2. If the maximum capacity of Caspian Company is changed from 140 to 170 persons-hour of labor per day, there would not be feasible solution regarding this new production constraint, and there is no single point to satisfy all constraints. Figure 2.8 and Table 2.6 illustrate the feasible production combinations of piezometer productions by considering new values for the labor constraint.

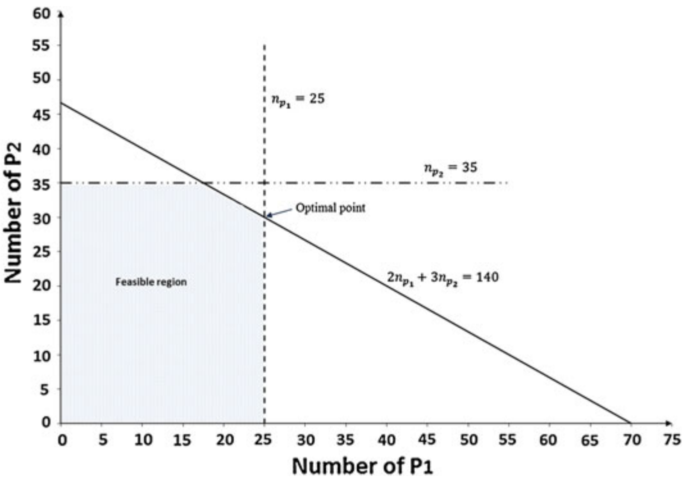


Fig. 2.7 The feasible production combinations for Caspian Company

Table 2.5 The possible combination of piezometer productions in Caspian Company

n_{p2}	$n_{p1} = (140 - 3n_{p2})/2$	R (\$)	n_{p2}	$n_{p1} = (140 - 3n_{p2})/2$	R (\$)
0	70	1,400	24	34	1,280
2	67	1,390	26	31	1,270
4	64	1,380	28	28	1,260
6	61	1,370	30	25	1,250
8	58	1,360	32	22	1,240
10	55	1,350	34	19	1,230
12	52	1,340	36	16	1,220
14	49	1,330	38	13	1,210
16	46	1,320	40	10	1,200
18	43	1,310	42	7	1,190
20	40	1,300	44	4	1,180
22	37	1,290	46	1	1,170

As can be seen from Table 2.6, there is no solution that satisfies all constraints, and hence, this problem is infeasible for the new value of labor constraint.

Using the Solver tool in Excel, the same results will be achieved and the program shows the following message as: *The Objective Cells values don't converge* (Fig. 2.9).

3. As mentioned previously, the unbounded solution happens when the feasible region (shaded area in Fig 2.7) is unbounded and so the value of objective

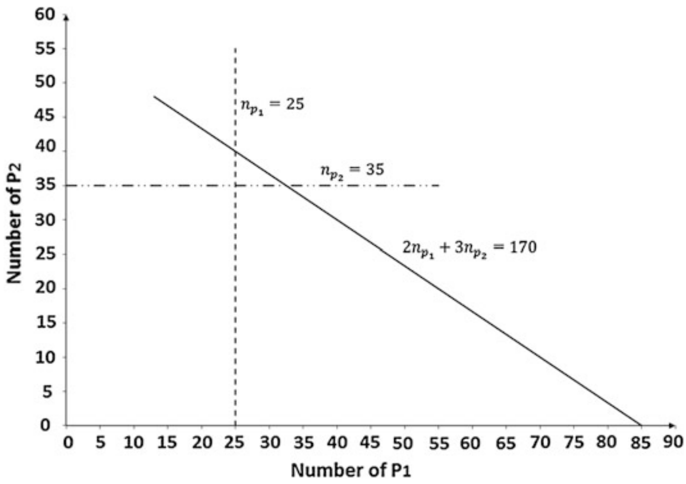


Fig. 2.8 The feasible production combinations for new labor constraint

Table 2.6 The possible combination of piezometer productions with new labor constraint

n_{p2}	$n_{p1} = (170 - 3n_{p2})/2$	R($)	n_{p2}	$n_{p1} = (170 - 3n_{p2})/2$	R($)
0	85	1,700	24	49	1,580
2	82	1,690	26	46	1,570
4	79	1,680	28	43	1,560
6	76	1,670	30	40	1,550
8	73	1,660	32	37	1,540
10	70	1,650	34	34	1,530
12	67	1,640	36	31	1,520
14	64	1,630	38	28	1,510
16	61	1,620	40	25	1,500
18	58	1,610	42	22	1,490
20	55	1,600	44	19	1,480
22	52	1,590	46	16	1,470

function can be increased or decreased to infinity without leaving the feasible region. For instance if we don't consider any constraint for n_{p1} (ignore constraints 1 and 3) its value can be varied from zero to infinity, and hence, the profit (R) simultaneously rises and approaches infinity (Fig. 2.10).

4. If the profit for each P_2 piezometer is changed from 25$ to $30, the profit function will be changed as follow:

$$R(\$) = \left(n_{p_1} \times 20\$\right) + \left(n_{p_2} \times 30\$\right)$$

In this case, there may be more than one combination of producing P_1 and P_2 piezometers to reach the maximum profit. As can be seen in Table 2.7, the

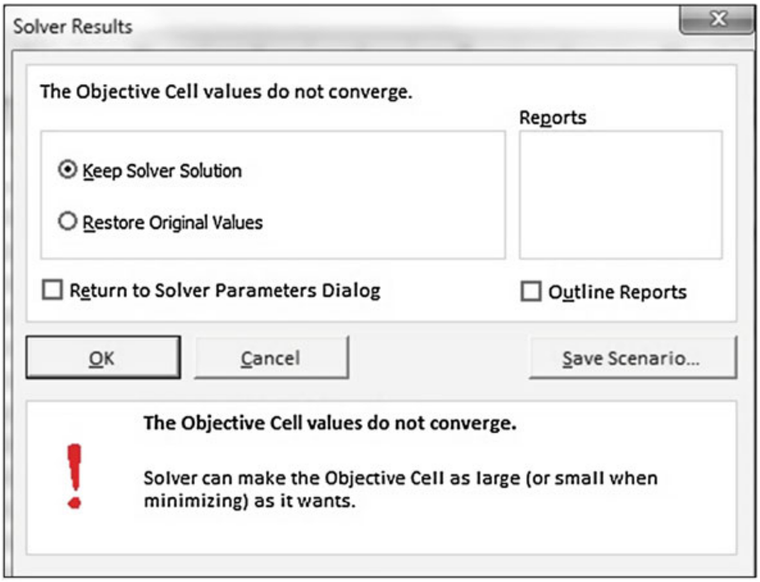


Fig. 2.9 The solver results

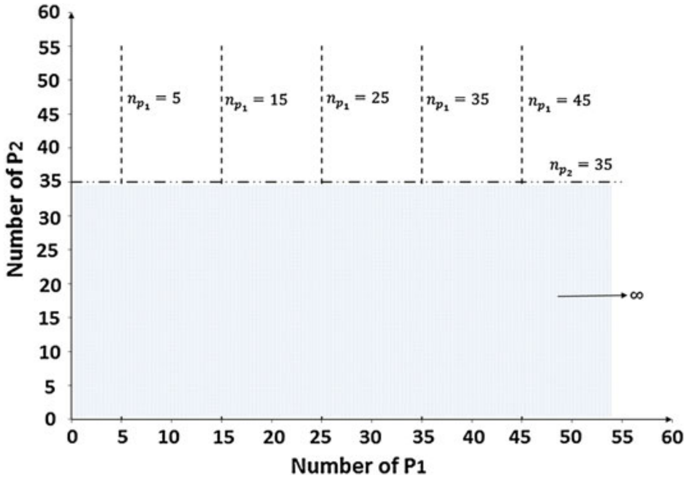


Fig. 2.10 The unbounded condition

production of different numbers of P_1 and P_2 piezometers resulted in same profit for Caspian Company. In other words, it is possible to produce several different combination of P_1 piezometer (e.g., 19, 22, or 25) or P_2 piezometer (e.g., 34, 32, or 30), while the profit is still constant and it is 1,400$. Hence, there are multiple optimal solutions to produce n_{p2} and n_{p1} in comparison to a unique optimal solution discussed above.

5. It is important to note that addition or subtraction of a positive constant value to or from the objective function will not change the optimum solution of the problem. This rule also is confirmed for multiplication or division of objective function by a positive constant value. In this section, the Caspian Company's profits are calculated for different profit functions as follows, and then the optimum solutions are determined in each case.

$$
\begin{aligned}
R(\$) &= \left[\left(n_{p_1} \times 20\right) + \left(n_{p_2} \times 25\right)\right] + 35 \\
R(\$) &= \left[\left(n_{p_1} \times 20\right) + \left(n_{p_2} \times 25\right)\right] - 20 \\
R(\$) &= \left[\left(n_{p_1} \times 20\right) + \left(n_{p_2} \times 25\right)\right] \times 1.5 \\
R(\$) &= \left[\left(n_{p_1} \times 20\right) + \left(n_{p_2} \times 25\right)\right]/5
\end{aligned}
$$

The results show that the optimum solutions in all cases are constant as $n_{p1} = 25$, and $n_{p2} = 30$ and they are not changed by changing the profit function with a positive constant value (Table 2.8).

Figure 2.11 shows any addition, subtraction, multiplication, and division of a constant value to or from desired objective function doesn't change the optimum solution for n_{p1}.

In the following section the application of simplex method for solving three types of common optimization problems in the field of water resources engineering are presented. The problems cover the process of linear programming for water distribution networks with and without pump stations, confined aquifer with one-dimensional steady-state flow, and confined aquifer with two-dimensional steady-state flow.

2.3.1 Optimization of Water Distribution Networks

A water distribution network is a major urban infrastructure which distributes water supply to residential, industrial, and commercial customers under various demand conditions at adequate pressures and flows. In general, water distribution systems are composed of pipes, pumps, distribution storages like reservoirs, and other hydraulic components. In addition to design and analysis of a water distribution system form a hydraulic point of view, a designer needs to determine the minimum cost of a distribution system to meet demands for all users at required pressure level. The overall cost of a water distribution system includes:

1. Cost of piping and appurtenances such as pumps, valves, flush hydrants, reservoirs, tanks, etc.,
2. Cost of energy for pumping the water to desired network connections to provide the minimum required pressure head elevation, and

Table 2.7 The possible combination of piezometer productions with multiple solutions

n_{p2}	$n_{p1} = (140 - 3n_{p2})/2$	R($)	n_{p2}	$n_{p1} = (140 - 3n_{p2})/2$	R($)
0	70	1,400	24	34	1,400
2	67	1,400	26	31	1,400
4	64	1,400	28	28	1,400
6	61	1,400	30	25	1,400
8	58	1,400	32	22	1,400
10	55	1,400	34	19	1,400
12	52	1,400	36	16	1,400
14	49	1,400	38	13	1,400
16	46	1,400	40	10	1,400
18	43	1,400	42	7	1,400
20	40	1,400	44	4	1,400
22	37	1,400	46	1	1,400

Table 2.8 The values of different profit function for Caspian Company

n_{p1}	n_{p2}	R($)	$R + 35$ ($)	$R -$ 20 ($)	$R \times 1.5$ ($)	R/ 5 ($)
25	30	1,250	1,285	1,230	1,875	250
22	32	1,240	1,275	1,220	1,860	248
19	34	1,230	1,265	1,210	1,845	246

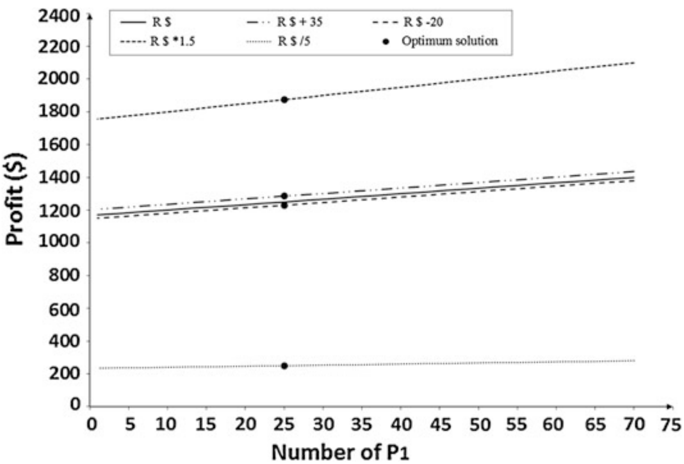


Fig. 2.11 Variation of profits versus n_{p1}

3. Operation and maintenance costs that includes administration and management of personnel, flushing the system at a particular time interval, repairing pipes, servicing of the pumps, billing customers, etc.

In this problem, we will optimize the piping cost (pipe length with a specific diameter of a simple branched water distribution system) with and without considering a pump station in the system. In other words, the main objective function here is obtaining the length of pipe with specific diameter (d) in each reach ($L_{i,j}$) between two connections of desired water distribution networks to minimize the cost of piping. A water distribution system tends to use a pipe or a combination of pipes in each reach to have minimum piping cost, while it has to satisfy the water demands and pressure requirements at all nodes as well as hydraulic constraints. The optimization process in the form of linear programming (LP) can be written as follow:

$$\min\ Z = \sum_{i,j} \sum_{d_{i,j}} C_{i,j,d} l_{i,j,d} + \sum_{n} C_{p_n} H_{p_n} \tag{2.9}$$

where, $C_{i,j,d}$ and $l_{i,j,d}$ are the cost per unit length, and the length of pipe between nodes i and j with diameter d, respectively: H_{pn} is the pumping head, n is the total number of pumps in the system, and C_{pn} is the cost per unit of pumping head. It is important to note that when there is no pump station in a water distribution system, the second term of Eq. (2.9) should be omitted. The main constraints for a water distribution network are energy and length constraints in conjunction with non-negativity of all pipes length and pumping head elevation. The constraints in this case can be written as:

1. Energy constraint:

$$H_{min,k} \le H_c + \sum_{n} H_{p_n} - \sum_{i,j} \sum_{d_{i,j}} I_{i,j,d} l_{i,j,d} \le H_{max,k} \tag{2.10}$$

where, $H_{min,k}$ and $H_{max,k}$ are the minimum and maximum required head at the demand point k, respectively: k is the total number of demand points, H_c is the constant elevation of piping system, and $I_{i,j,d}$ is the hydraulic gradient or gradient between two hydraulic head measurements over the length of the flow path. The energy loss for water flow in a pipe can be estimated using the Darcy-Weisbach equation as:

$$H_L = I \times l = \frac{8 \mathrm{f} Q^2}{\pi^2 g d^5} l \tag{2.11}$$

where, f is Darcy-Weisbach friction factor, Q is flow rate (cfs), and d is diameter of pipe (ft).

2. Length constraint:

$$\sum_{i,j,d} l_{i,j,d} = L_{i,j} \tag{2.12}$$

where, $L_{i,j}$ is the total reach length between each two connections that is a known variable in these types of problems. In other words, the total length of pipe in every reach, which can be a combination of pipes with different diameters, must be equaled to the total reach length between two connections.

3. Non-negativity conditions:

$$\begin{cases} l_{i,j,d} \geq 0 \\ H_{p_n} \geq 0 \end{cases} \tag{2.13}$$

Example 2.4 According to the above statements, determine the minimum cost of pipe and pump (head elevation) in various demand nodes in the following conditions:

1. When there is no pump in the system (Fig. 2.12), and
2. When there is a pump station in the system (Fig. 2.13).

Other necessary information includes:

(a) The unit cost of pipe for two standard diameters (Table 2.9),
(b) The minimum required pressure head elevations for all determined users (A, B, and C) are 550 (ft) and the demand discharges are presented in Table 2.10.
(c) Darcy-Weisbach friction factor is 0.02,
(d) The unit cost of pumping head is 220$,
(e) The total length of pipe between each connection is 1,000 ft.
(f) The constant elevation of piping system is assumed 650 ft when there is no pump in the system, and 555 ft when a pump is considered in the system.

Solution:

1. As noted above, the objective function is to minimize the cost, of pipes (smaller diameter) and it can be defined as follow:

$$\begin{aligned} \min Z = & \left(C_{0,1,1} l_{0,1,1} + C_{0,1,2} l_{0,1,2}\right) + \left(C_{1,2,1} l_{1,2,1} + C_{1,2,2} l_{1,2,2}\right) + \left(C_{2,3,1} l_{2,3,1} + C_{2,3,2} l_{2,3,2}\right) \\ & + \left(C_{2,4,1} l_{2,4,1} + C_{2,4,2} l_{2,4,2}\right) + \left(C_{1,5,1} l_{1,5,1} + C_{1,5,2} l_{1,5,2}\right) \\ = & \left(10 \times l_{0,1,1} + 15 \times l_{0,1,2}\right) + \left(10 \times l_{1,2,1} + 15 \times l_{1,2,2}\right) + \left(10 \times l_{2,3,1} + 15 \times l_{2,3,2}\right) \\ & + \left(10 \times l_{2,4,1} + 15 \times l_{2,4,2}\right) + \left(10 \times l_{1,5,1} + 15 \times l_{1,5,2}\right) \end{aligned}$$

Subject to

(a) The length constraints as:

$$\begin{aligned} l_{0,1,1} + l_{0,1,2} &= 1{,}000 \text{ ft} \\ l_{1,2,1} + l_{1,2,2} &= 1{,}000 \text{ ft} \\ l_{2,3,1} + l_{2,3,2} &= 1{,}000 \text{ ft} \\ l_{2,4,1} + l_{2,4,2} &= 1{,}000 \text{ ft} \\ l_{1,5,1} + l_{1,5,2} &= 1{,}000 \text{ ft} \end{aligned}$$

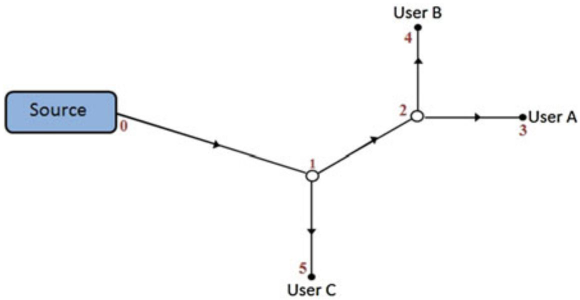


Fig. 2.12 The water distribution system without pumping station

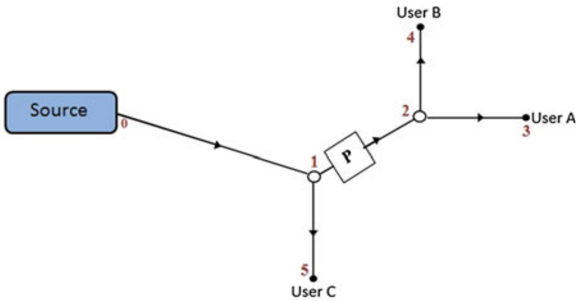


Fig. 2.13 The water distribution system with pumping station

Table 2.9 The cost information of pipes based on diameters

Diameter (*in*)	Diameter (ft)	Cost ($/ft)
21.0	1.75	10.0
24.0	2.00	15.0

Table 2.10 Demand discharges information

	Demand discharges					
	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
User A	$Q_A = 10$	$Q_A = 14$	$Q_A = 18$	$Q_A = 15$	$Q_A = 18$	$Q_A = 20$
User B	$Q_B = 12$	$Q_B = 18$	$Q_B = 20$	$Q_B = 17$	$Q_B = 18$	$Q_B = 21$
User C	$Q_C = 14$	$Q_C = 20$	$Q_C = 24$	$Q_C = 36$	$Q_C = 31$	$Q_C = 23$

(b) The energy constraints for all users is as follows:

For user A:

$$650 - \left(I_{0,1,1} \times l_{0,1,1} + I_{0,1,2} l_{0,1,2}\right) - \left(I_{1,2,1} l_{1,2,1} + I_{1,2,2} l_{1,2,2}\right) - \left(I_{2,3,1} l_{2,3,1} + I_{2,3,2} l_{2,3,2}\right) \geq 550$$

where,

$$I_{0,1,1} = \frac{8 f Q^2}{\pi^2 g d^5} = \frac{8 \times 0.02 \times (Q_A + Q_B + Q_C)^2}{\pi^2 \times 32.2 \times (1.75)^5}$$

The hydraulic gradient for Q_1 is:

$$I_{0,1,1} = \frac{8 \times 0.02 \times (36)^2}{\pi^2 \times 32.2 \times (1.75)^5} = 0.0398\,\text{ft/ft}$$

and for $I_{2,4,2}$ and Q_3 the hydraulic gradient is:

$$I_{2,4,2} = \frac{8 \times 0.02 \times (20)^2}{\pi^2 \times 32.2 \times (2)^5} = 0.0063\,\text{ft/ft}$$

Then, the hydraulic constraint for user A in Q_1 will be calculated as:

$$\begin{aligned} 650 - &\left(0.0398 \times l_{0,1,1} + 0.0204 \times l_{0,1,2}\right) - \left(0.0149 \times l_{1,2,1} + 0.0076 \times l_{1,2,2}\right) \\ &- \left(0.0031 \times l_{2,3,1} + 0.0016 \times l_{2,3,2}\right) \geq 550 \end{aligned}$$

and for user B in Q_1 is:

$$\begin{aligned} 650 &- \left(I_{0,1,1} \times l_{0,1,1} + I_{0,1,2} l_{0,1,2}\right) - \left(I_{1,2,1} l_{1,2,1} + I_{1,2,2} l_{1,2,2}\right) - \left(I_{2,4,1} l_{2,4,1} + I_{2,4,2} l_{2,4,2}\right) \\ &= 650 - \left(0.0398 \times l_{0,1,1} + 0.0204 \times l_{0,1,2}\right) - \left(0.0149 \times l_{1,2,1} + 0.0076 \times l_{1,2,2}\right) \\ &\quad - \left(0.0044 \times l_{2,4,1} + 0.0023 \times l_{2,4,2}\right) \geq 550 \end{aligned}$$

and finally for user C in Q_1 is:

$$\begin{aligned} 650 &- \left(I_{0,1,1} \times l_{0,1,1} + I_{0,1,2} l_{0,1,2}\right) - \left(I_{1,5,1} l_{1,5,1} + I_{1,5,2} l_{1,5,2}\right) \\ &= 650 - \left(0.0398 \times l_{0,1,1} + 0.0204 \times l_{0,1,2}\right) - \left(0.0060 \times l_{1,5,1} + 0.0031 \times l_{1,5,2}\right) \geq 550 \end{aligned}$$

The noted procedure above should be repeated for existing users in different demand points. The values of hydraulic gradient for all reaches and various demand discharges are shown in the Table 2.11

In the next step, the optimum values of pipes length are calculated by minimizing the objective function, Z. The simplex LP method (Excel|Data|Solver) is applied for optimization analysis and the results are presented in the Table 2.12.

2. In this section, the optimization problem is formulated for desired water distribution network by considering a pump station in the system (Fig. 2.13). Therefore, the objective function in this case can be written as:

$$\begin{aligned} \min Z = &\left(C_p H_p\right) + \left(C_{0,1,1} l_{0,1,1} + C_{0,1,2} l_{0,1,2}\right) + \left(C_{1,2,1} l_{1,2,1} + C_{1,2,2} l_{1,2,2}\right) \\ &+ \left(C_{2,3,1} l_{2,3,1} + C_{2,3,2} l_{2,3,2}\right) + \left(C_{2,4,1} l_{2,4,1} + C_{2,4,2} l_{2,4,2}\right) + \left(C_{1,5,1} l_{1,5,1} + C_{1,5,2} l_{1,5,2}\right) \\ = &\left(220 \times H_p\right) + \left(10 \times l_{0,1,1} + 15 \times l_{0,1,2}\right) + \left(10 \times l_{1,2,1} + 15 \times l_{1,2,2}\right) \\ &+ \left(10 \times l_{2,3,1} + 15 \times l_{2,3,2}\right) + \left(10 \times l_{2,4,1} + 15 \times l_{2,4,2}\right) \\ &+ \left(10 \times l_{1,5,1} + 15 \times l_{1,5,2}\right) \end{aligned}$$

The length constraints are the same as presented in the previous section, and the energy constraint for user A can be written as:

Table 2.11 The hydraulic gradient for all reaches and demand points

	Demand discharge (*cfs*)					
Hydraulic gradient	$Q_A = 10$ $Q_B = 12$ $Q_C = 14$	$Q_A = 14$ $Q_B = 18$ $Q_C = 20$	$Q_A = 18$ $Q_B = 20$ $Q_C = 24$	$Q_A = 18$ $Q_B = 18$ $Q_C = 31$	$Q_A = 15$ $Q_B = 17$ $Q_C = 36$	$Q_A = 20$ $Q_B = 21$ $Q_C = 23$
$I_{0,1,1}$	0.0398	0.0830	0.1180	0.1378	0.1420	0.1258
$I_{0,1,2}$	0.0204	0.0426	0.0605	0.0707	0.0728	0.0645
$I_{1,2,1}$	0.0149	0.0314	0.0443	0.0398	0.0314	0.0516
$I_{1,2,2}$	0.0076	0.0161	0.0227	0.0204	0.0161	0.0265
$I_{2,3,1}$	0.0031	0.0060	0.0099	0.0099	0.0069	0.0123
$I_{2,3,2}$	0.0016	0.0031	0.0051	0.0051	0.0035	0.0063
$I_{2,4,1}$	0.0044	0.0099	0.0123	0.0099	0.0089	0.0135
$I_{2,4,2}$	0.0023	0.0051	0.0063	0.0051	0.0046	0.0069
$I_{1,5,1}$	0.0060	0.0123	0.0177	0.0295	0.0398	0.0162
$I_{1,5,2}$	0.0031	0.0063	0.0091	0.0151	0.0204	0.0083

$$555 + H_p - \left(I_{0,1,1} \times l_{0,1,1} + I_{0,1,2} l_{0,1,2}\right) - \left(I_{1,2,1} l_{1,2,1} + I_{1,2,2} l_{1,2,2}\right) - \left(I_{2,3,1} l_{2,3,1} + I_{2,3,2} l_{2,3,2}\right) \geq 550$$

and for user B is:

$$555 + H_p - \left(I_{0,1,1} \times l_{0,1,1} + I_{0,1,2} l_{0,1,2}\right) - \left(I_{1,2,1} l_{1,2,1} + I_{1,2,2} l_{1,2,2}\right) - \left(I_{2,4,1} l_{2,4,1} + I_{2,4,2} l_{2,4,2}\right) \geq 550$$

and finally for user C has following form:

$$555 + H_p - \left(I_{0,1,1} \times l_{0,1,1} + I_{0,1,2} l_{0,1,2}\right) - \left(I_{1,5,1} l_{1,5,1} + I_{1,5,2} l_{1,5,2}\right) \geq 550$$

.

The optimized lengths for different demand points are presented in Table 2.13.

2.3.2 Optimization of One-Dimensional Confined Aquifers

The occurrence and movement of water beneath the surface of the Earth is called groundwater flow, and it when occurs in the saturated soil and rock below the water table, it is called saturated flow. Groundwater flow is an important part of the hydrologic cycle where different types of surface water such as reservoirs, rivers, streams, and overland flow from precipitation infiltrate into the earth crust and become subsurface water. A significant part of the subsurface water can be either stored or transmitted through a geological unit called *aquifer*. In other words, an aquifer is an underground water-saturated formation or layer consisting of permeable rock, sediment, or soil that yields usable amounts of water to wells and springs. Wells can be drilled into the aquifers to pump groundwater from the aquifer and deliver it to various demand points such as domestic, industrial, agricultural,

Table 2.12 The optimized lengths when there is no pumping station

Length of pipe (ft)

Pipe segment	$Q_A = 10$ $Q_B = 12$ $Q_C = 14$	$Q_A = 14$ $Q_B = 18$ $Q_C = 20$	$Q_A = 18$ $Q_B = 20$ $Q_C = 24$	$Q_A = 18$ $Q_B = 18$ $Q_C = 31$	$Q_A = 15$ $Q_B = 17$ $Q_C = 36$	$Q_A = 20$ $Q_B = 21$ $Q_C = 23$
$l_{0,1,1}$	1,000	396.21	0.00	0.00	0.00	0.00
$l_{0,1,2}$	0.00	603.79	1,000	1,000	1,000	1,000
$l_{1,2,1}$	1,000	1,000	205.42	0.00	142.02	0.00
$l_{1,2,2}$	0.00	0.00	794.57	1,000	857.97	1,000
$l_{2,3,1}$	1,000	1,000	1,000	781.85	1,000	454.29
$l_{2,3,2}$	0.00	0.00	0.00	218.14	0.00	545.70
$l_{2,4,1}$	1,000	1,000	1,000	781.85	1,000	314.15
$l_{2,4,2}$	0.00	0.00	0.00	218.14	0.00	685.84
$l_{1,5,1}$	1,000	1,000	1,000	985.69	349.02	1,000
$l_{1,5,2}$	0.00	0.00	0.00	14.30	650.97	0.00
min Z($)	50,000	53,018.95	58,972.86	62252.99	62,544.721	66,157.75

Table 2.13 The optimized lengths when there is a pumping station

Length of pipe (ft)

Pipe segment	$Q_A = 10$ $Q_B = 12$ $Q_C = 14$	$Q_A = 14$ $Q_B = 18$ $Q_C = 20$	$Q_A = 18$ $Q_B = 20$ $Q_C = 24$	$Q_A = 18$ $Q_B = 18$ $Q_C = 31$	$Q_A = 15$ $Q_B = 17$ $Q_C = 36$	$Q_A = 20$ $Q_B = 21$ $Q_C = 23$
$l_{0,1,1}$	1,000	0.00	0.00	0.00	0.00	0.00
$l_{0,1,2}$	0.00	1,000	1,000	1,000	1,000	1,000
$l_{1,2,1}$	1,000	1,000	1,000	1,000	1,000	0.00
$l_{1,2,2}$	0.00	0.00	0.00	0.00	0.00	1,000
$l_{2,3,1}$	1,000	1,000	1,000	1,000	1,000	1,000
$l_{2,3,2}$	0.00	0.00	0.00	0.00	0.00	0.00
$l_{2,4,1}$	1,000	1,000	1,000	1,000	1,000	1,000
$l_{2,4,2}$	0.00	0.00	0.00	0.00	0.00	0.00
$l_{1,5,1}$	1,000	1,000	1,000	1,000	1,000	1,000
$l_{1,5,2}$	0.00	0.00	0.00	0.00	0.00	0.00
Optimum of Hp	54.077	78.977	112.160	115.441	108.140	99.524
min Z($)	61,897.00	72,374.83	79,675.28	80,396.92	78,790.82	81,895.20

or environmental segments. Based on the physical characteristics of aquifers, they can be categorized into confined and unconfined aquifers. The confined or artesian aquifer is one in which the groundwater is sandwiched between two layers with low permeability and it is under pressure greater than atmospheric. On the other hand, the unconfined aquifers contain a water table instead of impermeable layer above the saturation zone (Fig. 2.14). It is important to note that when a well is drilled into confined aquifers, the groundwater rises above the upper boundary of aquifer and may even flow from the well onto the land surface, while, the water level in wells will be at the same elevation as the water table in an unconfined aquifer.

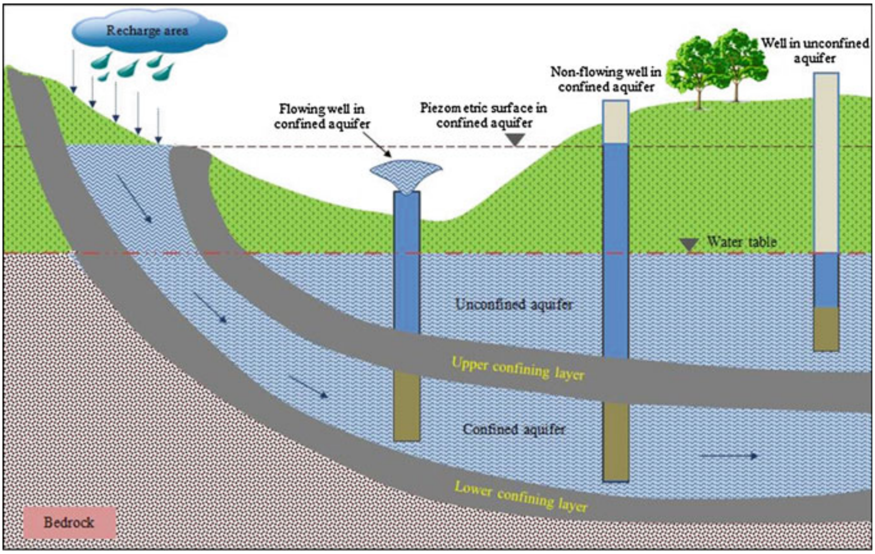


Fig. 2.14 Confined and unconfined aquifers

The general form of a two-dimensional diffusion equation for flow through a heterogeneous anisotropic media considering recharge or discharge (for example from a well) can be written as:

$$T_x \frac{\partial^2 h}{\partial x^2} + T_y \frac{\partial^2 h}{\partial y^2} = S \frac{\partial h}{\partial t} + W \tag{2.14}$$

where $T(L^2/T)$ is transmissivity, $h(L)$ is hydraulic head, S is the storage coefficient (dimensionless), t is time, and $W(L/T)$ is a sink term. The transmissivity is an important hydraulic property of aquifers which shows the capability of aquifer to transmit water through its whole saturated thickness. In other words, transmissivity can be defined as the rate of water flow through a cross-sectional area of an aquifer with a unit width and thickness b under unit hydraulic gradient. This parameter in a confined aquifer is calculated, as follow:

$$T = K.b \tag{2.15}$$

where, $K(L/T)$ and $b(L)$ are the hydraulic conductivity and the saturated thickness of aquifer respectively. For unconfined aquifers the saturated thickness can be replaced with the hydraulic head h as:

$$T = K.h \tag{2.16}$$

The other variable in Eq. (2.14) is the dimensionless factor storage coefficient or storativity that is defined as volume of water the aquifer will store or release per unit surface area and per unit decrease or increase in hydraulic head. This factor shows the ability of aquifer to store water and can be computed as:

$$S = b.S_s \tag{2.17}$$

where, S_s (L^{-1}) is the specific storage and it is defined as amount of water that an aquifer releases from storage per unit volume of saturated area per unit decline or raise in hydraulic head while remaining fully saturated. It is important to know that the values of storage coefficient in confined aquifers are less than 5×10^{-3} and more than 5×10^{-5} (Todd 1980). For unconfined aquifer, the storage coefficient varies from 0.01 to 0.30. The sink term W is the net discharge (e.g., withdrawal from a well) or recharge (q) from the control volume and is equal to:

$$W_{i,j} = \frac{q_{i,j}}{\Delta x_i \Delta y_j} \tag{2.18}$$

The positive and negative values of q represents pumping and recharge, respectively.

Example 2.5 Consider a confined aquifer with one-dimensional steady-state flow and fixed hydraulic heads along the boundaries, as is shown in Fig. 2.15. Develop an LP model to maximize the hydraulic heads for various pumping rates and determine the optimum head in each well for the following conditions:

1. The minimum value of the total desired discharge (W_{min}) from all wells is equal to 4 ft/day,
2. The minimum value of the desired discharge (W_{min}) from each well is equal to 4 ft/day.

The necessary information to solve this problem are: $W_{min} = 4$ ft/day, $\Delta x = 100$ ft, $T = 10{,}000$ ft^2/day, $h_0 = 125$ ft, $h_4 = 100$ ft.

Solution: The governing equation for the one-dimensional steady-state flow in(only x - direction) considering the pumping wells in confined aquifer can be derived from Eq. (2.14) as follows:

$$T_x \frac{\partial^2 h}{\partial x^2} + \underbrace{T_y \frac{\partial^2 h}{\partial y^2}}_{\text{This term becomes 0}} = S\frac{\partial h}{\ } + W \Rightarrow \frac{\partial^2 h}{\partial x^2} = \frac{W}{T_x} \tag{2.19}$$

The implementation form of the Eq. (2.19) based on the central finite difference technique has the following form:

$$\frac{h_{i+1} - 2h_i + h_{i-1}}{(\Delta x)^2} = \frac{W_i}{T_x} \tag{2.20}$$

The objective function to maximize the hydraulic heads for various pumping rates can be written as (Aguado et al. 1974):

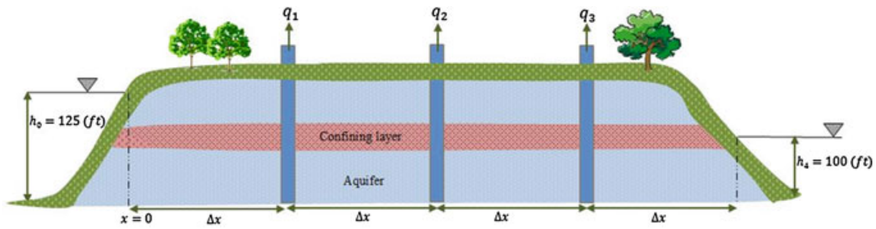


Fig. 2.15 A confined aquifer with one-dimensional steady-state flow and fixed hydraulic heads

$$\max Z = \sum_{i=1}^{n} h_i \tag{2.21}$$

where, n is the number of wells (in this example $n = 3$), and h_i is the hydraulic head in each well. The constraints that should be applied in this problem are:

$$\text{Subject to} \begin{cases} \dfrac{h_{i+1} - 2h_i + h_{i-1}}{(\Delta x)^2} = \dfrac{W_i}{T_x} \\ W_i \geq 0 \quad i = 1 \text{to} n \\ h_i \geq 0 \end{cases} \tag{2.22}$$

It should be noted that to provide a minimum specific pumping rates for all wells together, the following constraint also must be considered:

$$\sum_{i=1}^{n} W_i \geq W_{min} \tag{2.23}$$

in which, W_{min} is the minimum value of the total desired discharge from wells. The following supplementary constraint is useful for finding the optimum hydraulic heads in wells:

$$h_i \geq h_{i+1} \quad i = 0 \, \text{to} \;\; n \tag{2.24}$$

The developed LP model for having minimum value of 4 (ft/day) discharge from all wells together, can be written as:

$$\max\ Z = h_1 + h_2 + h_3$$

Subject to the following constraints as:

$$\begin{cases} i = 1 \rightarrow \dfrac{h_2 - 2h_1 + h_0}{(\Delta x)^2} = \dfrac{W_1}{T_x} \\ i = 2 \rightarrow \dfrac{h_3 - 2h_2 + h_1}{(\Delta x)^2} = \dfrac{W_2}{T_x} \\ i = 3 \rightarrow \dfrac{h_4 - 2h_3 + h_2}{(\Delta x)^2} = \dfrac{W_3}{T_x} \end{cases}$$

Which can be summarized as:

$$\begin{cases} i=1 \rightarrow h_0 = 125 = (2h_1 - h_2) + \left(\dfrac{W_1 \times \Delta x^2}{T_x}\right) \\ i=2 \rightarrow 0 = (2h_2 - h_1 - h_3) + \left(\dfrac{W_2 \times \Delta x^2}{T_x}\right) \\ i=3 \rightarrow h_4 = 100 = (2h_3 - h_2) + \left(\dfrac{W_3 \times \Delta x^2}{T_x}\right) \end{cases}$$

and the other constraints are:

$$\begin{aligned} & W_1 + W_2 + W_3 \geq W_{\min} \\ & h_1, h_2, h_3 \geq 0 \\ & W_1, W_2, W_3 \geq 0 \\ & h_0 \geq h_1 \geq h_2 \geq h_3 \geq h_4 \end{aligned}$$

Therefore, the unknowns in this problem are h_1, h_2, h_3 and W_1, W_2, W_3 for all wells completed in the confined aquifer. The essential point here is considering negligible values for well losses and well diameters in this optimization analysis. This problem can be solved simply by using Excel (Data|Solver) and applying the simplex method. The achieved results are presented in the following Table 2.14

The developed LP model for having the minimum value of 4 (ft/day) discharge from each well is almost the same as the previous section except Eq. (2.23) which should be applied as:

$$W_i \geq W_{min}; \quad i = 1 \text{ to } n \tag{2.25}$$

The estimated hydraulic heads and discharge rates for all wells based on the new constraint are presented in Table 2.15.

2.3.3 *Optimization of Two-Dimensional Confined Aquifers*

The governing equation for steady-state flow in two-dimensional (x and y—directions) considering the pumping wells in the confined aquifer can be derived from Eq. (2.19) as follows:

$$\underbrace{T_x \frac{\partial^2 h}{\partial x^2} + T_y \frac{\partial^2 h}{\partial y^2}}_{T_x = T_y = T} = \underbrace{S \frac{\partial h}{\partial t}}_{0} + W \Rightarrow \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = \frac{W}{T} \tag{2.26}$$

The implementation form of Eq. (2.26) using the central finite difference technique can be written as:

Table 2.14 Hydraulic heads and discharge rates with a minimum specific discharge for all wells

Z(ft)	Hydraulic head (ft)					Discharge rate (ft/day)		
	h_0	h_1	h_2	h_3	h_4	W_1	W_2	W_3
331.5	125.0	117.75	110.5	103.25	100.0	0.0	0.0	4.0

Table 2.15 Hydraulic heads and discharge rates with a minimum specific discharge in each well

Z(ft)	Hydraulic head (ft)					Discharge rate (ft/day)		
	h_0	h_1	h_2	h_3	h_4	W_1	W_2	W_3
317.5	125.0	112.75	104.5	100.25	100.0	4.0	4.0	4.0

$$\frac{h_{i+1,j} - 2h_{i,j} + h_{i-1,j}}{(\Delta x)^2} + \frac{h_{i,j+1} - 2h_{i,j} + h_{i,j-1}}{(\Delta y)^2} = \frac{W_{i,j}}{T} \tag{2.27}$$

With the assumption of $\Delta x = \Delta y$, Eq. (2.27) can be presented as:

$$h_{i+1,j} - 4h_{i,j} + h_{i-1,j} + h_{i,j+1} + h_{i,j-1} = \frac{(\Delta x)^2}{T} W_{i,j} \tag{2.28}$$

The following problem shows the application of these equations in solving an optimization problem.

Example 2.6 Consider the plan view of steady-state flow in a two-dimensional (x. and y—directions) confined aquifer shown in Fig. 2.16. Develop a LP model to determine the maximum hydraulic heads of wells located at nodes (2, 1), (1, 2), and (2, 3) that are shown as solid red circle, and one well located at any one of nodes (1, 1), (2, 2), and (1, 3) which are shown as hashed circles on the Fig. 2.16. The boundaries (dark hexagon nodes) are considered as fixed hydraulic heads to prevent any drawdown in wells and dewatering of aquifer that can be resulted in aquifer deformation and soil layer compression/consolidation.

The necessary information for this problem are: $W_{min} = 0.5–2$ ft/day, $\Delta x = \Delta y = 500$ ft, $T = 10{,}000$ ft^2/day, and $h_{(0,1)} = h_{(0,2)} = h_{(0,3)} = h_{(1,0)} = h_{(1,4)} = h_{(2,0)} = h_{(2,4)} = h_{(3,1)} = h_{(3,2)} = h_{(3,3)} = 25$ ft.

In this problem, it is assumed that the aquifer is homogeneous, and so, its hydraulic properties are the same at any point of aquifer ($T_x = T_y$). It is good to know that the terms homogeneous and heterogeneous are related to hydraulic conductivity of the aquifer at different locations. If the hydraulic conductivity remains constant, the aquifer is homogeneous, while, the aquifer is heterogeneous (or non-homogeneous), if hydraulic conductivity varies throughout the aquifer.

Solution: The objective function for this problem is (Aguado et al. 1974):

$$\max Z = h_{(1,2)} + h_{(2,1)} + h_{(2,3)} + h_{(1,1)} + h_{(2,2)} + h_{(1,3)}$$

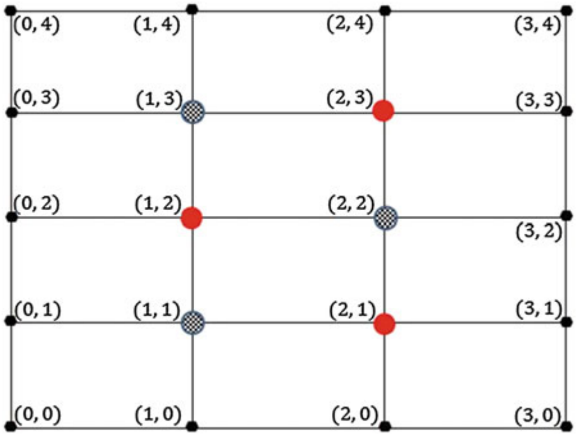


Fig. 2.16 Plan view a two-dimensional confined aquifer

The constraints at desired nodes can be written as

For node (1, 1):

$$h_{2,1} - 4h_{1,1} + h_{0,1} + h_{1,2} + h_{1,0} = \frac{(\Delta x)^2}{T} W_{1,1}$$

$$\Rightarrow 4h_{1,1} - h_{1,2} - h_{2,1} + \frac{(\Delta x)^2}{T} W_{1,1} = h_{0,1} + h_{1,0}$$

For node (2, 1):

$$h_{3,1} - 4h_{2,1} + h_{1,1} + h_{2,2} + h_{2,0} = \frac{(\Delta x)^2}{T} W_{2,1}$$

$$\Rightarrow 4h_{2,1} - h_{1,1} - h_{2,2} + \frac{(\Delta x)^2}{T} W_{2,1} = h_{3,1} + h_{2,0}$$

For node (1, 2):

$$h_{2,2} - 4h_{1,2} + h_{0,2} + h_{1,3} + h_{1,1} = \frac{(\Delta x)^2}{T} W_{1,2}$$

$$\Rightarrow 4h_{1,2} - h_{2,2} - h_{1,3} - h_{1,1} + \frac{(\Delta x)^2}{T} W_{1,2} = h_{0,2}$$

For node (2, 2):

$$h_{3,2} - 4h_{2,2} + h_{1,2} + h_{2,3} + h_{2,1} = \frac{(\Delta x)^2}{T} W_{2,2}$$

$$\Rightarrow 4h_{2,2} - h_{1,2} - h_{2,3} - h_{2,1} + \frac{(\Delta x)^2}{T} W_{2,2} = h_{3,2}$$

For node (1, 3):

$$h_{2,3} - 4h_{1,3} + h_{0,3} + h_{1,4} + h_{1,2} = \frac{(\Delta x)^2}{T} W_{1,3}$$
$$\Rightarrow 4h_{1,3} - h_{1,2} - h_{2,3} + \frac{(\Delta x)^2}{T} W_{1,3} = h_{0,3} + h_{1,4}$$

For node (2, 3):

$$\text{Node } (2,3) \rightarrow h_{3,3} - 4h_{2,3} + h_{1,3} + h_{2,4} + h_{2,2} = \frac{(\Delta x)^2}{T} W_{2,3}$$
$$\Rightarrow 4h_{2,3} - h_{2,2} - h_{1,3} + \frac{(\Delta x)^2}{T} W_{2,3} = h_{3,3} + h_{2,4}$$

These constraints also can be written in the form of matrix as follow:

$$\begin{bmatrix} 4 & -1 & -1 & 0 & 0 & 0 \\ -1 & 4 & 0 & -1 & 0 & 0 \\ -1 & 0 & 4 & -1 & -1 & 0 \\ 0 & -1 & -1 & 4 & 0 & -1 \\ 0 & 0 & -1 & 0 & 4 & -1 \\ 0 & 0 & 0 & -1 & -1 & 4 \end{bmatrix} \times \begin{bmatrix} h_{1,1} \\ h_{2,1} \\ h_{1,2} \\ h_{2,2} \\ h_{1,3} \\ h_{2,3} \end{bmatrix} + \frac{(\Delta x)^2}{T} \begin{bmatrix} w_{1,1} \\ w_{2,1} \\ w_{1,2} \\ w_{2,2} \\ w_{1,3} \\ w_{2,3} \end{bmatrix}$$
$$= \begin{bmatrix} h_{0,1} + h_{1,0} = 50 \\ h_{3,1} + h_{2,0} = 50 \\ h_{0,2} = 25 \\ h_{3,2} = 25 \\ h_{0,3} + h_{1,4} = 50 \\ h_{3,3} + h_{2,4} = 50 \end{bmatrix}$$

Additional constraints for solving this problem are:

$$\begin{cases} W_{1,1} + W_{2,2} + W_{1,3} \geq W_{min} \\ W_{1,2} \geq W_{min} \\ W_{2,1} \geq W_{min} \\ W_{2,3} \geq W_{min} \\ h_{i,j} \geq 0 \end{cases}$$

The following table shows the hydraulic heads and discharge rates at desired nodes of two-dimensional confined aquifer for different minimum value of the total discharge from internal wells. As it can be seen from the Table 2.16, when the minimum discharge from the well reaches 2 (ft/day) wells cannot meet the requirement, and so, the LP problem is infeasible. To find the optimum value of

Table 2.16 The Hydraulic heads and discharge rates at all internal nodes in various W_{min}

	$W_{min} = 0.5$ (ft/day)	$W_{min} = 1.0$ (ft/day)	$W_{min} = 1.5$ (ft/day)	$W_{min} = 2.0$ (ft/day)
$h_{(1,1)}$	22.16	19.33	16.49	No feasible solution
$h_{(2,1)}$	20.10	15.20	10.30	No feasible solution
$h_{(1,2)}$	18.56	12.11	5.67	No feasible solution
$h_{(2,2)}$	20.73	16.46	12.19	No feasible solution
$h_{(1,3)}$	18.83	12.66	6.49	No feasible solution
$h_{(2,3)}$	19.27	13.53	7.80	No feasible solution
$W_{(1,1)}$	0.00	0.00	0.00	No feasible solution
$W_{(2,1)}$	0.50	1.00	1.50	No feasible solution
$W_{(1,2)}$	0.50	1.00	1.50	No feasible solution
$W_{(2,2)}$	0.00	0.00	0.00	No feasible solution
$W_{(1,3)}$	0.50	1.00	1.50	No feasible solution
$W_{(2,3)}$	0.50	1.00	1.50	No feasible solution
Z	119.64	89.29	58.93	No feasible solution

Table 2.17 Hydraulic heads and discharge rates at all internal nodes in various Δx

	$\Delta x = 45$ (ft)	$\Delta x = 200$ (ft)	$\Delta x = 500$ (ft)
$h_{(1,1)}$	24.90	24.09	19.33
$h_{(2,1)}$	24.91	23.43	15.20
$h_{(1,2)}$	24.89	22.94	12.11
$h_{(2,2)}$	24.93	23.63	16.46
$h_{(1,3)}$	24.95	23.02	12.66
$h_{(2,3)}$	24.92	23.16	13.53
$W_{(1,1)}$	1.00	0.00	0.00
$W_{(2,1)}$	1.00	1.00	1.00
$W_{(1,2)}$	1.00	1.00	1.00
$W_{(2,2)}$	0.00	0.00	0.00
$W_{(1,3)}$	0.00	1.00	1.00
$W_{(2,3)}$	1.00	1.00	1.00
Z	149.50	140.28	89.29

hydraulic heads at all internal nodes, you can simply use Excel (DatalSolver) and choose the simplex method.

In addition to considering the effect of different values of minimum discharge (W_{min}) on the hydraulic heads (discharge from the wells and desired objective function in $\Delta x = 500$ ft), the effect of decreasing Δx on those parameters also are considered in $W_{min} = 1.0$, and $\Delta x = 45$ and 200 ft. The results of this part of example are shown in the Table 2.17.

2.4 Problems

Problem 2.1 Minimize the function f(x) using graphical method.

$$\mathrm{f}(\mathrm{x}) = 6x_1 + 3x_2$$

Subject to the following constraints:

$$\begin{aligned} 3x_1 + 2x_2 &\le 21 \\ x_1 - x_2 &\le 4.5 \\ x_1 + 2x_2 &\ge 3 \\ 4x_1 + x_2 &\ge 5.5 \\ x_1 \ge 0 \quad \text{and} \quad x_2 &\ge 0 \end{aligned}$$

Problem 2.2 Convert the following LP problem in standard form.

$$\min \mathrm{f} = x_1 + 3x_2 - 7x_3$$

Subject to

$$\begin{aligned} x_1 + x_2 + x_3 &\le 5 \\ 5x_1 - 4x_2 &\ge -11 \\ x_2 + x_3 &\ge -2 \end{aligned}$$

Problem 2.3 Maximize the following objective function using the simplex method.

$$\max \ \mathrm{f}(x) = x_1 + 8x_2$$

Subject to the below constraints:

$$\begin{aligned} x_1 - 2x_2 &\le 11 \\ 2x_1 + 6x_2 &\le 13 \\ x_1 - x_2 &\ge 6 \\ x_1 \ge 0 \quad \text{and} \quad x_2 &\ge 0 \end{aligned}$$

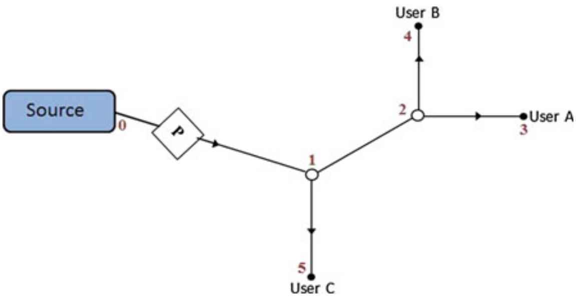
Problem 2.4 Minimize the following objective function using the simplex method.

$$\min \ \mathrm{f}(x) = 0.35x_1 - x_2 + 2.5x_3$$

Subject to

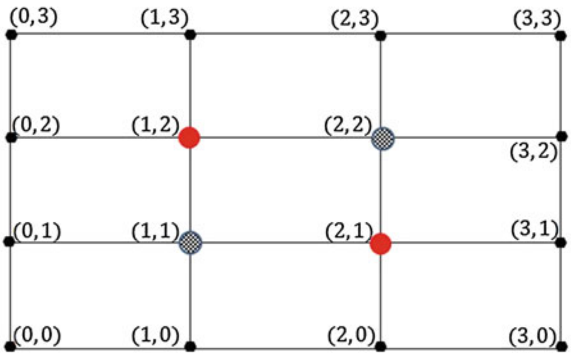
$$x_1 - 3x_2 \geq 4$$
$$x_2 + 1.5x_3 \geq 1$$
$$x_1 - x_3 \leq 5$$
$$x_1 \geq 0 \quad \text{and} \quad x_2 \geq 0$$

Problem 2.5 Determine the minimum cost of pipe and pump (head elevation) in various demand nodes in Example 2.4 by changing the location of pump station from pipe section 1–2 to the pipe section 0–1, and then, compare the optimization results.



Problem 2.6 Determine the optimal pumpage for a confined aquifer with one-dimensional steady-state flow and fixed hydraulic heads along the boundaries in Example 2.5 where $W_{min} = 50$ ft/day, $\Delta x = 25$ ft, $T = 8{,}000$ ft^2/day, $h_0 = 85$ ft, $h_4 = 75$ ft.

Problem 2.7 Consider the plan view of steady-state flow in a two-dimensional (x and y—directions) confined aquifer shown in the following figure. Develop a LP model to determine the maximum hydraulic heads of wells located at nodes (2, 1) and (1, 2), that are shown as solid red circle, and one well located at any one of nodes (1, 1) and (2, 2), which are shown as hashed circles on the figure below. The boundaries (dark hexagon nodes) are considered as fixed hydraulic heads to prevent any drawdown in wells and dewatering of aquifer that can be resulted in aquifer deformation and soil layer compression/consolidation.



The necessary information for this problem are: $W_{min} = 90$ ft/day, $\Delta x = \Delta y = 240$ ft, $T = 10{,}000$ ft^2/day, and $h_{(0,1)} = h_{(1,0)} = h_{(2,0)} = h_{(3,1)} = h_{(0,2)} = h_{(1,3)} = h_{(2,3)} = h_{(3,2)} = 105$ft. In this problem, it is assumed that the aquifer is homogeneous, and so, its hydraulic properties are the same at any point of aquifer ($T_x = T_y$).

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Chapter 3
Nonlinear Optimization

Abstract This chapter begins with an introduction to nonlinear programming and continues with introducing the conceptual framework of nonlinear optimization problems as well as their applications in water resources engineering. In addition, different nonlinear optimization methods including one-dimensional optimization techniques, unconstrained and constrained optimization methods in conjunction with a number of useful examples are provided to indicate why and how non-linearities arise in wide ranges of water resources optimization problems.

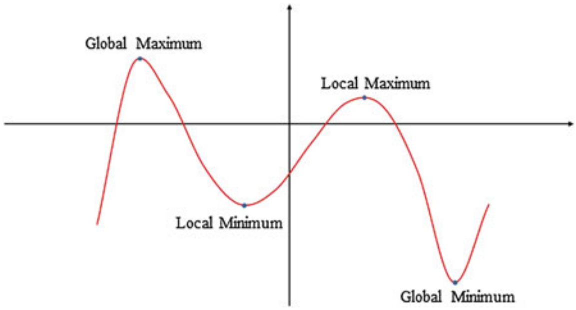
3.1 Introduction

Although linear programming is a powerful and widely applicable tool in modeling a variety of practical optimization problems, but many problems are inherently nonlinear and they can only be modeled using nonlinear functions. An optimization problem is a nonlinear problem if some of the objective function or some of the constraints are nonlinear. The nonlinear programming (NLP) or nonlinear optimization is an extension of linear programming to find the optimal solution of nonlinear problems. In the case of nonlinearity, the problems are considerably more complex than the linear problems and finding the optimal solution is more difficult. A common difficulty regarding nonlinear problems is finding the global or absolute optimal solution; and often a local or relative solution is found for the nonlinear problems. A global optimum can be defined as the best minimum or maximum value of the objective function in the entire feasible region, whereas, a local optimum is an optimum value over a subset of the domain and it happens in an immediate neighborhood (Fig. 3.1). A global minimum for function f(x) at the point x^* over the space S occurs if;

$$\mathrm{f}(x^*) \leq \mathrm{f}(x) \quad \text{for } \forall\, x \in S \tag{3.1}$$

E. Goodarzi et al., *Introduction to Optimization Analysis in Hydrosystem Engineering*,
Topics in Safety, Risk, Reliability and Quality 25, DOI: 10.1007/978-3-319-04400-2_3,

Fig. 3.1 The local and global optimum solutions



and in the case of global maximum x^* should satisfies the below relationship as;

$$f(x^*) \geq f(x) \quad \text{for } \forall\, x \in S \tag{3.2}$$

A local minimum of function f(x) happens at the point x^* over the space S if the following relationship is satisfied for an $\varepsilon > 0$;

$$f(x^*) \leq f(x) \quad \text{for } \forall x \in S \text{ with } |x - x^*| < \varepsilon \tag{3.3}$$

And a local maximum occurs if;

$$f(x^*) \geq f(x) \quad \text{for } \forall x \in S \text{ with } |x - x^*| < \varepsilon \tag{3.4}$$

In general, there are multiple local solutions for nonlinear optimization problems and so, finding the global solution is a difficult task except for some special cases such as convex or unimodal functions. The function f(x) is a convex function on the space S if the line segment that connects any two pairs of function occurs entirely above the graph. On the other hand, the function f(x) is a concave function on the space S if the line segment joining two arbitrary points of f(x) is located wholly below the graph. A convex function can be defined mathematically as;

$$f[(1-\lambda)x_1 + \lambda x_2] \leq (1-\lambda)f(x_1) + \lambda f(x_2) \tag{3.5}$$

The function f(x) is strictly convex if the Eq. (3.5) hold with a less than sign (<) instead of a $\leq$.

On the other hand, the function f(x) is concave if;

$$f[(1-\lambda)x_1 + \lambda x_2] \geq (1-\lambda)f(x_1) + \lambda f(x_2) \tag{3.6}$$

In this case, the function is strictly concave if the above equation holds only the greater sign (>).

Figure 3.2 shows a convex and a concave function, respectively. It is important to note that the convex function is always a bowl-shaped up, while the concave one is always a bowl-shaped down. Any local minimum and maximum of a convex and concave function is also a global minimum and maximum.

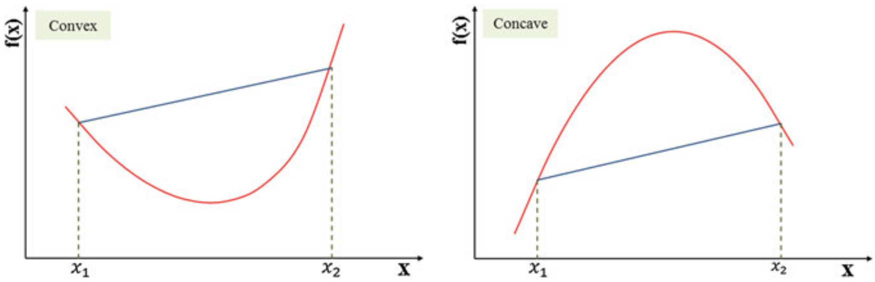


Fig. 3.2 The convex and concave functions

In addition to applying Eqs. (3.5) and (3.6), the convexity and concavity of function f(x) for $x \in (x_1, x_2, \ldots, x_n)$ can be determined using the Hessian matrix. This symmetric matrix is defined as;

$$H(x) = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_n} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & & \frac{\partial^2 f}{\partial x_2 \partial x_n} \\ \vdots & & \ddots & \vdots \\ \frac{\partial^2 f}{\partial x_n \partial x_1} & & \cdots & \frac{\partial^2 f}{\partial x_n^2} \end{bmatrix} \tag{3.7}$$

Consider f(x) as continuous second partial derivatives, then the convexity and concavity can be obtained as;

1. The function f(x) is convex, if and only if H(x) is positive semidefinite,
2. The function f(x) is strictly convex, if and only if H(x) is positive definite,
3. The function f(x) is concave, if and only if H(x) is negative semidefinite,
4. The function f(x) is strictly concave, if and only if H(x) is negative definite.

The definitions of positive definite and semidefinite, and negative definite and semidefinite of Hessian matrix H are defined as;

1. Positive semidefinite:

$$x^T Hx \geq 0 \quad \text{for all } x \tag{3.8}$$

2. Positive definite:

$$x^T Hx > 0 \quad \text{for all } x \neq 0 \tag{3.9}$$

3. Negative semidefinite:

$$x^T Hx \leq 0 \quad \text{for all } x \tag{3.10}$$

4. Negative definite:

$$x^T Hx < 0 \quad \text{for all } x \neq 0 \tag{3.11}$$

5. Indefinite:

$$\begin{aligned} x^T Hx > 0 \quad & \text{for some } x \\ x^T Hx < 0 \quad & \text{for some other } x \end{aligned} \tag{3.12}$$

where,

$$x^T Hx = (x_1, \ldots, x_n) \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \tag{3.13}$$

Example 3.1 Classify the following 2 × 2 matrix.

$$H = \begin{bmatrix} 7 & -3 \\ -4 & 8 \end{bmatrix}$$

Solution: To classify matrix A, we need to estimate $x^T Ax$ as follow;

$$\begin{aligned} x^T Hx &= \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} 7 & -3 \\ -4 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\ &= \begin{bmatrix} 7x_1 - 4x_2 & -3x_1 + 8x_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\ &= 7x_1^2 - 4x_1x_2 - 3x_1x_2 + 8x_2^2 \\ &= 7x_1^2 + 8x_2^2 - 7x_1x_2 \end{aligned}$$

As the final function is positive for all $x_1 \neq 0$, and $x_2 \neq 0$, the Hessian matrix is positive definite.

Another way to test the status of H(x) is using its eigenvalues. The Hessian matrix is positive definite if all of its eigenvalues are positive, and it is negative definite if all of its eigenvalues are negative. To find the eigenvalues (λ) of an $n \times n$ Hessian matrix $H(x)$, the following condition should be satisfied;

$$|H - \lambda I_n| = 0 \tag{3.14}$$

where, the sign | | is determinant of (.), and I_n is the identity matrix.

Example 3.2 Find the Eigenvalues of the presented matrix in Problem 3.1, and classify the matrix.

$$H = \begin{bmatrix} 7 & -3 \\ -4 & 8 \end{bmatrix}$$

Solution: Based on Eq. (3.14), we need to solve the following equation;

$$
\begin{aligned}
|H - \lambda I_2| &= 0 \\
&= \left| \begin{bmatrix} 7 & -3 \\ -4 & 8 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| \\
&= \left| \begin{bmatrix} 7 & -3 \\ -4 & 8 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| \\
&= \left| \begin{bmatrix} 7-\lambda & -3 \\ -4 & 8-\lambda \end{bmatrix} \right| \\
&= [(7-\lambda)(8-\lambda)] - [(-3) \times (-4)] = \lambda^2 - 15\lambda + 44 = 0
\end{aligned}
$$

The matrix has two Eigenvalues $\lambda = 4$ and $\lambda = 11$. As all of Eigenvalues are positive, the Hessian matrix is positive definite.

Another important way to test the status of Hessian matrix is using the concept of *leading principal submatrix* and *leading principal minors*. A $k \times k$ submatrix of an $n \times n$ matrix H, which is known as leading principal submatrix, can be determined by removing the last $n - k$ columns and rows from matrix H. The determinant of this submatrix is called leading principal minor of H. Some leading principal minors of an $n \times n$ can be written as follow;

- First leading principle minors;

$$H_1 = |a_{11}|$$

- Second leading principle minors;

$$H_2 = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

- Third leading principle minors;

$$H_3 = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

- The nth leading principle minors;

$$H_n = \begin{vmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} \\ . & . & . & . & . \\ . & . & . & . & . \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{vmatrix}$$

The matrix H is a positive definite matrix if and only if all of its leading principal minors are positive, and it is positive semidefinite if its entire principal minors are non-negative. On the other hand, the matrix H is a negative definite if and only if the sign of leading principal minors H_i is negative for odd values of i and positive for even values of i.

Example 3.3 Determine if the following functions are convex or concave.

$$\mathrm{f}_1(x) = 7x_1^2 - 5x_1x_2 + 3x_2^2$$
$$\mathrm{f}_2(x) = -7x_1^2 - 5x_1x_2 - 3x_2^2$$
$$\mathrm{f}_3(x) = -5x^2 + 6$$

Solution: To obtain the convexity and concavity of each function, we need to calculate the leading principal minors as;

$$H(x) = \begin{bmatrix} \dfrac{\partial^2 \mathrm{f}_1}{\partial x_1^2} & \dfrac{\partial^2 \mathrm{f}_1}{\partial x_1 \partial x_2} \\ \dfrac{\partial^2 \mathrm{f}_1}{\partial x_2 \partial x_1} & \dfrac{\partial^2 \mathrm{f}_1}{\partial x_2^2} \end{bmatrix} = \begin{bmatrix} 14 & -5 \\ -5 & 6 \end{bmatrix}$$

Therefore,

$$H_{i=1} = |14| = 14$$
$$H_{i=2} = \begin{vmatrix} 14 & -5 \\ -5 & 6 \end{vmatrix} = (14 \times 6) - [(-5) \times (-5)] = 59$$

As all of the leading principal minors are positive, the matrix H is positive definite and so, the function $\mathrm{f}_1(\mathrm{x})$ is strictly convex. For the second function, we have;

$$H(x) = \begin{bmatrix} \dfrac{\partial^2 \mathrm{f}_2}{\partial x_1^2} & \dfrac{\partial^2 \mathrm{f}_2}{\partial x_1 \partial x_2} \\ \dfrac{\partial^2 \mathrm{f}_2}{\partial x_2 \partial x_1} & \dfrac{\partial^2 \mathrm{f}_2}{\partial x_2^2} \end{bmatrix} = \begin{bmatrix} -14 & -5 \\ -5 & -6 \end{bmatrix}$$

Hence,

$$H_{i=1} = |-14| = -14$$
$$H_{i=2} = \begin{vmatrix} -14 & -5 \\ -5 & -6 \end{vmatrix} = [(-14) \times (-6)] - [(-5) \times (-5)] = 59$$

As the sign of leading principal minors H_i is negative for odd values of i ($i = 1$), and it is positive for even values of i ($i = 2$), the matrix H is a negative definite and so, the function f_2 is strictly concave. For a single variable function like $\mathrm{f}_3(\mathrm{x})$ we can simply examine the sign of second derivative in which a positive sign shows the function is convex, while, a negative sign demonstrates a concave function. In the case of this problem, we have;

$$\frac{\partial^2 \mathrm{f}_3}{\partial x} = -10 < 0$$

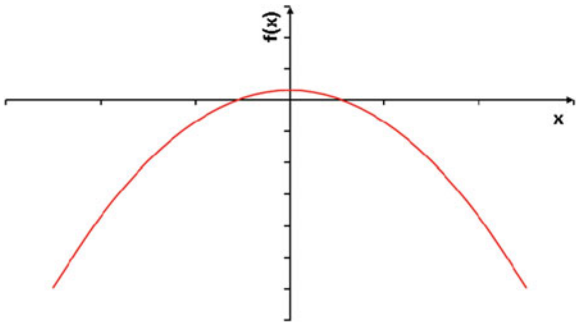


Fig. 3.3 The concave function $f_3(x)$

Therefore, it can be concluded that the function f_3 is a concave function (Fig. 3.3).

Nonlinear optimization techniques can be divided into three main classes as; (1) one-dimensional optimization methods, (2) unconstrained optimization techniques, and (3) constrained optimization methods. Each of these three classes includes a number of important optimization techniques that are useful in finding the optimum solution of nonlinear functions; and the most important of them are explained in the following sections.

The one-dimensional search methods use an iterative process to find the optimum values and it is classified into two main categorizes as; elimination and approximation approaches. These two categorizes also are divided into several useful methods in which some of them are presented in Table 3.1. In this chapter the Fibonacci and golden section methods from elimination category, and the Newton method from approximation groups with appropriate examples are described in the next sections.

The second nonlinear optimization technique is unconstrained optimization methods which are classified into two main types as Direct Search Methods and Indirect Search (or Descent) Methods. The direct search methods do not use the derivatives of the desired objective functions and only the values of objective function that should be minimized or maximized are used here, while, indirect search methods need both values of objective functions and the derivatives of objective functions. Some of the most important direct and indirect search methods for solving nonlinear optimization problems are presented in the Table 3.2. In this chapter, the random search and univariate methods from direct search group and the steepest descent method from indirect category are illustrated.

As noted above, the third category of nonlinear optimization approaches is constrained optimization methods that include the following techniques to solve an optimization problem; penalty function method, Lagrange multiplier, quadratic programming, and generalized reduced gradient (GRG) method. In the following sections, the Lagrange multiplier and GRG methods in conjunction with a number of useful examples are presented.

Table 3.1 Different types of one-dimensional search methods (Rao 2009)

Elimination methods	Approximation methods
Unrestricted search method	Quadratic interpolation method
Exhaustive search method	Cubic interpolation method
Dichotomous search method	Direct root methods:
Interval halving method	• Newton method
Fibonacci method	• Quasi–Newton method
Golden section method	• Secant method

Table 3.2 Direct and indirect search methods

Direct search methods	Indirect search methods
Random search method:	Steepest descent (Cauchy) method
• Random jump method	
• Random walk method	
• Random walk method with direction exploitation	
Grid search method	Conjugate gradient (Fletcher–Reeves) method
Univariate method	Newton's method
Pattern direction method	Marquardt method
Rosen Brock's method of rotating coordinates	Quasi–Newton method
Simplex method	

3.2 One-Dimensional Search Methods

The nonlinear objective functions are usually very complicated and finding an analytical solution for them is very cumbersome or even impossible. In these cases, one may apply trial based techniques, such as one-dimensional search methods that use a trial procedure to estimate an initial solution for the objective function, and improve the calculated solution consecutively until the pre-determined convergence criterion is met. The main objective function in one-dimensional optimization problem is;

$$\min F = \mathrm{f}(x) \tag{3.15}$$

in which, f(x) is function of a single variable x, and it has a unique solution if f(x) is a unimodal function over some ranges of a closed interval like $[x_l, x_u]$. The closed interval here is called a *bracket*. A unimodal function is a function with unique minimum or maximum in a region that is going to be searched, and for x_1 and x_2 which are placed in the interval $[x_l, x_u]$ it can be mathematically defined as;

1. A unimodal function with minimum value (Fig. 3.4a); if the points x_1 and x_2 are both on the same side of the optimum point (x^*), and $x_l < x_1 < x_2 < x^*$, then the point near the optimum has the lower values in which $\mathrm{f(x^*)} < \mathrm{f(x_2)} < \mathrm{f(x_1)}$.

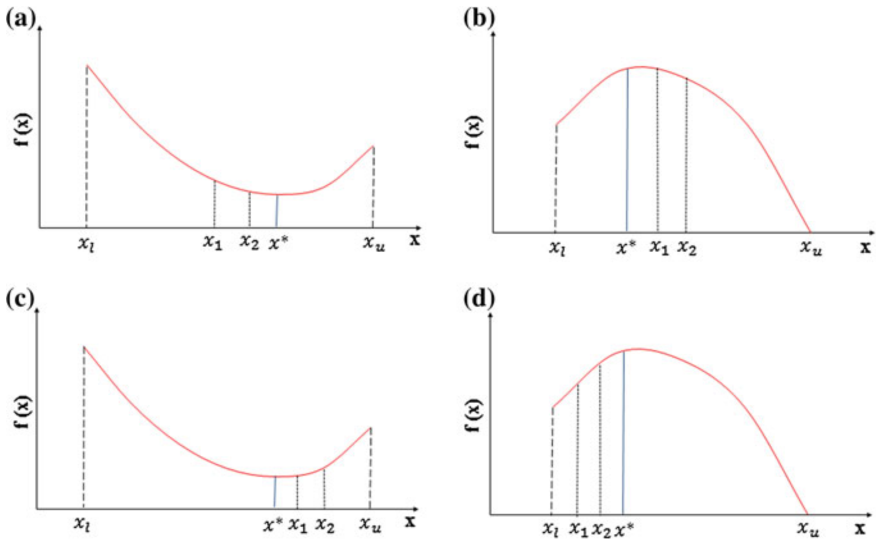


Fig. 3.4 Two arbitrary unimodal functions

2. A unimodal function with maximum value (Fig. 3.4b); if the points x_1 and x_2 are both on the same side of the optimum point (x^*), and $x^* < x_1 < x_2 < x_u$, then the point near the optimum has the higher values as $\mathrm{f}(\mathrm{x}^*) > \mathrm{f}(\mathrm{x}_1) > \mathrm{f}(\mathrm{x}_2)$.
3. A unimodal function with minimum value (Fig. 3.4c); if the points x_1 and x_2 are both on the same side of the optimum point (x^*), and $x^* < x_1 < x_2 < x_u$, then the point near the optimum has the lower values as $\mathrm{f}(\mathrm{x}^*) < \mathrm{f}(\mathrm{x}_1) < \mathrm{f}(\mathrm{x}_2)$.
4. A unimodal function with maximum value (Fig. 3.4d); if the points x_1 and x_2 are both on the same side of the optimum point (x^*), and $x_l < x_1 < x_2 < x^*$, then the point near the optimum has the higher values as $\mathrm{f}(\mathrm{x}^*) > \mathrm{f}(\mathrm{x}_2) > \mathrm{f}(\mathrm{x}_1)$.

The interval $[x_l, x_u]$ also is known as *range of uncertainty* that includes the optimum value and it is established for desired unimodal function regardless of whether the function is continuous or discontinuous; or differentiable or non-differentiable. Based on this method, a portion of the range of uncertainty continually eliminated on the basis of function evaluations until the remaining interval gets adequately small. In other words, for a unimodal function f(x) with a minimum at x^* and considering two points x_i and x_j in which $x_l < x_i < x_j < x_u$, we can write;

1. If $\mathrm{f}(\mathrm{x}_i) > \mathrm{f}(\mathrm{x}_j)$, the minimum of f(x) is not in the interval $[x_l, x_i]$, and so, $x^* \in [x_i, x_u]$. Hence, some ranges of the interval can be eliminated and narrowed down into two different points in the range (Fig. 3.5a).
2. If $\mathrm{f}(\mathrm{x}_i) < \mathrm{f}(\mathrm{x}_j)$, the minimum of f(x) is not in the interval $[x_j, x_u]$, and so, $x^* \in [x_l, x_j]$. Therefore, the range $[x_j, x_u]$ can be eliminated in searching for minimum value of objective function (Fig 3.5b).
3. If $\mathrm{f}(\mathrm{x}_i) = \mathrm{f}(\mathrm{x}_j)$, both intervals $[x_l, x_i]$ and $[x_j, x_u]$ can be eliminated.

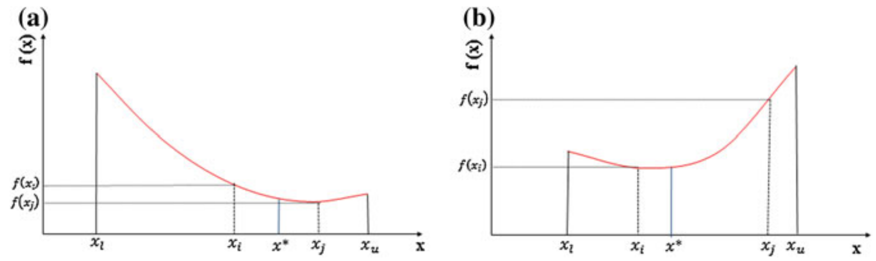


Fig. 3.5 Reducing the range of uncertainty

Table 3.3 Fibonacci numbers for m = 0–13

m	F_m
0	1
1	1
2	2
3	3
4	5
5	8
6	13
7	21
8	34
9	55
10	89
11	144
12	233
13	377

3.2.1 Fibonacci Method (Elimination Technique)

In mathematics, the Fibonacci numbers are the numbers that are defined as;

$$F_m = F_{m-1} + F_{m-2} \tag{3.16}$$

with the initial values of $F_0 = F_1 = 1$. Therefore, the Fibonacci series for m = 0–4 can be calculated as;

$$\begin{cases} F_{m=0} = F_{m=1} = 1 \\ F_{m=2} = F_{m=1} + F_{m=0} = 1 + 1 = 2 \\ F_{m=3} = F_{m=2} + F_{m=1} = 2 + 1 = 3 \\ F_{m=4} = F_{m=3} + F_{m=2} = 3 + 2 = 5 \end{cases}$$

The Fibonacci numbers for m = 0–13 are presented in the Table 3.3.

Assume two points x_i^1 and x_j^1 are located in the range of uncertainty (d_n) in which $d_n = [x_l, x_u]$, and $\mathrm{f}(\mathrm{x}_\mathrm{i}^1) > \mathrm{f}(\mathrm{x}_\mathrm{j}^1)$. As it can be seen in the Fig. 3.6, the minimum of

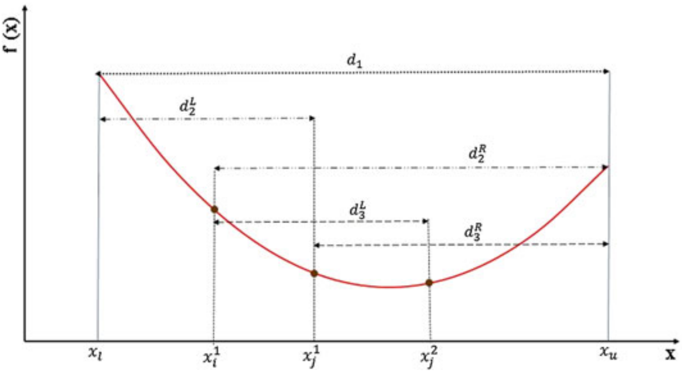


Fig. 3.6 The process of elimination intervals

f(x) does not lie in the interval $[x_l, x_i^1]$ and so, the left interval $[x_l, x_i^1]$ can be eliminated and the uncertainty range will be narrowed down into the right interval d_{n+1}^R. It is important to note that, after eliminating the interval $[x_l, x_i^1]$, the point x_i^1 is the starting point of new reduced range of uncertainty and so $d_n = [x_i^1, x_u]$. In the case of $f(x_i^1) < f(x_j^1)$, the interval $[x_i^1, x_u]$ will be discarded. In the next step, another interior point, say x_j^2, can be selected to repeat the process of reducing uncertainty range getting closer to the optimum value. Since $f\left(x_j^1\right) > f(x_j^2)$, the minimum value is placed in the interval d_{n+2}^R and the uncertainty range of $\left[x_i^1, x_j^1\right]$ can be deleted. This procedure should be repeated in order to find the final reduced uncertainty range and the optimum value.

Based on Fig. 3.6, the following relation can be written as;

$$d_1 = d_2^L + d_3^R$$

If we assume all the sub-intervals are equal (e.g., $d_2^L = d_2^R = d_2$ and $d_3^L = d_3^R = d_3$), it can be concluded that;

$$d_1 = d_2^L + d_3^R = d_2 + d_3 \tag{3.17}$$

Based on this equation, a sequence of intervals for n experiments can be generated as follows;

$$\begin{cases} d_1 = d_2 + d_3 \\ d_2 = d_3 + d_4 \\ d_3 = d_4 + d_5 \\ \quad \vdots \\ d_n = d_{n+1} + d_{n+2} \end{cases} \tag{3.18}$$

According to the presented sequence of intervals for n experiments, there are n equations with $n + 2$ unknown variables, and so, it is possible to generate unlimited sequences by considering some additional assumptions. To generate the Fibonacci sequences, it can be assumed that the interval d_{n+2} will be eliminated at step n, and hence;

$$\begin{aligned} d_{n+1} &= d_n - \underbrace{d_{n+2}}_{0} \\ \Rightarrow d_{n+1} &= d_n = F_0 d_n \end{aligned} \tag{3.19}$$

Therefore, the following relations can be established;

$$\begin{aligned} d_n &= d_{n+1} + \underbrace{d_{n+2}}_{0} = d_n = F_1 d_n \\ d_{n-1} &= d_n + d_{n+1} = F_1 d_n + F_0 d_n = 2d_n = F_2 d_n \\ d_{n-2} &= d_{n-1} + d_n = F_2 d_n + F_1 d_n = 3d_n = F_3 d_n \\ d_{n-3} &= d_{n-2} + d_{n-1} = F_3 d_n + F_2 d_n = 3d_n + 2d_n = 5d_n = F_4 d_n \\ &\vdots \\ d_{n-m} &= d_k = d_{k+1} + d_{k+2} = F_{m+1} d_n = F_{n-k+1} d_n \\ &\vdots \\ d_1 &= d_2 + d_3 = F_n d_n \end{aligned} \tag{3.20}$$

According to the above equations, the last interval of uncertainty that happens at $n = m$ is;

$$d_n = \frac{d_1}{F_n} \tag{3.21}$$

Based on above equation, the location of nth experiments (d_n^*) can be calculated as;

$$d_n^* = \frac{F_{m-n}}{F_{m-(n-2)}} d_{n-1} \tag{3.22}$$

and the range of uncertainty at the end of this experiment (d_n) is;

$$d_n = d_{n-1} - d_n^* = \frac{F_{m-(n-1)}}{F_m} d_1 \tag{3.23}$$

For example in the case of $n = 2$, the location of 2nd and the range of uncertainty at this location are;

$$d_2^* = \frac{F_{m-2}}{F_m} d_1$$

$$d_2 = d_1 - d_2^* = \frac{F_{m-1}}{F_m} d_1$$

It is important to note that the interior points in one-dimensional search methods can be calculated based on the following relations;

$$\left.\begin{aligned} x_1 &= x_l + d_2^* \\ x_2 &= x_u - d_2^* \end{aligned}\right. \Rightarrow x_2 = x_l + (x_u - x_1) \tag{3.24}$$

The following example provides all necessary steps for obtaining the minimum value of a unimodal function based on Fibonacci technique.

Example 3.4 Minimize f(x) in the interval [0.0,4.0] using the Fibonacci method for the total number of experiments $n = 8$.

$$\mathrm{f}(x) = 3x^2 - 4x + 5.5$$

Solution: The required steps to find the minimum value of desired objective function based on the above principles are presented in the following sections. It is important to note that the initial range is $d_1 = x_u - x_l = 4.0 - 0.0 = 4$, $m = 8$ and n varies from 2 to m.

Step 1: Determine d_n^* for $n = 2$ from the following equation;

$$d_2^* = \frac{F_{8-2}}{F_{8-(2-2)}} d_{2-1} = \frac{F_6}{F_8} d_1 = \frac{13}{34} \times 4 = 1.529$$

Now, the interior points x_1 and x_2 in conjunction with $f(x_1)$ and $f(x_2)$ can be calculated as (Fig. 3.7);

$$x_1 = x_l + d_2^* = 0 + 1.529 = 1.529 \rightarrow f(x_1) = 6.399$$
$$x_2 = x_u - d_2^* = 4.0 - 1.529 = 2.470 \rightarrow f(x_2) = 13.929$$

Step 2: Compare the values of $f(x_1)$ and $f(x_2)$ to determine the range of interval that should be eliminated. As $f(x_1) < f(x_2)$, the interval $[x_2, x_u] = [2.470, 4.0]$ is discarded using the unimodality assumption (Fig. 3.8). Therefore, the range of uncertainty at the end of this experiment will be;

$$d_2 = d_1 - d_2^* = 4.0 - 1.529 = 2.470$$

Or,

$$d_2 = \frac{F_7}{F_8} d_1 = \frac{21}{34} \times 4 = 2.470$$

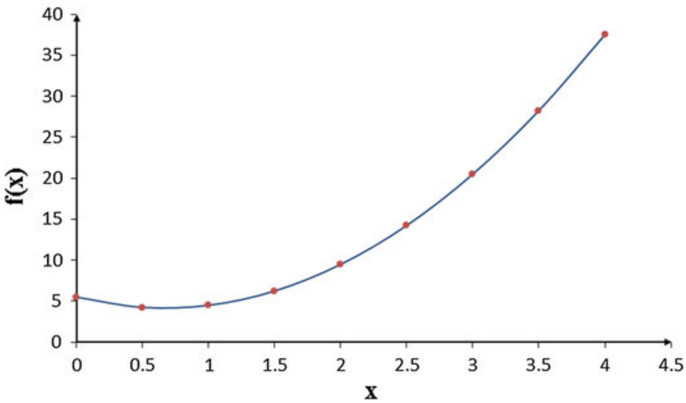


Fig. 3.7 The objective function f(x)

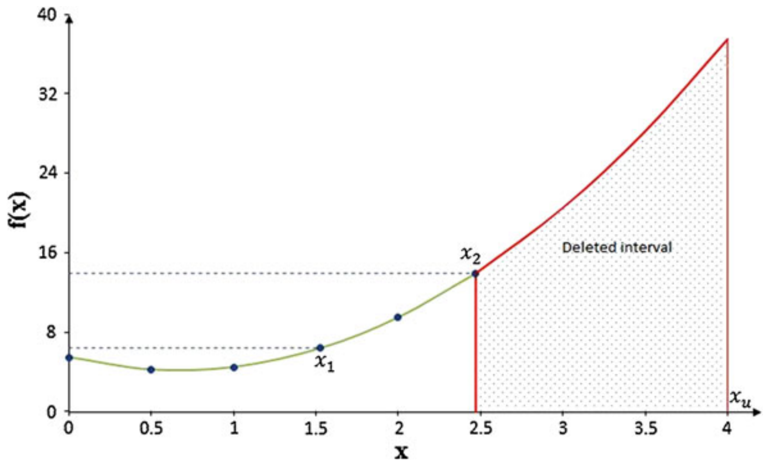


Fig. 3.8 The eliminated interval in step one

By discarding the interval $[2.470, 4]$, the range of uncertainty reduces to $[0.0, 2.470]$, and hence, $x_l = 0.0$, and $x_{u_{new}} = x_2 = 2.470$.

Step 3: In this step, the value of d_3^*, the interior point and x_3 should be computed, and then, the functions $\mathrm{f}(\mathrm{x}_1)$ and $\mathrm{f}(\mathrm{x}_3)$ will be compared.

$$d_3^* = \frac{F_5}{F_7} d_2 = \frac{8}{21} \times 2.470 = 0.941$$

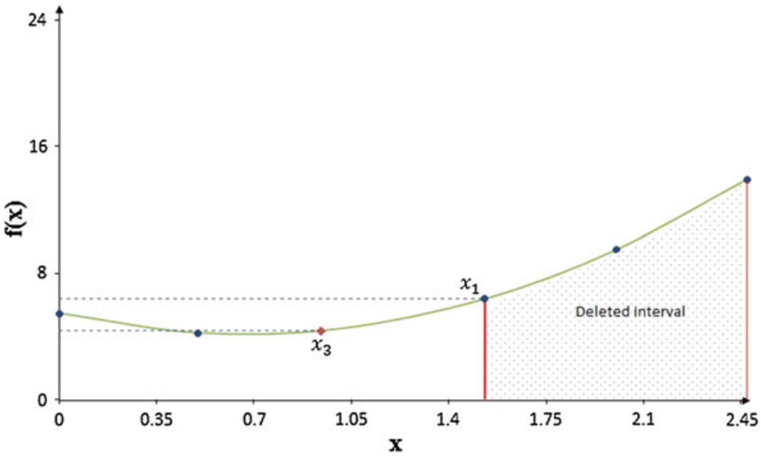


Fig. 3.9 The second eliminated interval

And

$$x_3 = x_l + (x_u - x_1) = 0 + (2.470 - 1.529)$$
$$= 0.941 \rightarrow f(x_3) = 4.392$$

As $f(x_1) > f(x_3)$, the interval $[x_1, x_{u_{new}}]$ which is $[1.529, 2.470]$ will be eliminated (Fig. 3.9). So, the range of uncertainty in this step is;

$$d_3 = d_2 - d_3^* = 2.470 - 0.941 = 1.529$$

Or

$$d_3 = \frac{F_6}{F_8} d_1 = \frac{13}{34} \times 4 = 1.529$$

The new interval reduces to $[0.0, 1.529]$, and hence, $x_l = 0.0$, and $x_{u_{new}} = x_1 = 1.529$.

Step 4: The process of elimination is continued based on the new evaluated interval as follows;

$$d_4^* = \frac{F_4}{F_6} d_3 = \frac{5}{13} \times 1.529 = 0.588$$

And,

$$x_4 = x_l + (x_u - x_3) = 0.0 + (1.529 - 0.941)$$
$$= 0.588 \rightarrow f(x_4) = 4.185$$

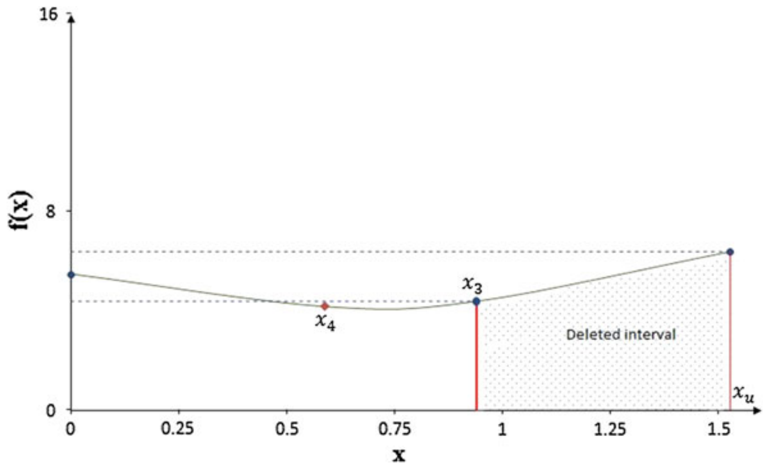


Fig. 3.10 The third eliminated interval

As $f(x_3) > f(x_4)$, the interval $[x_3, x_{u_{new}}]$ which is $[0.941, 1.529]$ is deleted (Fig. 3.10). The range of uncertainty here is;

$$d_4 = d_3 - d_4^* = 1.529 - 0.588 = 0.941$$

Therefore, the interval reduces to $[0.0, 0.941]$, and x_l and $x_{u_{new}}$ are 0.0 and 0.941, respectively.

Step 5: The necessary calculations in this step are;

$$d_5^* = \frac{F_3}{F_5} d_4 = \frac{3}{8} \times 0.941 = 0.352$$

And,

$$\begin{aligned} x_5 &= x_l + (x_u - x_4) = 0.0 + (0.941 - 0.588) \\ &= 0.352 \rightarrow f(x_5) = 4.461 \end{aligned}$$

As $f(x_5) > f(x_3)$, the interval $[0, x_5]$ which is $[0.0, 0.352]$ should be excluded (Fig. 3.11), and the range of uncertainty is computed as;

$$d_5 = d_4 - d_5^* = 0.941 - 0.352 = 0.588$$

In this step, the interval reduces to $[0.352, 0.941]$, and so, $x_{l_{new}} = 0.352$ and $x_u = 0.941$. The remaining steps for calculating the minimum value of desired objective function are presented in Table 3.4.

The final range of uncertainty due to Fibonacci search can be calculated as;

$$d_n = \frac{d_1}{F_m} \tag{3.25}$$

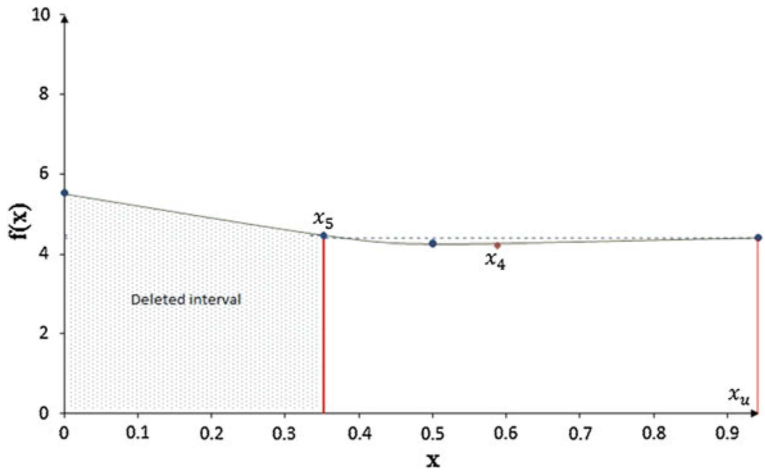


Fig. 3.11 The fourth eliminated interval

Table 3.4 The Fibonacci method procedure

n	d_n^*	x_i	x_j	f(x_i)	f(x_j)	Greater f(x)	d_n	x_l	x_u
2	1.5294	1.5294(x_1)	2.4706(x_2)	6.3997	13.9291	f(x_2)	2.4706	0.0000	2.4706
3	0.9412	1.5294(x_1)	0.9412(x_3)	6.3997	4.3927	f(x_1)	1.5294	0.0000	1.5294
4	0.5882	0.9412(x_3)	0.5882(x_4)	4.3927	4.1851	f(x_3)	0.9412	0.0000	0.9412
5	0.3529	0.5882(x_4)	0.3529(x_5)	4.1851	4.4619	f(x_5)	0.5882	0.3529	0.9412
6	0.2353	0.5882(x_4)	0.7059(x_6)	4.1851	4.1713	f(x_4)	0.3529	0.5882	0.9412
7	0.1176	0.7059(x_6)	0.8235(x_7)	4.1713	4.2405	f(x_7)	0.2353	0.5882	0.8235
8	0.1176	0.7059(x_6)	0.7059(x_8)	4.1713	4.1713	–	0.1176	0.5880	0.7059

Now, the final interval in Table 3.4 can be compared with Eq. (3.25), as;

$$d_8 = \frac{d_1}{F_8} = \frac{4}{34} = 0.118$$

In addition, at the point $n = m$ the ratio of the final interval d_n to the initial range of uncertainty can be compared with the ratio $1/F_m$, as;

$$\frac{d_n}{d_0} = \frac{1}{F_m} \tag{3.26}$$

Therefore;

$$\frac{d_8}{d_0} = \frac{0.118}{4.0} = 0.0294$$

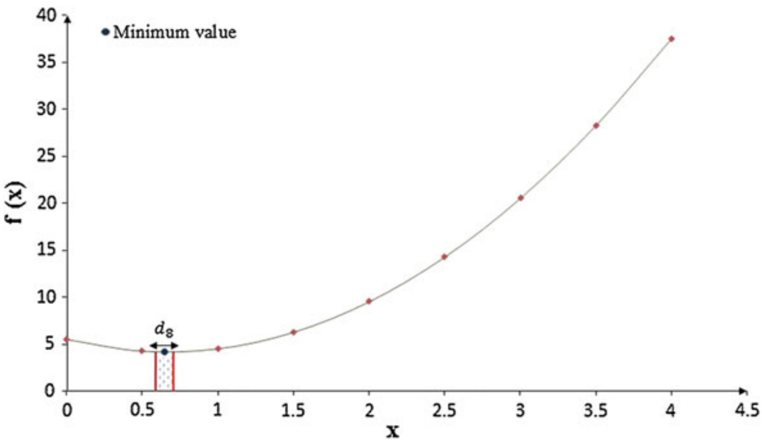


Fig. 3.12 The final interval

And,

$$\frac{1}{F_8} = \frac{1}{34} = 0.0294$$

The minimum value f(x^*) occurs at the center of interval [0.5882,0.7059], and so (Fig. 3.12);

$$x^* = \frac{0.5882 + 0.7059}{2} = 0.6471$$
$$\Rightarrow \mathrm{f}(x^*) = \mathrm{f}(0.6471) = 4.1678$$

To see how we are close to the exact optimum value, the minimum value can be obtained simply by setting the first derivative equal to zero as;

$$\frac{d\mathrm{f}(x)}{dx} = 0 \rightarrow f'(x) = 6x - 4 = 0$$
$$\Rightarrow x = x^* = \frac{4}{6} = 0.6666$$

and so, $\mathrm{f}(\mathrm{x}^*) = 4.1666$.

Comparing the results of analytical method with the Fibonacci technique shows the answers are very close.

3.2.2 Golden Section Method (Elimination Technique)

In general, the entire procedure of finding an optimum value using Fibonacci and Golden Section Methods (GSM) are almost the same and their basic idea is reducing the range of uncertainty and generating the new smaller interval for minimizing the desired unimodal function. However, the total number of iterations (n) must be obtained before beginning calculations in the Fibonacci approach, while, this is not an initial condition in GSM and the search process is stopped when an acceptable accuracy is achieved and the bracketing interval is reasonably small. The fundamental rule in the GSM is based on splitting an interval in which the ratio of every two nearby segments remains constant as;

$$\frac{d_1}{d_2} = \frac{d_2}{d_3} = \frac{d_3}{d_4} = \cdots \frac{d_n}{d_{n+1}} = \omega \tag{3.27}$$

Based on the Eq. (3.27), we can write;

$$\left.\begin{array}{r} \frac{d_1}{d_2} = \omega \rightarrow d_2 = \frac{d_1}{\omega} \\ \frac{d_2}{d_3} = \omega \rightarrow d_2 = \omega \cdot d_3 \end{array}\right\} \Rightarrow \frac{d_1}{d_3} = \omega^2 \tag{3.28}$$

And,

$$\left.\begin{array}{r} \frac{d_2}{d_3} = \omega \rightarrow d_3 = \frac{d_2}{\omega} \\ \frac{d_3}{d_4} = \omega \rightarrow d_3 = \omega \cdot d_4 \end{array}\right\} \Rightarrow \frac{d_2}{d_4} = \omega^2 \rightarrow \frac{d_1}{d_4} = \omega^3 \tag{3.29}$$

Therefore, the ratio of the first interval to the last one is;

$$\frac{d_1}{d_{n+1}} = \omega^n \tag{3.30}$$

On the other hand, according to the evaluated sequence of intervals for n experiments which are presented in Eq. (3.18) as $d_1 = d_2 + d_3$, we can write;

$$\frac{d_1}{d_3} = \frac{d_2}{d_3} + 1 \tag{3.31}$$

By applying Eq. (3.28) into Eq. (3.31), it can be concluded that;

$$\omega^2 = \omega + 1$$

That is;

$$\omega^2 - \omega - 1 = 0 \tag{3.32}$$

As the positive root of this equation is $\omega = 1.618$, we can say the ratio of each interval to the next interval is the constant value of 1.618. Therefore, the length of uncertainty range at this location is a multiple of d_1, as shown below;

$$d_2 = \frac{d_1}{\omega} = \frac{d_1}{1.618} = 0.618\ d_1$$

And so,

$$d_{n+1} = \frac{d_n}{\omega} = \frac{d_n}{1.618} \Rightarrow d_{n+1} = 0.618\ d_n \tag{3.33}$$

In addition to the range of uncertainty, the location of the *n*th experiment can be evaluated as;

$$d_{n+1} = d_n - d^*_{n+1} \tag{3.34}$$

where,

$$d^*_{n+1} = 0.382\ d_n \tag{3.35}$$

For example, the range of uncertainty for $n = 2$ is $d_2 = 0.618\ d_1$, and based on Eq. (3.34), we have; $d_2 = d_1 - d^*_2$. Therefore, it can be concluded that $d^*_2 = 0.382\ d_1$.

All necessary steps to find an optimum value based on the GSM are presented in the following example.

Example 3.5 Using the GSM with the convergence criterion of $\delta = 0.045$, solve example 3.4 and compare the results with the outcome of Fibonacci approach.

$$\begin{aligned} &\mathrm{f}(x) = 3x^2 - 4x + 5.5 \\ &x \in [0.0, 4.0] \end{aligned}$$

Solution: The procedure in the GSM is same as the Fibonacci approach and includes the following steps;
Step 1: Put $n = 1$ and determine d^*_2 and d_2 as;

$$d^*_2 = 0.382 d_1 = 0.382 \times 4.0 = 1.528$$

And,

$$\begin{aligned} d_2 &= d_1 - d^*_2 = 4.0 - 1.528 = 2.472 \\ d_2 &= 0.618 d_1 = 0.618 \times 4 = 2.472 \end{aligned}$$

Now, the values of x_1 and x_2 should be calculated as follows;

$$\begin{aligned} x_1 &= x_l + d^*_2 \rightarrow x_1 = 0 + 1.528 = 1.528 \\ x_2 &= x_u - d^*_2 \rightarrow x_1 = 4.0 - 1.528 = 2.472 \end{aligned}$$

And so,

$$\begin{aligned} f(x_1) &= f(1.528) = 6.392 \\ f(x_2) &= f(2.472) = 13.944 \end{aligned}$$

Comparing the two functions $f(x_1)$ and $f(x_2)$ we see $f(x_1)<f(x_2)$, and so, the interval $[x_2, x_u] = [2.472, 4.0]$ is discarded by using the unimodality assumption and the range of uncertainty reduces to $[0.0, 2.472]$. As the convergence criterion (δ) is still larger than specified criteria, we have to continue calculations to approach closer to the specified convergence criterion.

$$\delta = x_u - x_l = 2.472 - 0.0 = 2.472 > 0.045$$

Step 2: In this step, d_3^*, d_3, the interior point x_3, and the function $f(x_3)$ are evaluated as follows;

$$d_3^* = 0.382 d_2 = 0.382 \times 2.472 = 0.944$$

And,

$$d_3 = d_2 - d_3^* = 2.472 - 0.944 = 1.528$$
$$d_3 = 0.618 d_2 = 0.618 \times 2.472 = 1.528$$

And then,

$$\begin{aligned} x_3 &= x_l + (x_u - x_1) = 0.0 + (2.472 - 1.528) \\ &= 0.944 \rightarrow f(x_3) = 4.397 \end{aligned}$$

As $f(x_1) > f(x_3)$, the interval [1.528,2.472] is eliminated and the new interval is [0.1,528]. Now, we need to check the convergence criterion (δ) as follows;

$$\delta = 1.528 - 0.0 = 1.528 > 0.04$$

Based on the evaluated value of δ, we still need to continue the calculations. All necessary steps to solve this problem using the GSM are presented in the Table 3.5.

The minimum value $f(x^*)$ can be computed as;

$$\begin{aligned} x^* &= \frac{0.6400 + 0.6800}{2} = 0.6600 \\ &\Rightarrow f(x^*) = f(0.6600) = 4.1668 \end{aligned}$$

and the convergence criterion (δ) in the last step is;

$$\delta = 0.68 - 0.64 = 0.04 < 0.045$$

As it can be seen, the convergence criterion is met and the answer is in an acceptable range. To compare the results of Fibonacci and GSM, the ratio of the final to the initial range of uncertainty in the same number of iteration (here $n = 8$) can be applied as;

For Fibonacci method

$$\frac{d_n}{d_1} = \frac{d_8}{d_1} = \frac{0.1176}{4.0} = 0.0294$$

Table 3.5 The golden section method procedure

n	d_n^*	d_n	x_i	x_j	f(x$_i$)	f(x$_j$)	Greater f(x)	x_l	x_u
2	1.5280	2.4720	1.5280(x_1)	2.4720(x_2)	6.3924	13.9444	f(x_2)	0.0000	2.4720
3	0.9443	1.5277	1.5280(x_1)	0.9440 (x_3)	6.3924	4.3974	f(x_1)	0.0000	1.5280
4	0.5836	0.9441	0.9440(x_3)	0.5840 (x_4)	4.3974	4.1872	f(x_3)	0.0000	0.9440
5	0.3607	0.5835	0.5840(x_4)	0.3600(x_5)	4.1872	4.4488	f(x_5)	0.3600	0.9440
6	0.2229	0.3606	0.5840(x_4)	0.7200(x_6)	4.1872	4.1752	f(x_4)	0.5840	0.9440
7	0.1377	0.2228	0.7200(x_6)	0.8080(x_7)	4.1752	4.2266	f(x_7)	0.5840	0.8080
8	0.0851	0.1377	0.7200(x_6)	0.6720(x_8)	4.1752	4.1668	f(x_6)	0.5840	0.7200
9	0.0526	0.0851	0.6720(x_8)	0.6320(x_9)	4.1668	4.1703	f(x_9)	0.6320	0.7200
10	0.0325	0.0526	0.6720(x_8)	0.6800(x_{10})	4.1668	4.1672	f(x_{10})	0.6320	0.6800
11	0.0201	0.0325	0.6720(x_8)	0.6400(x_{11})	4.1668	4.1688	f(x_{11})	0.6400	0.6800

and for Golden Section Method

$$\frac{d_n}{d_1} = \frac{d_8}{d_1} = \frac{0.138}{4.0} = 0.0344$$

Based on the computed values, the Fibonacci technique is more efficient than the GSM technique in reducing the range of uncertainty. However, the GSM allows to continue the searching process to meet the convergence criterion, and so, the ratio of length of final interval to the initial one in this method is;

$$\frac{d_n}{d_1} = \frac{d_{11}}{d_1} = \frac{0.033}{4.0} = 0.0081$$

3.2.3 Newton Method (Approximation Technique)

Consider the polynomial function f(x) as differentiable function with relative minimum or maximum at $x = x^*$, we can write;

$$\left.\frac{d\mathrm{f}(x)}{dx}\right|_{x=x^*} = \mathrm{f}'(x^*) = 0 \tag{3.36}$$

In other words, the first derivative of f(x) demonstrates how whether a function is decreasing or increasing. The second derivative tells us whether the first derivative is raising or declining, and so, it can be concluded that;

$$\begin{aligned} \left.\frac{d^2\mathrm{f}(x)}{dx^2}\right|_{x=x_0} &= \mathrm{f}''(x_0) > 0 \rightarrow \text{there is a minimum} \\ \left.\frac{d^2\mathrm{f}(x)}{dx^2}\right|_{x=x_0} &= \mathrm{f}''(x_0) < 0 \rightarrow \text{there is a maximum} \end{aligned} \tag{3.37}$$

Based on the Taylor's theorem, if function **f**(**x**) and its derivatives are known at the $\boldsymbol{x} = \boldsymbol{x_0}$, then;

$$\begin{aligned} \mathrm{f}(x) &= \frac{\mathrm{f}(x_0)}{0!} + \frac{\mathrm{f}'(x_0)}{1!}(x - x_0) + \frac{\mathrm{f}'(x_0)}{2!}(x - x_0)^2 + \cdots + \frac{\mathrm{f}^n(x_0)}{n!}(x - x_0)^n + R_n(x) \\ &= \left[\sum_{k=0}^{n} \frac{\mathrm{f}^k(x_0)}{k!}(x - x_0)^k\right] + R_n(x) \end{aligned} \tag{3.38}$$

where, R_n is the error term.

The Taylor series converge to the f(x), if and only if $\lim_{n\to\infty} R_n(x) = 0.0$. By using the first two terms of Taylor's series and setting the first derivative of this series equal to zero to find the minimum value function f(x), we can write;

$$\mathrm{f}'(x) = \mathrm{f}'(x_0) + \mathrm{f}''(x_0)(x - x_0) = 0$$

And so,

$$x = x_0 - \frac{\mathrm{f}'(x_0)}{\mathrm{f}''(x_0)}$$

$$\text{or} \tag{3.39}$$

$$x_{k+1} = x_k - \frac{\mathrm{f}'(x_k)}{\mathrm{f}''(x_k)}$$

where, x_0 (or x_k) denotes an approximation to the minimum value of f(x).

The Newton's method uses the Eq. (3.39) iteratively till meet the convergence criterion which is defined as a very small value such as ε.

Example 3.6 Apply Newton method to find the minimum of the function f(x) shown in Fig. 3.13 considering $\varepsilon = 0.01$, and two starting points $x_0 = 0.1$ and $x_0 = 2.5$.

$$\mathrm{f}(x) = 3x^2 - 4\ln(x)$$
$$x \in [0.1, 2.5]$$

Solution: Based on Eq. (3.39), we need to evaluate the first and second derivative of function f(x) as;

$$\mathrm{f}'(x) = 6x - \frac{4}{x}$$

$$\mathrm{f}''(x) = 6 + \frac{4}{x^2}$$

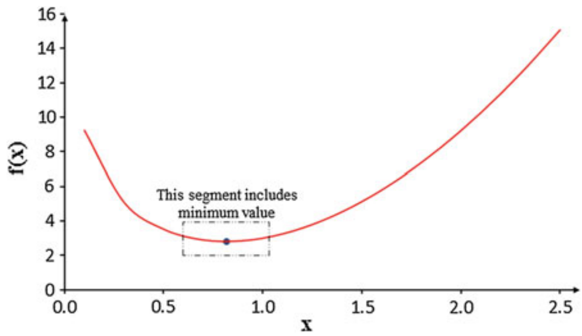


Fig. 3.13 Function f(x) in interval [0.1,2.5]

Step 1: Apply the first starting point $x_{k=0} = 0.2$ and calculate $x_{k+1} = x_1$ as;

$$f'(x) = 6x - \frac{4}{x} = 6 \times 0.1 - \frac{4}{0.1} = -39.4$$

$$f''(x) = 6 + \frac{4}{x^2} = 6 + \frac{4}{0.1^2} = 406.0$$

and so,

$$x_1 = x_0 - \frac{f'(x_0)}{f''(x_0)} = 0.1 - \left(\frac{-39.4}{406.0}\right) = 0.197$$

Now, we need to check the convergence criterion;

$$|f'(x_{k+1})| = |f'(0.197)| = |-19.118| > 0.01$$

Step 2: Put $k = 1$ and then calculate x_2 as follows;

$$f'(x) = 6 \times 0.197 - \frac{4}{0.197} = -19.118$$

$$f''(x) = 6 + \frac{4}{0.197^2} = 109.020$$

and so,

$$x_2 = x_1 - \frac{f'(x_1)}{f''(x_1)} = 0.197 - \left(\frac{-19.118}{109.020}\right) = 0.372$$

The convergence check shows the procedure should still be continued.

$$|f'(0.372)| = |-8.507| > 0.01$$

Table 3.6 and Fig. 3.14 show the entire procedure for evaluating minimum of f(x), and the path that the function approaches from the first starting point (0.1) to the minimum using the Newton method.

As it can be seen from this table, the convergence criterion is already met at $k = 5$, but the procedure is continued up to $k = 6$, where the $f'(x) = 0$.

Table 3.6 The entire procedure of evaluation minimum of $f(x)$ for starting point 0.1

k	x_k	f(x)	f′(x)	f″(x)	x_{k+1}	Convergence check
0	0.100	9.240	−39.400	406.000	0.197	Continue
1	0.197	6.614	−19.118	109.023	0.372	Continue
2	0.372	4.367	−8.507	34.843	0.617	Continue
3	0.617	3.075	−2.788	16.523	0.785	Continue
4	0.785	2.817	−0.382	12.486	0.816	Continue
5	0.816	2.811	−0.007	12.009	0.816	Stop
6	0.816	2.811	0.000	12.000	0.816	–

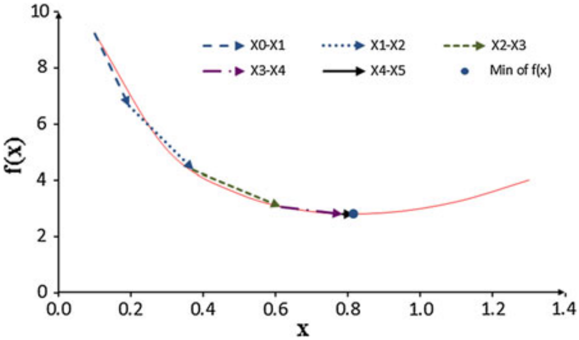


Fig. 3.14 The approaching routes from starting point 0.1 toward the minimum value

All of the necessary steps to calculate the minimum value and also the routes of approaching toward the minimum point using the Newton method based on the starting point 2.5 are presented in Table 3.7 and Fig. 3.15.

3.3 Unconstrained Optimization Methods

The main concern of unconstrained optimization is finding the optimum point of nonlinear functions when there is no constraint and restriction on the decision variables. These types of problems commonly arise in many areas of engineering and science including economic, physics, hydrosystem engineering, environmental engineering, mechanical engineering, and so on. The following section focuses on the unconstrained optimization problems and looks into three different methods to solve nonlinear and unconstrained optimization problems;

1. Random search methods that is placed in direct search category (see Table 3.2),
2. Univariate method which is a member of direct search methods (see Table 3.2),
3. Steepest descent method from descent category (see Table 3.2).

Table 3.7 The entire procedure of evaluation minimum of f(x) for starting point 2.5

k	x_k	f(x)	f′(x)	f″(x)	x_{k+1}	Convergence check
0	2.500	15.085	13.400	6.640	0.482	Continue
1	0.482	3.617	−5.408	23.223	0.715	Continue
2	0.715	2.876	−1.307	13.828	0.809	Continue
3	0.809	2.811	−0.086	12.107	0.816	Continue
4	0.816	2.811	0.000	12.000	0.816	Stop

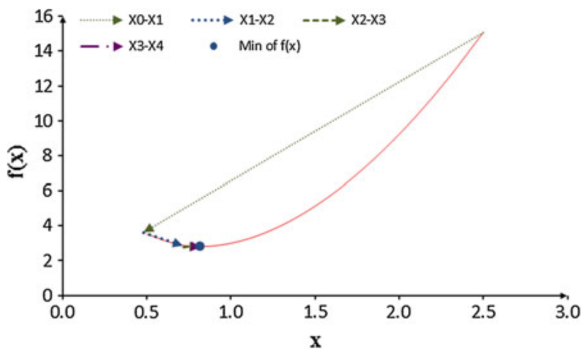


Fig. 3.15 The approaching routes from starting point 2.5 toward the minimum value

3.3.1 Random Search Method

Random search methods are known as numerical optimization techniques that use various types of randomness algorithms in the form of a pseudo-random number generator to find the optimum value for a continuous or discrete objective function. For example, Monte Carlo method is an important numerical technique that commonly is used to generate sequences of random numbers. This method is one of the most famous and widely used numerical methods from the early of 1940. With the remarkable increase in computer capabilities and the development of variance reduction schemes in recent years, applying this method has been increased in different scientific fields. The basic element of this method is iteration and generation of random variables from a specific range. In other words, it is a numerical simulation which replicates stochastic input random variables from desired probability distribution (Goodarzi et al. 2012). The random search methods are capable to handle large optimization problems, it converges quickly to the global optimal solutions, and they are relatively easy to use in various types of optimization problems. Three of the most important random search methods are; random jumping method, random walk method, and random walk method with directional exploitation. The random jumping method with appropriate example is illustrated in the following section.

The random jumping method is one of the simplest techniques to find the minimum value of function f(x) subject to the lower and upper bounds x_{l_i} and x_{u_i},

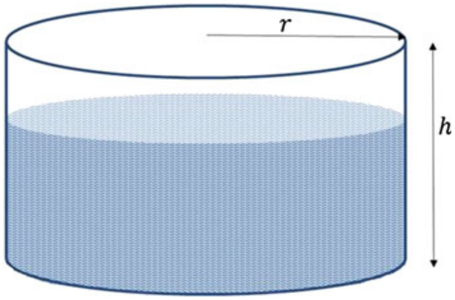


Fig. 3.16 Open-top water cylindrical tank

respectively, for design variable x_i, $i = 1, 2, 3, \ldots, n$ in which $x_{l_i} < x_i < x_{u_i}$. The main step in this technique is generating different sets of random numbers R_i which are uniformly distributed between 0 and 1, and then, evaluating function f(x) at the points x_i, $i = 1, 2, 3, \ldots, n$ to find the smallest f(x) as minimum value.

$$x_i = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} x_{l_1} + R_1(x_{u_1} - x_{l_1}) \\ x_{l_2} + R_2(x_{u_2} - x_{l_2}) \\ \vdots \\ x_{l_n} + R_n(x_{u_n} - x_{l_n}) \end{bmatrix} \tag{3.40}$$

It is important to note that although the problem is unconstrained, two upper and lower bounds are considered on the vector x_i. The following example shows how this method works in finding the minimum value of the desired function.

Example 3.7 **Water Cylindrical Tank**
Consider an open-top water cylindrical tank that can be used for storing water in the emergency cases in small urban area. Assume the required volume (V) for this cylindrical tank is 200 m^3 with the dimensions r and h as radius and height of the cylinder, respectively (Fig. 3.16). As the cost of building the open-top cylindrical water tank is a function of the amount of material used, find the minimum dimensions of the tank based on the random jumping method to minimize the cost of building the water tank.

Solution: To minimize the cost of building an open-top cylindrical water tank, we need to minimize the outside surface area of the tank which is function of radius and height as follows;

$$\mathrm{f}(r, h) = A = \pi r^2 + 2\pi r h$$

In other words, the objective function here is the outside surface area and the decision variables are r and h. As the volume of this cylindrical tank is already known, the variable h also can be written based on radius of cylinder r as follow;

$$V = \pi r^2 h \rightarrow h = \frac{V}{\pi r^2} = \frac{200}{\pi r^2}$$

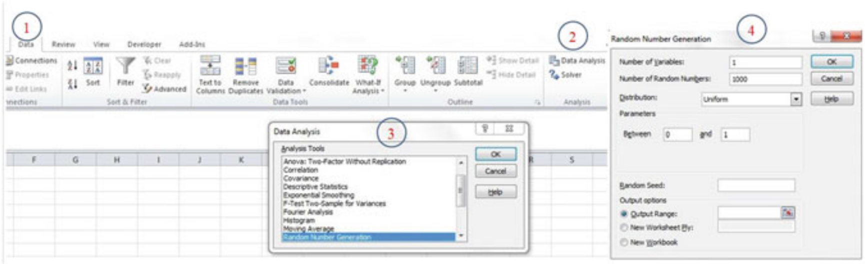


Fig. 3.17 Generate random numbers using Excel

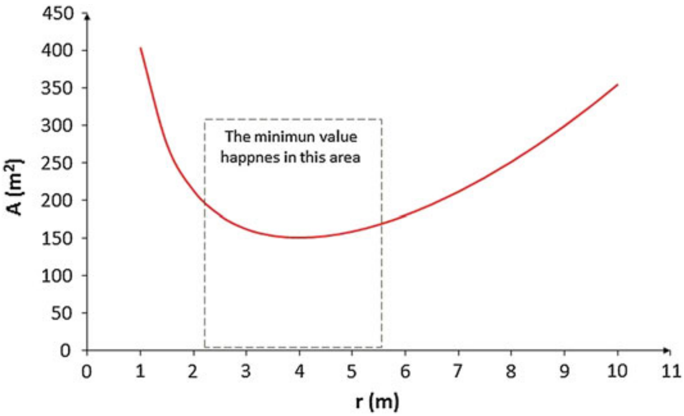


Fig. 3.18 The outside surface area versus radius of cylindrical tank

Therefore, the number of decision variables is reduced to one variable and so, the objective function can be written as;

$$\mathrm{f}(r) = A = \pi r^2 + 2\pi r\left(\frac{200}{\pi r^2}\right) = \pi r^2 + \frac{400}{r}$$

When the objective function is established, we need to generate a large number of random numbers that are uniformly distributed between 0 and 1. The Data Analysis tool in Excel can be used to produce a specified numbers of random variables. Figure 3.17 shows the necessary steps to apply available tool in Excel for generating random numbers based on the uniform distribution.

Figure 3.18 is presented to show the trend of varying the outside surface area of cylindrical water tank versus the radius of cylinder as the main decision variable in this problem, and also to graphically track the approximate location of the minimum point.

This problem is solved for six different conditions to see the effect of number of generated random numbers and different lower and upper bound of the problem on

Table 3.8 The results for 100 generated random numbers and $r_l = 1$ and $r_u = 10$

Upper bound (r_l)	Lower bound (r_u)	i (iteration)	Generated random numbers (sorted)	$r = x_l + R_i(x_u - x_l)$	f(r)
1	10	1	0.037	1.337	304.790
		2	0.049	1.442	283.987
		3	0.059	1.533	268.377
			⋮	⋮	⋮
		22	0.315	3.832	150.515
		23	0.334	4.010	150.268
		24	0.347	4.120	150.414
			⋮	⋮	⋮
		98	0.981	9.833	344.449
		99	0.995	9.954	351.436
		100	0.998	9.981	353.046
			minimum		150.268

the optimal solution in random jumping method. To compare the estimated results with the analytic solution, the minim value of the outside surface area is evaluated as;

$$\frac{df(r)}{dr} = 0 \rightarrow 2\pi r - \frac{400}{r^2} = 0$$
$$\Rightarrow r = 3.993(\text{m}) \quad \text{and} \quad f(3.993) = 150.265\ (\text{m}^2)$$

In addition, the height of cylinder simply can be calculated as;

$$h = \frac{200}{\pi r^2} = 3.993$$

The following tables show the achieved results based on the random jumping method in various conditions. The presented results in Table 3.8 are based on 100 generated random numbers considering 1 and 10 as the lower and upper bounds, respectively.

Table 3.9 illustrates the results of optimization analysis for 1,000 random numbers considering 1 as the lower bound and 10 as the upper bound.

The final outcomes for all considered condition in this problem are briefly presented in the Table 3.10. According to the results, using larger numbers of sample sizes resulted in increasing the precision of calculations, while, expanding the interval of upper and lower bounds gives the results with lower precision.

In general, the random search methods have lots of advantages including their simplicity to use, applicability for discontinuous and non-differentiable objective functions in addition to continues functions, and capability to find the global minimum while there are several relative minima.

Table 3.9 The results for 1,000 generated random numbers and $r_l = 1$ and $r_u = 10$

Upper bound (r_l)	Lowe bound (r_u)	i (iteration)	Generated random numbers (sorted)	$r = x_l + R_i(x_u - x_l)$	f(r)
1	10	1	0.0006	1.005	401.098
		2	0.0008	1.007	400.991
		3	0.0012	1.011	400.350
			⋮	⋮	⋮
		346	0.3315	3.983	150.266
		347	0.3318	3.986	150.265
		348	0.3330	3.997	150.265
			⋮	⋮	⋮
		998	0.9961	9.965	352.095
		999	0.9981	9.983	353.158
		1,000	0.9987	9.988	353.449
			minimum		150.265

Table 3.10 The final results of random jumping method in various conditions

Upper bound (r_l)	Lower bound (r_u)	n (number of iteration)	Decision variable r	min f(r)
1	10	100	4.009	150.268
1	20	100	4.041	150.287
1	40	100	3.672	151.292
1	10	1,000	3.996	150.265
1	20	1,000	3.991	150.265
1	40	1,000	4.001	150.266

3.3.2 Univariate Method

The univariate method is a simple direct search method that is based on choosing n fixed search directions for the objective function f(x) and performs a series of iterations to reach the optimal point. Based on this technique, a multivariable function is reduced to a single variable function and a sequence of one-dimensional minimization search will be performed by changing one variable at the time while holding the remaining variables constant. Afterward, the selected starting point of non-constant variable is improved and the objective function is estimated using this new value. In the next step, the process is repeated for the second variable to improve its initial estimation whereas keeping the other constants. This process should be continued for all variables and change the objective function step by step to meet the convergence criterion to reach the minimum point. It is important to note that each iteration of a univariate search method includes a search direction d_k and a step length α_k in the kth iteration in which;

$$x_{i,k+1} = x_{i,k} + \alpha_k d_k \tag{3.41}$$

In general, the search direction needs to be a descent direction to reduce the objective function in the direction of search. Therefore, the value of objective function for the new points should be less than or equal to the previous points. The search direction can be obtained as;

$$[d_k]^T = \begin{cases} (1, 0, 0, \ldots, 0) & \text{for } k = 1 \\ (0, 1, 0, \ldots, 0) & \text{for } k = 2 \\ \vdots & \\ (0, 0, 0, \ldots, 1) & \text{for } k = n \end{cases} \tag{3.42}$$

After this, we need to determine whether the decreasing trend of objective function is in the positive or negative direction by choosing an examination length (ε), and estimating;

$$\begin{aligned} \mathrm{f}_k &= \mathrm{f}\left(x_{i,k}\right) \\ \mathrm{f}_+ &= \mathrm{f}(x_{i,k} + \varepsilon d_k) \\ \mathrm{f}_- &= \mathrm{f}(x_{i,k} - \varepsilon d_k) \end{aligned} \tag{3.43}$$

In the next step, the following conditions should be checked;

1. If $\mathrm{f}_+ < \mathrm{f}_\mathrm{k}$, d_k is the correct direction and so, $\mathrm{f}_k = \mathrm{f}(x_{i,k} + \alpha_k d_k)$,
2. If $\mathrm{f}_- < \mathrm{f}_\mathrm{k}$, $-d_k$ is the correct direction and hence, $\mathrm{f}_k = \mathrm{f}(x_{i,k} + \alpha_k d_k)$, and
3. If $\mathrm{f}_\mathrm{k} < \mathrm{f}_+$ and $\mathrm{f}_k < \mathrm{f}_-$, the search procedure should be stopped.

The step length α_k which is a positive scalar should be determined based on an optimization analysis as follows;

$$\alpha_k^* = \min_{\alpha} \mathrm{f}(x_{i,k} + \alpha_k d_k) \tag{3.44}$$

Then, the previous point is replaced with the estimated new value;

$$\begin{aligned} x_{i,k+1} &= x_{i,k} + \alpha_k^* d_k \\ \mathrm{f}_{k+1} &= f\left(x_{i,k+1}\right) \end{aligned} \tag{3.45}$$

The process of finding minimum value will be terminated if the convergence condition is satisfied.

Example 3.8 Minimize f(x) using univariate method with the starting points $x_1 = 0$ and $x_2 = 0$, and $\varepsilon = 0.05$.

$$\mathrm{f}(x) = 1.25x_1 - 0.45x_2 + x_1^4 + x_1x_2 + x_2^2$$

Solution: The following sections include the necessary steps in details to find the optimal solution.

Step 1: $K = 1$

(a) The first direction for search is chosen as $d_{k=1} = (1, 0)$ by considering the starting points $x_{i,k} = (0, 0)$. Then, the functions f_1, f_+, and f_- are computed as;

$$f_1 = f(x_{i,1}) = 1.25(0) - 0.45(0) + (0)^4 + (0) \times (0) + (0)^2 = 0$$

and

$$\begin{cases} x_{1,1} + \varepsilon d_1 = 0 + 0.05 \times 1 = 0.05 \\ x_{2,1} + \varepsilon d_1 = 0 + 0.05 \times 0 = 0 \end{cases} \Rightarrow f_+ = f(0.05, 0) = 0.0625$$

$$\begin{cases} x_{1,1} + \varepsilon d_1 = 0 - 0.05 \times 1 = -0.05 \\ x_{2,1} + \varepsilon d_1 = 0 - 0.05 \times 0 = 0 \end{cases} \Rightarrow f_- = f(-0.05, 0) = -0.0624$$

(b) Now, we need to determine the correct direction of d_k. As $f_- < f_1$, the correct direction is $-d_1$ and so, $f_1 = f(x_{i,1} - \alpha_1 d_1)$.

$$\begin{cases} x_{1,1} - \alpha_1 d_1 = 0 - \alpha_1 \times 1 = -\alpha_1 \\ x_{2,1} - \alpha_1 d_1 = 0 - \alpha_1 \times 0 = 0 \end{cases}$$

and so,

$$\begin{aligned} f(-\alpha_1, 0) &= 1.25(-\alpha_1) - 0.45(0) + (-\alpha_1)^4 + (-\alpha_1)(0) + (0)^2 \\ &= -1.25\alpha_1 + \alpha_1^4 \end{aligned}$$

The optimum value of α can be obtained by setting the first derivative equal to zero as;

$$\frac{df}{d\alpha} = 0 \Rightarrow -1.25 + 4\alpha_1^3 = 0 \rightarrow \alpha_1 = \alpha_1^* = -0.6786$$

(c) In this step, the calculated step length α_1 is substituted in the appropriate equations and the optimum value of f(x) will be estimated in the first iteration.

$$\begin{cases} x_{1,2} = 0 - (-0.6786) \times 1 = 0.6786 \\ x_{2,2} = 0 - (-0.6786) \times 0 = 0 \end{cases} \Rightarrow f_2 = f(0.6786, 0) = 1.0603$$

As it can be seen the variable x_2 is held constant and the variable x_1 is improved in the first iteration.

Step 2: $K = 2$

(a) The second direction is selected as $d_2 = (0,1)$ by considering the starting point of $x_{i,2} = (0.6786, 0)$. Therefore, the functions, f_+, and f_- are;

$$\begin{cases} x_{1,2} + \varepsilon d_2 = 0.6786 + 0.05 \times 0 = 0.6786 \\ x_{2,2} + \varepsilon d_2 = 0 + 0.05 \times 1 = 0.05 \end{cases} \Rightarrow f_+ = f(0.6786, 0.05) = 1.0742$$

$$\begin{cases} x_{1,2} - \varepsilon d_2 = 0.6786 - 0.05 \times 0 = 0.6786 \\ x_{2,2} - \varepsilon d_2 = 0 - 0.05 \times 1 = -0.05 \end{cases} \Rightarrow f_- = f(-0.05, 0) = 1.0513$$

(b) As $f_- < f_2$, the correct direction is $-d_2$ and so, $f_2 = f\left(x_{i,2} - \alpha_2 d_2\right)$.

$$\begin{cases} x_{1,2} - \alpha_2 d_2 = 0.6786 - \alpha_2 \times 0 = 0.6786 \\ x_{2,2} - \alpha_2 d_2 = 0 - \alpha_2 \times 1 = -\alpha_2 \end{cases}$$

and so,

$$f(0.6786, -\alpha_2) = 1.25(0.6786) - 0.45(-\alpha_2) + (0.6786)^4 + (0.6786)(-\alpha_2) + (-\alpha_2)^2 = \alpha_2^2 - 0.2286\alpha_2 + 1.0603$$

After that, the optimum value of α can be obtained as;

$$\frac{df}{d\alpha} = 0 \Rightarrow 2\alpha_2 - 0.2286 = 0 \rightarrow \alpha_2 = \alpha_2^* = 0.1143$$

(c) Finally, the optimum of f(x) can be estimated as;

$$\begin{cases} x_{1,3} = 0.6786 - 0.1143 \times 0 = 0.6786 \\ x_{2,3} = 0 - 0.1143 \times 1 = -0.1143 \end{cases} \Rightarrow f_3 = f(0.6786, -0.1143) = 1.0472$$

As it can be seen, the variable x_1 is fixed in the second iteration while, the variable x_2 has been changed. The entire procedure of finding the minimum value of function f(x) are shown in the Table 3.11. The values of f_+ and f_- are both greater than f_k in the iteration 8, and so, the optimization process is terminated in this step and the value of -0.9831 is considered as minimum of f(x) at the $x_1 = -0.7745$ and $x_2 = 0.6084$.

Table 3.11 The process of finding minimum value using univariate search method

k	d_k	$x_{i,k}$	f_k	f_+	f_-	Correct direction	α_k^*	$x_{i,k+1}$	f_{k+1}
1	(1,0)	(0,0)	0	0.0625	−0.0624	$-d_1$	−0.6786	(0.6786, 0)	1.0603
2	(0,1)	(0.6786, 0)	1.0603	1.0742	1.0513	$-d_2$	0.1143	(0.6786, −0.1143)	1.0472
3	(1,0)	(0.6786, −0.1143)	1.0472	1.1737	0.9345	$-d_3$	1.3358	(−0.6572, −0.1143)	−0.4953
4	(0,1)	(−0.6572, −0.1143)	−0.4953	−0.5596	−0.4260	d_4	0.6679	(−0.6572, 0.5536)	−0.9414
5	(1,0)	(−0.6572, 0.5536)	−0.9414	−0.9018	−0.9680	$-d_5$	0.1096	(−0.7668, 0.5536)	−0.9799
6	(0,1)	(−0.7668, 0.5536)	−0.9799	−0.9829	−0.9719	d_6	0.0548	(−0.7668, 0.6084)	−0.9829
7	(1,0)	(−0.7668, 0.6084)	−0.9829	−0.9717	−0.9764	$-d_7$	0.0077	(−0.7745, 0.6084)	−0.9831
8	(0,1)	(−0.7745, 0.6084)	−0.9831	−0.9810	−0.9802	–	–	–	–

3.3.3 Steepest Descent Method

The steepest descent method or the Cauchy's method is a well-known and old optimization technique which was proposed by Cauchy (1847) for solving unconstrained and non-linear minimization problems. The main idea behind this approach is considering the negative of gradient of objective function f(x) at any individual point as search direction *d*, in which this direction results in maximum descent in the function f(x). Hence, this method is called the steepest descent or gradient descent method. Although this method is very simple and also recognized as fundamental approach in the developing optimization theory, it is not known as very efficient method and the convergence rate also can be very slow in the most of real problems. The necessary steps to find the minimum of continuously differentiable function f(x_i) in which $x_i = x_1, x_1, \ldots, x_n$, using the steepest descent method are;

1. Choose an initial starting point $x_{i,k}$ in the first iteration ($k = 1$),
2. Determine the search direction d_k as;

$$d_{i,k} = -\nabla \mathrm{f}\left(x_{i,k}\right) = \left[\frac{\partial \mathrm{f}}{\partial x_{1,k}} \frac{\partial \mathrm{f}}{\partial x_{2,k}} \frac{\partial \mathrm{f}}{\partial x_{3,k}} \cdots \frac{\partial \mathrm{f}}{\partial x_{n,k}}\right]^T \tag{3.46}$$

3. Compute the optimal step length α_k along a given search direction as follow;

$$\begin{aligned} x_{i,k+1} &= x_{i,k} + \alpha_k^* d_k \\ \alpha_k^* &= \min_{\alpha} \mathrm{f}\left(x_{i,k} + \alpha_k d_k\right) \end{aligned} \tag{3.47}$$

4. Check the convergence criterion based on the following formula;

$$\left|\frac{\mathrm{f}\left(x_{i,k+1}\right) - \mathrm{f}\left(x_{i,k}\right)}{\mathrm{f}\left(x_{i,k}\right)}\right| \leq \varepsilon \tag{3.48}$$

where, ε is the convergence tolerance. If the convergence criterion is not met, the estimated points become the starting point and we need to lunch the search from step 2 using this new evaluated point.

Example 3.9 Apply the steepest descent method to solve the Problem 3.7 by considering $\varepsilon = 0.001$.

Solution: Based on the Problem 3.7, the objective function is;

$$\mathrm{f}(r) = \pi r^2 + \frac{400}{r}$$

Firstly, an initial starting point is chosen at the first iteration as $r_{k=1} = 0.4$ and then, the value of f(r) has been evaluated in this point as;

$$\mathrm{f}(r) = \pi \times 0.4^2 + \frac{400}{0.4} = 1{,}000.503$$

The gradient of f(r) in this point can be calculated as;

$$\nabla \mathrm{f}(r) = \frac{\partial f}{\partial r} = 2\pi r - \frac{400}{r^2}$$

and

$$\nabla \mathrm{f}(r_{k=1}) = \frac{\partial \mathrm{f}}{\partial r_1} = 2\pi(0.4) - \frac{400}{(0.4)^2} = -2{,}497.49$$

Therefore,

$$d_k = -\nabla \mathrm{f}(r_1) = 2,497.49$$

Now, we need to calculate the optimal step length α_k^* by minimizing $\mathrm{f}(\mathrm{r_k} + \alpha_\mathrm{k} \mathrm{d_k})$ as follow;

$$\alpha_1^* = \min_{\alpha} \mathrm{f}(r_1 + \alpha_1 d_1) = \min_{\alpha} \mathrm{f}(0.4 + \alpha_1 2{,}497.49)$$

in which the minimum value can be evaluated as;

$$\begin{aligned}
&\frac{d\mathrm{f}(0.4 + \alpha_1 2{,}497.49)}{d\alpha_1} = 0 \\
&\rightarrow \frac{d}{d\alpha_1}\left[\pi(0.4 + \alpha_1 2{,}497.49)^2 + \frac{400}{(0.4 + \alpha_1 2{,}497.49)}\right] = 0 \\
&\Rightarrow \alpha_1 = \alpha_1^* = 0.00143
\end{aligned}$$

and so,

$$\begin{aligned}
r_2 = r_1 + \alpha_1 d_1 &= 0.4 + 0.00143 \times 2{,}497.49 = 3.9714 \\
\Rightarrow \mathrm{f}(r_1 + \alpha_1 d_1) &= 150.2694
\end{aligned}$$

The convergence criterion is not met yet since;

$$\left|\frac{150.2694 - 1{,}000.503}{1{,}000.503}\right| = 0.8498 > 0.001$$

In the second iteration, set $k = 2$ and $r_2 = 3.9714$ and then, calculate the gradient as follow;

$$\nabla \mathrm{f}(r_{k=2}) = \frac{\partial \mathrm{f}}{\partial r_2} = 2\pi(3.9714) - \frac{400}{(3.9714)^2} = -0.4082$$
$$\Rightarrow d_2 = -\nabla \mathrm{f}(r_2) = 0.4082$$

Afterward,

$$\frac{df(3.9714 + \alpha_2 0.4082)}{d\alpha_2} = 0 \Rightarrow \alpha_2 = \alpha_2^* = 0.0537$$

Hence,

$$r_3 = r_2 + \alpha_2 d_2 = 3.9714 + 0.0537 \times 0.4082 = 3.9933$$
$$\Rightarrow \mathrm{f}(r_2 + \alpha_2 d_2) = 150.2650$$

The convergence criterion is met here as;

$$\left| \frac{150.2650 - 150.2694}{150.2694} \right| = 2.9193 \times 10^{-5} < 0.001$$

Therefore, it can be concluded that the minimum of f(r) is 150.2650 that happens in $r = 3.9933$.

Example 3.10 Apply the steepest descent method to solve the Problem 3.8 with the starting points $x_1 = 0$ and $x_2 = 0$, and $\varepsilon = 0.0005$.

$$\mathrm{f}(x) = 1.25x_1 - 0.45x_2 + x_1^4 + x_1 x_2 + x_2^2$$

Solution: The value of function f(x) in the selected starting points is;

$$\mathrm{f}(0,0) = 1.25(0) - 0.45(0) + (0)^4 + (0)(0) + (0)^2 = 0$$

Then, the gradient of $\mathrm{f}(\mathrm{x}_{\mathrm{i,k}})$ can be computed as;

$$\nabla f(x_{i,k}) = \frac{\partial \mathrm{f}}{\partial x} = \begin{bmatrix} \dfrac{\partial \mathrm{f}}{\partial x_{1,1}} \\ \dfrac{\partial \mathrm{f}}{\partial x_{2,1}} \end{bmatrix} = \begin{bmatrix} 4x_1^3 + x_2 + 1.25 \\ x_1 + 2x_2 - 0.45 \end{bmatrix}$$

Now, we need to evaluate the gradient of $\mathrm{f}(\mathrm{x}_{\mathrm{i,k}})$ at the selected points as follow;

$$\nabla \mathrm{f}(0,0) = \begin{bmatrix} 1.25 \\ -0.45 \end{bmatrix}$$

Therefore,

$$-\nabla \mathrm{f} = \begin{bmatrix} -d_{1,1} \\ -d_{2,1} \end{bmatrix} = \begin{bmatrix} -1.25 \\ 0.45 \end{bmatrix}$$

To estimate the optimal step length α_k^*, the minimum of $\mathrm{f}\left(\mathrm{x}_{\mathrm{i,k}} + \alpha_{\mathrm{k}}\mathrm{d}_{\mathrm{k}}\right)$ should be calculated as;

$$\alpha_1^* = \min_{\alpha} \mathrm{f}\left(x_{1,1} + \alpha_1 d_{1,1}, x_{2,1} + \alpha_1 d_{2,1}\right) = \min_{\alpha} \mathrm{f}(-1.25\alpha_1, 0.45\alpha_1)$$

and

$$\begin{aligned}
&\frac{d\mathrm{f}(-1.25\alpha_1, 0.45\alpha_1)}{d\alpha_1} = 0 \\
&\rightarrow \frac{d}{d\alpha_1}\left[2.4414\alpha_1^4 - 0.36\alpha_1^2 - 1.765\alpha_1\right] = 0 \\
&\Rightarrow \alpha_1 = \alpha_1^* = 0.6087
\end{aligned}$$

and hence,

$$\begin{cases} x_{1,2} = x_{1,1} + \alpha_1 d_{1,1} = 0 + 0.6087 \times (-1.25) = -0.7608 \\ x_{2,2} = x_{2,1} + \alpha_1 d_{2,1} = 0 + 0.6087 \times (0.45) = 0.2739 \end{cases}$$

and this calculations resulted in;

$$\mathrm{f}(-0.7608, 0.2739) = -0.8726$$

The convergence criterion is not met yet, and we need to proceed to the next iteration. All necessary calculations to find the minim value are presented in the Table 3.12.

3.4 Constrained Optimization Methods

A large class of optimization problems lies in constrained optimization problems that can be formulated as constrained maximization or minimization problems with complex constraints. In a constraint optimization problem, the feasible areas are bounded and there are restrictions on the points that the decision variables may be taken and we are interested in. In this case, the optimum value should be determined such that all constraints are simultaneously satisfied. In the following sections the Lagrange Multiplier and Generalized Reduced Gradient methods as two important constrained optimization techniques with a few examples associated with each method are presented.

3.4.1 Lagrange Multiplier Method

The Lagrange multiplier is an optimization technique to find the local minima or maxima of desired objective function which are subject to equality constraints. In other words, it is a mathematical tool to find the optimum value of differentiable

Table 3.12 The procedure of finding minimum value based on steepest descent method

k	x_1	x_2	$f(x_1,x_2)$	$d_{1,k}$	$d_{2,k}$	α_k^*	$x_{1,k+1}$	$x_{2,k+1}$	$f(x_{1,k+1}, x_{2,k+1})$	Convergence criterion
1	0.0000	0.0000	0.0000	−1.2500	0.4500	0.6087	−0.7609	0.2739	−0.8726	–
2	−0.7609	0.2739	−0.8726	0.2381	0.6630	0.3201	−0.6847	0.4861	−0.9514	0.0903
3	−0.6847	0.4861	−0.9514	−0.4522	0.1624	0.1914	−0.7712	0.5172	−0.9744	0.0242
4	−0.7712	0.5172	−0.9744	0.0677	0.1868	0.3110	−0.7502	0.5753	−0.9805	0.0063
5	−0.7502	0.5753	−0.9805	−0.1366	0.0496	0.1745	−0.7740	0.5840	−0.9824	0.0019
6	−0.7740	0.5840	−0.9824	0.0209	0.0561	0.3061	−0.7676	0.6011	−0.9829	0.0006
7	−0.7676	0.6011	−0.9829	−0.0419	0.0154	0.1705	−0.7748	0.6037	−0.9831	0.0002

functions by converting a constrained non-linear problem into an unconstrained problem by applying an augmented function known as Lagrangian function. The Lagrangian function for a minimization problem can be defined as;

$$L(x, \lambda) = \mathrm{f}(x) + \sum_{k=1}^{m} \lambda_k \mathrm{g}_k(x) = \mathrm{f}(x)\lambda^T \mathrm{g}(x) \tag{3.49}$$

where, λ is an $m \times 1$ vector of Lagrange multipliers, and g(x) is a vector of constraint equations.

To find the minimum of f(x) based on the Lagrange multiplier method, we need to set the following partial derivatives equal to zero;

$$\begin{aligned} &\frac{\partial L}{\partial x_i}(x^*, \lambda^*) = \frac{\partial f}{\partial x_i} + \sum_{k=1}^{m} \lambda_k \frac{\partial \mathrm{g}}{\partial x_i} = 0 \quad \text{for } i = 1, 2, \ldots, n \\ &\frac{\partial L}{\partial \lambda_k}(x^*, \lambda^*) = \mathrm{g}_k(x) = 0 \quad \text{for } k = 1, 2, \ldots, m \end{aligned} \tag{3.50}$$

in which, x^* is the optimum solution and λ^* is the associated Lagrange multipliers.

The following problems explain the procedure of using Lagrange multiplier method in finding the optimal solution of unconstrained optimization problems.

Example 3.11 Find the minimum of function f(x) based on the Lagrange multiplier method;

$$\min \mathrm{f}(x) = 6.34(x_1 - 0.5)^2 + 4.5(x_2 - 1.5)^2$$

Subject to:

$$x_1 + 3.35(x_2 - 1) = 0$$

Solution: In the first step, the Lagrangian function should be established as;

$$L(x, \lambda) = \left[6.34(x_1 - 0.5)^2 + 4.5(x_2 - 1.5)^2\right] + \lambda[x_1 + 3.35(x_2 - 1)]$$

Based on the Eq. (3.50), the derivatives can be calculated as;

$$\begin{aligned} \frac{\partial L}{\partial x_1} &= 12.68x_1 + \lambda - 6.34 = 0 \\ \frac{\partial L}{\partial x_2} &= 9x_2 + 3.35\lambda - 13.5 = 0 \\ \frac{\partial L}{\partial \lambda} &= x_1 + 3.35(x_2 - 1) = 0 \end{aligned}$$

Solving the above equations simultaneously resulted in $x_1^* = 0.3706$, $x_2^* = 0.8893$, and $\lambda^* = 1.6405$. The minimum of f(x) associated to these optimal solutions is 1.7840.

Example 3.12 Apply the Lagrange multiplier method to solve the Problem 3.7.

$$\mathrm{f}(r,h) = \pi r^2 + 2\pi rh$$

Subject to:

$$h - \frac{200}{\pi r^2} = 0$$

Solution: The Lagrangian function can be written as;

$$L(r,h,\lambda) = \left(\pi r^2 + 2\pi rh\right) + \lambda\left(h - \frac{200}{\pi r^2}\right)$$

Now, we need to calculate the derivatives as follows;

$$\begin{aligned}
\frac{\partial L}{\partial r} &= 2\pi(h+r) + \frac{400\lambda}{\pi r^3} = 0 \\
\frac{\partial L}{\partial h} &= 2\pi r + \lambda = 0 \\
\frac{\partial L}{\partial \lambda} &= h - \frac{200}{\pi r^2} = 0
\end{aligned}$$

The optimal solution for this problem, which are the minimum dimensions of tank, are; r= 3.992, h = 3.992, $\lambda = -25.088$. And the value of f(r) is 150.265.

3.4.2 Generalized Reduced Gradient Method

The Generalized Reduced Gradient (GRG) method is a popular optimization method to solve constrained nonlinear optimization problems. This method includes an implicit variable elimination procedure and it can be considered as extensions of the Simplex method for linear programming that is presented in Chap. 2. Based on the GRG algorithm, the inequality constraints are converted into equality constraints by adding nonnegative slack variables to constraint equations, and express the basic (or dependent) variables in terms of non-basic (or independent) variables to solve desired nonlinear optimization problem. Consider the following non-linear optimization problem as;

$$\min \mathrm{f}(x_i) \quad i = 1, 2, \ldots, n \tag{3.51}$$

Subject to:

$$\begin{aligned}
&g_k(x_i) \le 0 \quad k = 1, 2, \ldots, m \\
&x_l \le x_i \le x_u
\end{aligned} \tag{3.52}$$

where, $f(x_i)$ and $g_k(x_i)$ are continuously differentiable, *n* is the number of independent decision variables, *m* is the number of constrains, and *l* and *u* are the lower and upper bounds, respectively. In the first step, we need to add a non-negative slack variable to each of the inequality constraints. Therefore, the Eq. (3.51) and Eq. (3.52) can be written as;

$$\min f(x_i) \quad i = 1, 2, \ldots, n \tag{3.53a}$$

Subject to:

$$\begin{aligned} & g_k(x_i) + x_{n+k} = 0 \quad k = 1, 2, \ldots, m \\ & x_l \le x_i \le x_u \\ & x_{n+k} \ge 0 \end{aligned} \tag{3.53b}$$

The idea behind the GRG method is to convert a constraint problem into an unconstrained problem and eliminate some variables using the equality constraints. Based on this method, the independent variables are expressed in terms of *m* basic variable (x_b) and *n–m* non-basic variable (x_{nb}), and then, *m* constraint equations is solved for the all basic variables in terms of non-basic variables. All necessary steps to solve a non-linear optimization problem using the GRG method are presented in the following section, as;

1. Add nonnegative slack variables to all inequalities except the non-negativity inequalities,
2. Choose initial feasible trial values for non-basic variables,
3. Present objective function in terms of non-basic variables,
4. Determine the search direction ***d*** in each iteration as;

$$\boldsymbol{d} = \nabla_{nb} f = \left[\frac{\partial f}{\partial x_{nb}}\right] - \pi^T \left[\frac{\partial g}{\partial x_{nb}}\right] \tag{3.54}$$

in which ∇ is the gradient operator. Based on the steepest decent method, the search direction when applied to a function f is ∇f in which ∇ is the gradient operator and shows the direction of the maximum rate of increase in the objective function. It is important to note that $-\nabla f$ should be applied for minimization problems. The parameter in Eq. (3.54) π^T can be computed as;

$$\pi^T = \left[\frac{\partial f}{\partial x_b}\right]^T \left[\frac{\partial g}{\partial x_b}\right]^{-1} \tag{3.55}$$

in which, $[\partial g/\partial x_b] = \boldsymbol{B}$ is a nonsingular or regular matrix, and $[\partial F/\partial x_b]^T$ is the transpose of matrix $[\partial F/\partial x_b]$. It is important to note that nonsingular matrix such as $\boldsymbol{B}$ is a square invertible matrix which its inverse is denoted by $\boldsymbol{B}^{-1}$. Therefore Eq. (3.55) can be written as;

$$\pi^T = \left[\frac{\partial f}{\partial x_b}\right]^T \boldsymbol{B}^{-1} \tag{3.56}$$

5. Determine the optimal feasible step size β for the line search procedure by substituting the non-basic variables using the following equation to obtain the new objective function $f\left[(x_{nb})_{new}\right]$

$$\begin{aligned} (x_{nb})_{new} &= (x_{nb})_{old} + \beta \boldsymbol{d} \\ f\left[(x_{nb})_{new}\right] &= f\left[(x_{nb})_{old} + \beta \boldsymbol{d}\right] \end{aligned} \tag{3.57}$$

Now, we need to determine a basic variable which should be became non-basic one based on the optimal and feasible value of β. In other words, different values of β in a desired step must be used till one of the basic variables drops to zero, and then, use that variable as non-basic variable in the next iteration to find the new reduced objective function.

All of the aforementioned steps must be repeated till the optimum value is reached. In order to be more familiar with the main concepts behind GRG technique, the following problem discusses the principles of GRG technique to solve a non-linear optimization problem.

Example 3.13 Solve the following maximization problem using GRG method;

$$\max \, \mathrm{f}(x) = x_1^2 + 3x_1 - x_2$$

Subject to the below constraints;

$$\begin{cases} x_1^2 + 4x_2 \le 15 \\ 2x_1^2 - 3x_2 \le 20 \\ x_1 \ge 0, \ x_2 \ge 0 \end{cases}$$

Solution: The solution procedure which is including the necessary steps of GRG method, are presented in the following section.

Step 1: At the first, nonnegative slack variables are added to all inequalities except the non-negativity inequalities. Here there are two inequalities, and so, two slack variables like x_3 and x_4 should be added to the constraint equations as;

$$\begin{cases} g_1(x_{nb}, x_b) = x_1^2 + 4x_2 + x_3 - 15 = 0 \\ g_2(x_{nb}, x_b) = 2x_1^2 - 3x_2 + x_4 - 20 = 0 \end{cases}$$

Hence, the non-basic variables (x_{nb}) and basic variables (x_b) are (x_1, x_2) and (x_3, x_4), respectively, that can be shown as;

$$X = \begin{bmatrix} x_{nb} \\ x_b \end{bmatrix} = \begin{bmatrix} x_1, x_2 \\ x_3, x_4 \end{bmatrix}$$

Step 2: The desired initial feasible trial values should be selected for the current non-basic variables. The initial selected values in the first iteration are $x_{nb}^{i=1} = (x_1 = 2, x_2 = 2)$. (Fig. 3.19).

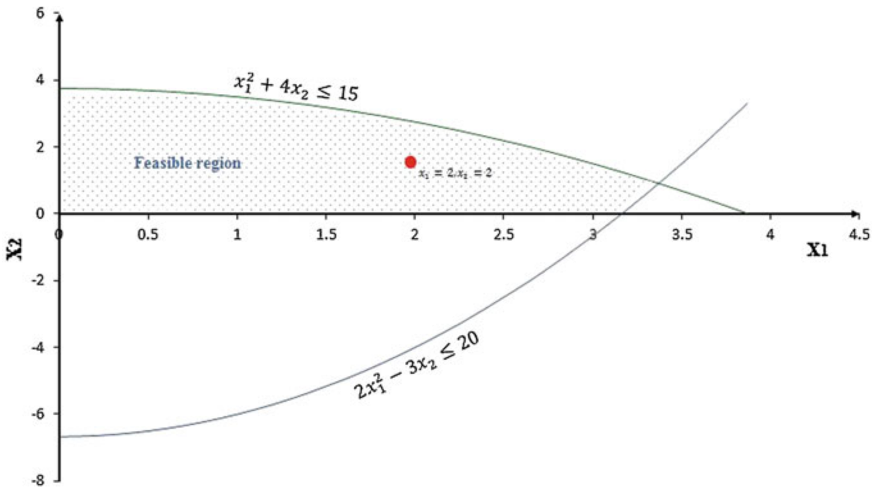


Fig. 3.19 The feasible region and the initial selected values for non-basic variables

The values of objective function and slack variables based on the selected initial values can be calculated as;

$$\mathrm{f}(x) = x_1^2 + 3x_1 - x_2 = 2^2 + (3 \times 2) - 2 = 8.0$$
$$x_3 = 15 - x_1^2 - 4x_2 = 15 - 2^2 - (4 \times 2) = 3.0$$
$$x_4 = 20 - 2x_1^2 + 3x_2 = 20 - \left(2 \times 2^2\right) + (3 \times 2) = 18.0$$

Step 3: In this step, the objective function should be presented in terms of non-basic variables as;

$$\max \mathrm{f}(x_{nb}) = \mathrm{f}(x_1, x_2) = x_1^2 + 3x_1 - x_2$$

It is important to note that the objective function which is expressed in terms of non-basic variables is called the *reduced objective*.

Step 4: To improve the current solution using the selected initial values for non-basic variables, we need to determine the search direction ***d*** in each step. The reduced gradient of f(x) (or ∇f), and consequently the search direction ***d*** are computed as;

$$\boldsymbol{d} = \nabla_{nb}\mathrm{f} = \left[\frac{\partial \mathrm{f}}{\partial x_{nb}}\right] - \pi^T \left[\frac{\partial g}{\partial x_{nb}}\right] \tag{3.58}$$

where,

$$\left[\frac{\partial \mathrm{f}}{\partial x_{nb}}\right] = \begin{bmatrix} \frac{\partial \mathrm{f}}{\partial x_1} \\ \frac{\partial \mathrm{f}}{\partial x_2} \end{bmatrix}_{(x_1=2, x_2=2)} = \begin{bmatrix} 2x_1 + 3 \\ -1.0 \end{bmatrix}_{(2,2)} = \begin{bmatrix} 7.0 \\ -1.0 \end{bmatrix}$$

and

$$\left[\frac{\partial g}{\partial x_{nb}}\right] = \begin{bmatrix} \frac{\partial g_1}{\partial x_1} & \frac{\partial g_1}{\partial x_2} \\ \frac{\partial g_2}{\partial x_1} & \frac{\partial g_2}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 2x_1 & 4 \\ 4x_1 & -3 \end{bmatrix}_{(2,2)} = \begin{bmatrix} 4 & 4 \\ 8 & -3 \end{bmatrix}$$

and the value of π^T in this problem is;

$$\pi^T = \begin{bmatrix} \frac{\partial f}{\partial x_3} & \frac{\partial f}{\partial x_4} \end{bmatrix} \begin{bmatrix} \frac{\partial g_1}{\partial x_3} & \frac{\partial g_1}{\partial x_4} \\ \frac{\partial g_2}{\partial x_3} & \frac{\partial g_2}{\partial x_4} \end{bmatrix}^{-1} = [0 \quad 0] \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^{-1} = [0 \quad 0] \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = (0, 0)$$

One of the simplest tools which can be used to estimate the inverse of desired matrices is the *WolframAlpha* computational knowledge engine. It is an online service that answers many mathematical questions that can be reached using the following address; http://www.wolframalpha.com. According to the above computations, the search direction vector $\boldsymbol{d}$ is;

$$\boldsymbol{d} = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = \begin{bmatrix} 7.0 \\ -1.0 \end{bmatrix} - (0, 0) \begin{bmatrix} 4 & 4 \\ 8 & -3 \end{bmatrix} = \begin{bmatrix} 7.0 \\ -1.0 \end{bmatrix}$$

Step 5: To find the optimal feasible step size β for the line search procedure, the non-basic variables should be substituted as $(x_{nb})_{new} = (x_{nb})_{old} + \beta \boldsymbol{d}$, and so, the new objective function is;

$$\max \quad f\left[(x_{nb})_{new}\right] = f(x_1 + \beta \boldsymbol{d}_1, x_2 + \beta \boldsymbol{d}_2) = f\left(\underbrace{x_1 + \beta \boldsymbol{d}_1}_{x_{1new}}, \underbrace{x_2 + \beta \boldsymbol{d}_2}_{x_{2new}}\right)$$

and so,

$$\begin{aligned} \max f\left[(x_{nb})_{new}\right] &= f(2.0 + 7\beta, 2.0 - \beta) = (2.0 + 7\beta)^2 + 3(2.0 + 7\beta) - (2.0 - \beta) \\ &= 49\beta^2 + 50\beta + 8 \end{aligned}$$

Afterward, we need to determine a basic variable which should be became non-basic variable for an optimal and feasible value of β. As mentioned above, various values of β must be used till one of the basic variables drops to zero, and then, pick that variable as non-basic variable in the next iteration. The estimated results for current basic and non-basic variables and $F(x_{nb})$ associated with various β are presented in the Table 3.13.

As it can be seen from the table, by increasing β both basic variables x_3 and x_4 are decreasing and approaching to zero. The negative value for x_3 (-1.70) at $\beta = 0.15$ demonstrates this solution is an infeasible solution since the slack variables should be nonnegative, and also it shows x_3 drops to zero for a particular

Table 3.13 The optimal value of β

β	x_1	x_2	x_3	x_4	$F(x_1, x_2)$
0.00	2.00	2.00	3.00	18.00	8.00
0.05	2.35	1.95	1.68	14.81	10.62
0.10	2.70	1.90	0.11	11.12	13.49
0.15	3.05	1.85	−1.70	6.95	16.60
0.1032	2.72	1.90	0.00	10.87	13.68

value of β between 0.1 and 0.15. The feasible β is simply found as 0.1032 and the associated values of other variables and objective function are shown in the last row of Table 3.13. The new solution points here are $(x_1 = 2.72, x_2 = 1.90)$.

Step 6: As the variable x_3 is dropped to zero, it should be selected as new non-basic variable, while, x_2 will be the new basic variable. Therefore, we have;

$$X = \begin{bmatrix} x_{nb} \\ x_b \end{bmatrix} = \begin{bmatrix} x_1, x_3 \\ x_2, x_4 \end{bmatrix}$$

The new basic variables in terms of new non-basic variables can be expressed as;

$$x_2 = \frac{1}{4}\left(15 - x_1^2 - x_3\right)$$

and

$$x_4 = 20 - 2x_1^2 + 3x_2 = 20 - 2x_1^2 + 3\left[\frac{1}{4}\left(15 - x_1^2 - x_3\right)\right]$$
$$= \frac{1}{4}\left(-11x_1^2 - 3x_3 + 125\right)$$

The reduced objective function in terms of new non-basic variables x_1 and x_3 is;

$$\max f(x_{nb}) = f(x_1, x_3) = x_1^2 + 3x_1 - x_2$$
$$= x_1^2 + 3x_1 - \left[\frac{1}{4}\left(15 - x_1^2 - x_3\right)\right] = \frac{5}{4}x_1^2 + 3x_1 + \frac{x_3}{4} - \frac{15}{4}$$

Step 7: All calculations in Step 4 should be repeated for the new reduced objective function at point $x_1 = 2.72$ and $x_3 = 0$, as;

$$\left[\frac{\partial f}{\partial x_{nb}}\right] = \begin{bmatrix} \dfrac{\partial f}{\partial x_1} \\ \dfrac{\partial f}{\partial x_3} \end{bmatrix}_{(x_1=2.72, x_3=0.0)} = \begin{bmatrix} \dfrac{10}{4}x_1 + 3 \\ \dfrac{1}{4} \end{bmatrix}_{(2.72, 0.0)} = \begin{bmatrix} 9.8 \\ 0.25 \end{bmatrix}$$

Table 3.14 The optimal value of β

β	x_1	x_3	x_2	x_4	$f(x_{1new}, x_{3_{new}})$
0.00	2.72	0.00	1.90	10.90	13.66
0.05	3.21	0.01	1.17	2.90	18.76
0.10	3.70	0.03	0.32	−6.42	24.47
0.06637	3.37	0.02	0.91	0.00	20.57

and

$$\left[\frac{\partial g}{\partial x_{nb}}\right] = \begin{bmatrix} \frac{\partial g_1}{\partial x_1} & \frac{\partial g_1}{\partial x_3} \\ \frac{\partial g_2}{\partial x_1} & \frac{\partial g_2}{\partial x_3} \end{bmatrix} = \begin{bmatrix} 2x_1 & 1 \\ 4x_1 & 0 \end{bmatrix}_{(2.72,0.0)} = \begin{bmatrix} 5.44 & 1.0 \\ 10.88 & 0.0 \end{bmatrix}$$

and

$$\pi^T = \begin{bmatrix} \frac{\partial f}{\partial x_2} & \frac{\partial f}{\partial x_4} \end{bmatrix} \begin{bmatrix} \frac{\partial g_1}{\partial x_2} & \frac{\partial g_1}{\partial x_4} \\ \frac{\partial g_2}{\partial x_2} & \frac{\partial g_2}{\partial x_4} \end{bmatrix}^{-1} = [0 \quad 0] \begin{bmatrix} 4 & 0 \\ -3 & 1 \end{bmatrix}^{-1} = [0 \quad 0] \begin{bmatrix} 4 & 0 \\ -3 & 1 \end{bmatrix}$$
$$= (0, 0)$$

Based on the above computations, the new search direction vector $\boldsymbol{d}$ in this step is;

$$\boldsymbol{d} = \begin{bmatrix} d_1 \\ d_3 \end{bmatrix} = \begin{bmatrix} 9.8 \\ 0.25 \end{bmatrix} - (0, 0) \begin{bmatrix} 4 & 0 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 9.8 \\ 0.25 \end{bmatrix}$$

Step 8: The new reduced objective function is calculated based on the new search direction vector as;

$$\begin{aligned} \max \quad f\left[(x_{nb})_{new}\right] &= f\left[\underbrace{x_1 + \beta \boldsymbol{d}_1}_{x_{1new}}, \underbrace{x_3 + \beta \boldsymbol{d}_3}_{x_{3new}}\right] \\ &= f(2.72 + 9.8\beta, 0.0 + 0.25\beta) \\ &= \frac{5}{4}(2.72 + 9.8\beta)^2 + 3(2.72 + 9.8\beta) + \frac{0.25}{4} - \frac{15}{4} \\ &= 120.05\beta^2 + 96.10\beta + 13.66 \end{aligned}$$

The results of line search to determine the feasible and optimal β is shown in Table 3.14.

The new solution in this step are $x_1 = 3.37$, $x_2 = 0.91$, and $f(x_{1new}, x_{3new}) = 20.57$.

Step 9: As the variable x_4 drops to zero, it is considered as new non-basic variable, and x_1 will be a new basic variable. Therefore, we can write;

$$X = \begin{bmatrix} x_{nb} \\ x_b \end{bmatrix} = \begin{bmatrix} x_3, x_4 \\ x_1, x_2 \end{bmatrix}$$

Hence, the basic variables in terms of non-basic variables can be written as;

$$x_1 = (15 - 4x_2 - x_3)^{\frac{1}{2}}$$
$$x_2 = \frac{1}{3}\left(2x_1^2 + x_4 - 20\right)$$

We need to eliminate x_1 and x_2 from right sides of the above equation to express the basic variables only in terms of non-basic variables. The easier way is to solve the following system of equations one time for x_2 and x_1.

$$\begin{cases} g_1(x) = x_1^2 + 4x_2 + x_3 - 15 = 0 \\ g_2(x) = 2x_1^2 - 3x_2 + x_4 - 20 = 0 \end{cases}$$

Solve for x_2 resulted in;

$$\begin{cases} -2\left(x_1^2 + 4x_2 + x_3 - 15\right) = 0 \\ \qquad\qquad + \\ 2x_1^2 - 3x_2 + x_4 - 20 = 0 \end{cases} \Rightarrow x_2 = \frac{1}{11}(-2x_3 + x_4 + 10)$$

and solving the system for x_1 resulted in;

$$\begin{cases} \frac{3}{4}\left(x_1^2 + 4x_2 + x_3 - 15\right) = 0 \\ \qquad\qquad - \\ 2x_1^2 - 3x_2 + x_4 - 20 = 0 \end{cases} \Rightarrow x_1 = \left(\frac{-3}{11}x_3 - \frac{11}{4}x_4 + \frac{125}{11}\right)^{\frac{1}{2}}$$

The reduced objective in terms of x_3 and x_4 can be written as;

$$\begin{aligned} \max \quad f(x_{nb}) = F(x_3, x_4) &= x_1^2 + 3x_1 - x_2 \\ &= \left[\left(\frac{-3}{11}x_3 - \frac{11}{4}x_4 + \frac{125}{11}\right)^{\frac{1}{2}}\right]^2 + 3\left(\frac{-3}{11}x_3 - \frac{11}{4}x_4 + \frac{125}{11}\right)^{\frac{1}{2}} - \left[\frac{1}{11}(-2x_3 + x_4 + 10)\right] \\ &= \frac{1}{44}\left[6\sqrt{11}\sqrt{-12x_3 - 121x_4 + 500} - 4x_3 - 125x_4 + 540\right] \end{aligned}$$

Step 10: All computations in Step 4 should be repeated based on the new reduced objective function at point $x_3 = 0$ and $x_4 = 0$, as follow;

$$\left[\frac{\partial f}{\partial x_{nb}}\right] = \begin{bmatrix} \frac{\partial f}{\partial x_3} \\ \frac{\partial f}{\partial x_4} \end{bmatrix}_{(x_3=0,0x_4=0.0)} = \begin{bmatrix} \frac{-9}{\sqrt{11}\sqrt{-12x_3 - 121x_4 + 500}} - \frac{1}{11} \\ \frac{33\sqrt{11}}{4\sqrt{-12x_3 - 121x_4 + 500}} - \frac{125}{44} \end{bmatrix}_{(0.0,0.0)} = \begin{bmatrix} -0.21 \\ -1.61 \end{bmatrix}$$

As it can be seen $\nabla_{nb}f$ has negative value for the current non-basic variables as -0.21 and -1.61. The negative gradient points for a maximization problem that the function is expected to increase, indicates moving the parameters in this direction will be resulted in a lower value of desired function. In other words, an increase in one of the non-basic variables will decrease the value of objective function. Therefore, the optimal solution for this nonlinear problem occurs in

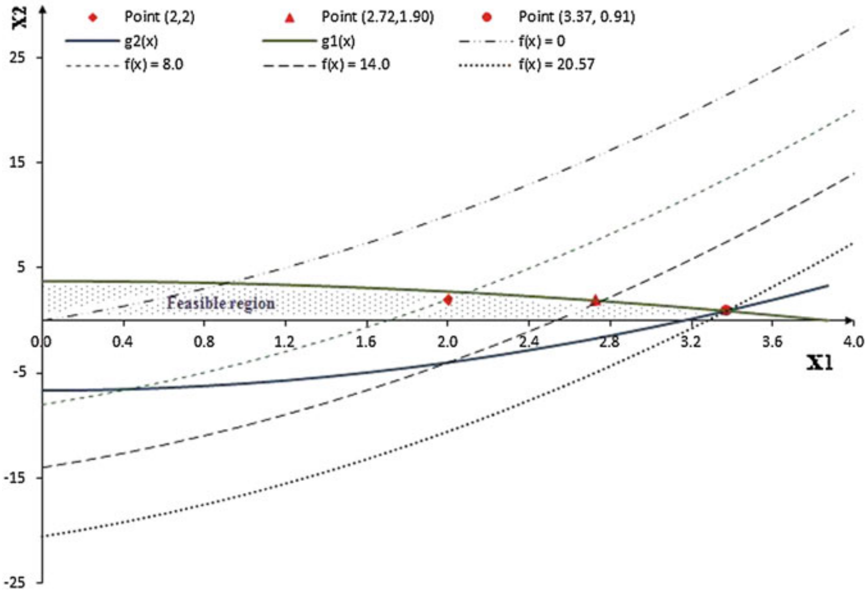


Fig. 3.20 The solution points, constraints, feasible region, and the objective functions associated with its different values

$x_1 = 3.37$, $x_2 = 0.91$, and $f(x_{1new}, x_{2new}) = 20.57$. Figure 3.20 shows the solution points in different steps, constraints and feasible region, and the objective functions associated with its different values.

Example 3.14 An unconfined aquifer, shown in Fig. 3.21, includes three observation wells which are located 15 m apart in the direction of flow. The constant upstream and downstream heads are 100 and 95 m, respectively; and the aquifer made up of gravelly course sand with the hydraulic conductivity of 40 m/day. The typical values of hydraulic conductivity for various materials are presented in Table 3.15.

Determine the optimum head in each well for various minimum values of the total discharge (W_{min}) from 20 to 80 (m/day).

Solution: The governing equation for steady-state flow in a one-dimensional unconfined aquifer by considering the pumping wells can be written as;

$$T_x \frac{\partial^2 h}{\partial x^2} = W \tag{3.59}$$

As mentioned in the Chap. 2, the saturated thickness will be replaced with the hydraulic head h in unconfined cases ($T_x = Kh$), and so,

$$\frac{\partial}{\partial x}\left[h \frac{\partial h}{\partial x}\right] = \frac{W}{K} \tag{3.60}$$

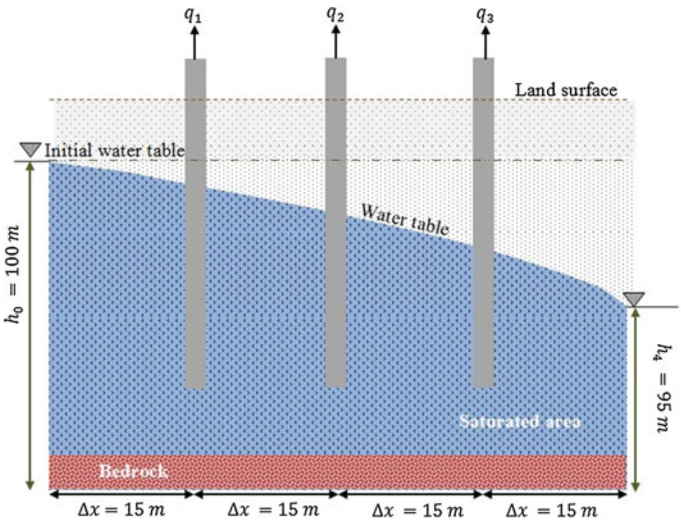


Fig. 3.21 Schematic view of unconfined aquifer

Table 3.15 Hydraulic conductivity for different materials (Smedema and Rycroft 1983)

Material	Hydraulic conductivity (m/day)
Gravelly course sand	10–50
Medium sand	1–5
Sandy loam, fine sand	1–3
Loam, well-structured clay loam and clay	0.5–2
Very fine sandy loam	0.2–0.5
Poorly structured clay loam and clay	0.002–0.2
Dense clay (no cracks, no pores)	<0.002

Apply the following derivative trick as;

$$\frac{dh}{dx}h = \frac{1}{2}\frac{dh^2}{dx} \tag{3.61}$$

The implementation form of the Eq. (3.60) can be written as follow;

$$\frac{d^2h^2}{dx^2} = \frac{2W}{K} \tag{3.62}$$

Based on the central differencing technique Eq. (3.62) can be discretized as;

$$\frac{h_{i+1}^2 - 2h_i^2 + h_{i-1}^2}{(\Delta x)^2} = \frac{2W_i}{K} \tag{3.63}$$

In this case, the objective function to maximize the hydraulic heads for various pumping rates is (Aguado et al. 1974);

$$\max Z = \sum_{i=1}^{n} h_i^2 \tag{3.64}$$

where n is the number of wells (here $n = 3$), and h_t is the hydraulic head in each well. The constraints that should be applied in this problem are;

$$\begin{cases} \dfrac{h_{i+1}^2 - 2h_i^2 + h_{i-1}^2}{(\Delta x)^2} = \dfrac{2W_i}{K} & \\ W_i \geq 0 & i = 1 \text{ to } n \\ \sum\limits_{i=1}^{n=3} W_i \geq W_{min} & \\ h_i \geq h_{i+1} & i = 0 \text{ to } n \end{cases} \tag{3.65}$$

Therefore, the developed optimization model for having minimum value of 20–80 m/day from all wells simultaneously, can be written as;

$$\max Z = h_1^2 + h_2^2 + h_3^2$$

and the constraints are;

$$\begin{cases} i = 1 \rightarrow \dfrac{h_2^2 - 2h_1^2 + h_0^2}{(\Delta x)^2} = \dfrac{2W_1}{K} \\ i = 2 \rightarrow \dfrac{h_3^2 - 2h_2^2 + h_1^2}{(\Delta x)^2} = \dfrac{2W_2}{K} \\ i = 3 \rightarrow \dfrac{h_4^2 - 2h_3^2 + h_2^2}{(\Delta x)^2} = \dfrac{2W_3}{K} \end{cases}$$

$$\Rightarrow \begin{cases} i = 1 \rightarrow h_0^2 = 100^2 = \left(2h_1^2 - h_2^2\right) + \left(\frac{2\times\Delta x^2}{K} W_1\right) \\ i = 2 \rightarrow 0 = \left(2h_2^2 - h_1^2 - h_3^2\right) + \left(\frac{2\times\Delta x^2}{K} W_2\right) \\ i = 3 \rightarrow h_4^2 = 95^2 = \left(2h_3^2 - h_2^2\right) + \left(\frac{2\times\Delta x^2}{K} W_3\right) \end{cases}$$

And,

$$W_1 + W_2 + W_3 \geq W_{min}$$
$$h_1, h_2, h_3 \geq 0$$
$$W_1, W_2, W_3 \geq 0$$
$$h_0 \geq h_1 \geq h_2 \geq h_3 \geq h_4$$

The unknowns in this problem are h_1, h_2, h_3 and W_1, W_2, W_3 in all wells of desired aquifer. As noted in previous chapter, the amount of well losses and well diameters are considered as negligible value. This problem is solved simply by using Excel (Data|Solver) and applying the GRG method. The achieved results are presented in the Table 3.16.

Table 3.16 Hydraulic heads and discharge rates in various W_{min} using GRG method

	$W_{min} = 20$ (m/day)	$W_{min} = 40$ (m/day)	$W_{min} = 60$ (m/day)	$W_{min} = 80$ (m/day)
h_0	100.00	100.00	100.00	100.00
h_1	98.20	97.54	96.44	95.35
h_2	96.95	96.37	95.79	95.20
h_3	95.70	95.19	95.13	95.05
h_4	95.00	95.00	95.00	95.00
W_1	10.26	23.01	50.95	78.20
W_2	0.00	0.00	0.00	0.00
W_3	9.74	16.99	9.05	1.80
Z	28200	27,862.5	27,525	27,187.5

It is important to note that this problem also can be solved using simplex method by applying a simple linearization procedure for hydraulic head. As the only non-linear variable here is h^2, the substitution $m = h^2$ can be applied, and so, the Eq. (3.65) will be changed as;

$$\frac{m_{i+1} - 2m_i + m_{i-1}}{(\Delta x)^2} = \frac{2W_i}{K} \tag{3.66}$$

Therefore, the objective function of problem in this case is;

$$\max Z = m_1 + m_2 + m_3$$

and the constraints are;

$$\begin{cases} i = 1 \rightarrow \dfrac{m_2 - 2m_1 + m_0}{(\Delta x)^2} = \dfrac{2W_1}{K} \\ i = 2 \rightarrow \dfrac{m_3 - 2m_2 + m_1}{(\Delta x)^2} = \dfrac{2W_2}{K} \\ i = 3 \rightarrow \dfrac{m_4 - 2m_3 + m_2}{(\Delta x)^2} = \dfrac{2W_3}{K} \end{cases}$$

$$\Rightarrow \begin{cases} i = 1 \rightarrow m_0 = 100^2 = (2m_1 - m_2) + \left(\dfrac{2 \times \Delta x^2}{K} W_1\right) \\ i = 2 \rightarrow 0 = (2m_2 - m_1 - m_3) + \left(\dfrac{2 \times \Delta x^2}{K} W_2\right) \\ i = 3 \rightarrow m_4 = 95^2 = (2m_3 - m_2) + \left(\dfrac{2 \times \Delta x^2}{K} W_3\right) \end{cases}$$

And,

$$W_1 + W_2 + W_3 \geq W_{min}$$
$$m_1, m_2, m_3 \geq 0$$
$$W_1, W_2, W_3 \geq 0$$
$$m_0 \geq m_1 \geq m_2 \geq m_3 \geq m_4$$

Table 3.17 Hydraulic heads and discharge rates in various W_{min} using simplex method

	$W_{min} = 20$ (m/day)	$W_{min} = 40$ (m/day)	$W_{min} = 60$ (m/day)	$W_{min} = 80$ (m/day)
m_0	10,000.00	10,000.00	10,000.00	10,000.00
m_1	9,700.00	9,643.75	9,587.50	9,531.25
m_2	9,400.00	9,287.50	9,175.00	9,062.50
m_3	9,100.00	8,931.25	8,762.50	8,593.75
m_4	9,025.00	9,025.00	9,025.00	9,025.00
h_0	100.00	100.00	100.00	100.00
h_1	98.49	98.20	97.92	97.63
h_2	96.95	96.37	95.79	95.20
h_3	95.39	94.51	93.61	92.70
h_4	95.00	95.00	95.00	95.00
W_1	0.00	0.00	0.00	0.00
W_2	0.00	0.00	0.00	0.00
W_3	20.00	40.00	60.00	80.00
Z	28,200	27,862.5	27,525	27,187.5

The unknowns in this problem are m_1, m_2, m_3 and W_1, W_2, W_3 in all wells of unconfined aquifer. This problem is solved simply by using Excel (Data|Solver) and applying the Simplex method, and the achieved results are presented in Table 3.17

As it can be seen from Table 3.17, the values of objective function Z have not been changed using both GRG and Simplex methods, while, the values of h and W are different regarding difficulties in the complexities of nonlinear functions.

3.5 Problems

Problem 3.1 Find the eigenvalues of the following matrices and then classify them.

(a) $H = \begin{bmatrix} 5 & -2 \\ -3 & 6 \end{bmatrix}$

(b) $H = \begin{bmatrix} 12 & 5 \\ 1 & 8 \end{bmatrix}$

(c) $H = \begin{bmatrix} -4 & 9 \\ 1 & 3 \end{bmatrix}$

Problem 3.2 Determine the concavity or convexity of the following functions.

(a) $\mathrm{f}_1(x) = x_1^4 - x_1x_2 + 3x_2^3$

(b) $\mathrm{f}_2(x) = -x_1^3 - x_2^2 + 5$

(c) $\mathrm{f}_3(x) = x_1x_2 - x^2$

Problem 3.3 Explain how one-dimensional search method works.
Problem 3.4 Define the Fibonacci sequence and calculate the 15th, 21th, and 50th Fibonacci number.

Problem 3.5 Minimize f(x) on the interval [−4,2] using the Fibonacci method for the total number of experiments $n = 10$.

$$f(x) = (x+1)^2 + 0.25x + 1.5$$

Problem 3.6 Solve problem 3.5 using the golden section method with the convergence criterion of $\delta = 0.065$, and then compare your results with the outcome of Fibonacci approach.

Problem 3.7 Apply Newton method to find the minimum of function f(x) by considering $\varepsilon = 0.01$, and starting point $x_0 = 0.2$.

$$f(x) = x^4 - 2.45\exp(x)$$
$$x \in [0.2, 2]$$

Problem 3.8 Find the minimum of function f(x) in Problem 3.7 using random jumping method.

Problem 3.9 Minimize f(x) using univariate method with the starting points $x_1 = 0$ and $x_2 = 0$, and $\varepsilon = 0.1$.

$$f(x) = x_1^2 + x_2^2 - 3.8x_1 - 2.11x_2$$

Problem 3.10 Apply the steepest descent method to solve Problem 3.9 with the starting points $x_1 = 0$ and $x_2 = 0$, and $\varepsilon = 0.005$.

Problem 3.11 Explain main difference between constrained and unconstrained optimization problems.

Problem 3.12 Find the minimum of function f(x) based on the Lagrange multiplier method.

$$f(x) = x_1^2 + (x_2 - 3)^2 - x_1 - x_2$$

Subject to:

$$x_1^2 - x_2 < 11$$

Problem 3.13 Apply GRG method to find the maximum of function f(x).

$$-x_1^4 + x_2^2 - x_1x_2$$

Subject to the following constraints;

$$\begin{cases} x_1^2 - 2x_2 \leq 31 \\ x_1 + 3x_2 \leq 14 \end{cases}$$

Problem 3.14 Solve Example 3.9 by considering $\varepsilon = 0.01$ and $\varepsilon = 0.1$, and compare your results to see how changing ε will effect on the optimization results.

Problem 3.15 Solve Example 3.14 by assuming three observation wells are located 5, 10, and 30 m apart in the direction of flow.

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Chapter 4
Multiobjective Optimization

Abstract Many real-world problems require multiobjective evaluation that involves several minimizing or maximizing objective functions to be optimized simultaneously. In the case of multiobjective problems, a set of optimal solutions instead of a single solution will be determined. This chapter provides a basic introduction to the application of multiobjective optimization in water resources engineering and also provides a number of useful and applicable examples to better understand the concept of multiobjective optimization.

4.1 Basic Concepts

In some situations, the optimization problems include more than one objective function to be optimized simultaneously. In these cases, the process of optimizing a number of objective functions (at least two objectives) is called multiobjective optimizations (MOO). Based on the single objective optimization concept, a particular global maximum or minimum value is found in the desired search space, whereas, in the multi-objective problems a set of solutions instead of a single value will be determined. In other words, the main purpose of multiple objective optimizations is generating a set of optimal solutions, known as *Pareto front*, that show the best trade-off relations between all objectives. In general, there are conflicts between objectives of multiple objective problems in which improvement in one of them results in declining another one, and so, all desired objectives cannot be met simultaneously.

Most of the water resources optimization problems are naturally multiobjective and they are usually nonlinear and multi-dimensional. Some examples for multiple and conflictive objectives in the water resources engineering are; designing water distribution system with the highest efficiency, while, reducing the cost of design simultaneously; or minimizing cost of building hydraulic structures, whereas, the flow capacity should be maximized; or keep the water level in the dam reservoir as high as possible to generate maximum electricity, while keep it at appropriate level to prevent overtopping due to unexpected heavy rainfall.

E. Goodarzi et al., *Introduction to Optimization Analysis in Hydrosystem Engineering*, Topics in Safety, Risk, Reliability and Quality 25, DOI: 10.1007/978-3-319-04400-2_4,

The general form of a multiobjective problem, also known as a vector minimization problem, with n objectives is;

$$\min \quad [\mathrm{f}_1(X), \mathrm{f}_2(X), \ldots, \mathrm{f}_k(X)]; \quad X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \tag{4.1a}$$

Subject to;

$$g_j(X) \leq 0, \quad j = 1, 2, \ldots m \tag{4.1b}$$

where, $\mathrm{f}_1(\mathrm{x})$, $\mathrm{f}_2(\mathrm{x})$, …, $\mathrm{f}_\mathrm{k}(\mathrm{x})$ denote the objective functions, $\mathrm{g}_j(x_i)$ represent the constraints of the problem, and x is the decision variable. It is important to note that for $n = 1$, the MOO problem is reduced to a single objective function, and so, the traditional optimization method can be used to solve the problem.

The term “min” here means all of the objective functions are minimized simultaneously, while, it is not possible to find a single solution as an optimal solution for all of the functions together. In general, it is very difficult to find all possible solutions of all objectives simultaneously in a multiobjective optimization problem, and so, most engineers would like to make a balance among optimal solutions. In other words, the main objective of MOO is finding a collection of acceptable optimal solutions, known as *Pareto optimal set* or *Pareto efficiency*, and selects the best answer from the obtained *Pareto* set. Mathematically, a feasible solution X^* is a Pareto optimal solution if (1) for every feasible solution like X and $i = 1, \ldots, k$, $\mathrm{f_i(X)} \geq \mathrm{f_i(X^*)}$, which is known as strict Pareto optimum or a strict efficient solution; or (2) for at least one objective function $\mathrm{f_i(X)} > \mathrm{f_i(X^*)}$, that is known as weak Pareto optimum or a weak efficient solution (Caramia and Dell’olmo 2008). In other words, a vector solution $X = (x_1, x_2, \ldots, x_n)$ is considered as Pareto optimal solution if and only if reducing an objective function causes a simultaneous increase of one or more other objective functions. To be more familiar with this concept, consider two arbitrary objectives functions $\mathrm{f}_1(\mathrm{x})$ and $\mathrm{f}_2(\mathrm{x})$ as shown in Fig. 4.1. The Pareto optimal solutions here include all the values of x between 3 and 8 which are plotted as bold blue and red lines on the graph.

It is important to note that any optimal solution of Pareto optimal set cannot be better off without making at least one component of the other objective functions worse off. For example, when we are producing less of one product while producing more of another simultaneously, we can say an economy is productively efficient. It is interesting to know that the appropriate balance among the multiple objectives is called trade-off in the business and economic analyses and this term also is used in engineering optimization problems. Hence we can say, the Pareto set represents the trade-off among objective functions and a collection of efficient solutions instead of one solution in a desired optimization analysis. It should be noted that if any improvement in at least one objective function doesn’t make any other objective function worse off, the Pareto optimal is called *Pareto improvement*.

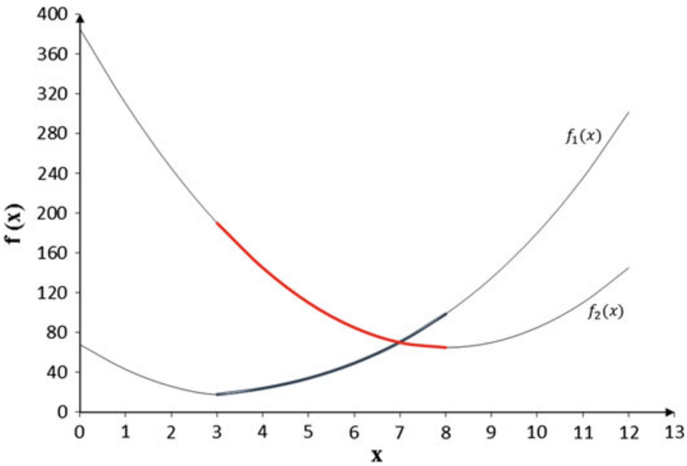

Fig. 4.1 The Pareto optimal solutions for two arbitrary objectives functions $f_1(x)$ and $f_2(x)$

The other important concept in this case is *Pareto frontier* or *Pareto curve* that simply shows the trade-off between all of the objective functions. When a set of solution of desired multiobjective optimization problem is plotted in the objective space, the Pareto optimal set is called Pareto frontier or Pareto curve and all the points on this curve are Pareto optimal (Alba et al. 2009). Figure 4.2 and Table 4.1 show the values of $f_1(x)$ and $f_2(x)$ for different values of $x \in [0, 13]$, and the Pareto frontier for those two arbitrary functions, respectively. All points on the Pareto frontier (between B and C) are Pareto optimal, since reduction of $f_1(x)$ on this curve causes a simultaneous increase of $f_2(x)$.

On the other hand, other points such as A and D are not Pareto optimal since decreasing value of $f_1(x)$ from point A to B results in simultaneous reduction of $f_2(x)$; or increasing $f_1(x)$ from point C to D causes reduction of values of $f_2(x)$ simultaneously. While, by increasing the values of $f_1(x)$ from B to C all values of $f_2(x)$ are decreased.

The Pareto curve shows all the combination of optimal solution for desired objective functions. However, still there are a number of significant questions about this curve as; what determines whether the points B or D or any point between B and D is the best answer? Or, how can we determine the right combination form the Pareto set? Or how decision makers can make the best decision based on the Pareto curve?

There are several different ways for solving multiobjective optimization problems in which some of them with a number of examples are presented in this chapter. In general, the classical methods can be categorized into three approaches as; Weighted method, Goal Programming, and ε-Constraint method. The ultimate goal of these techniques are maximizing or minimizing the new single objective based on the determined constraints of the system. In the following sections, solving a multi objectives problem based on weighted, and ε-constraints methods are presented.

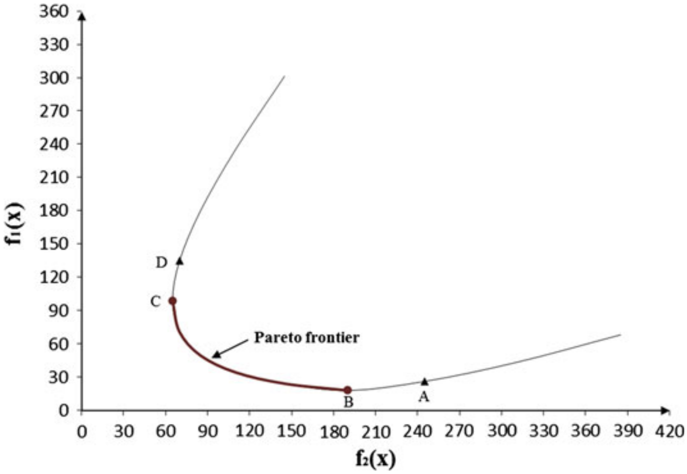

Fig. 4.2 The Pareto frontier for two arbitrary objectives functions $f_1(x)$ and $f_2(x)$

Table 4.1 The values of $f_1(x)$ and $f_2(x)$ for different values of x

x		$f_1(x)$	$f_2(x)$	x		$f_1(x)$	$f_2(x)$
0		68	385	7		**70.50**	*70*
1		43	310	8	*C*	**98.67**	*65*
2	A	26	245	9	D	134.83	70
3	*B*	**17.83**	**190**	10		180.00	85
4		**24.00**	**145**	11		235.17	110
5		**34.17**	**110**	12		301.33	145
6		**49.33**	**85**	13		379.50	190

4.2 Weighted Method

The Weighted method is one of the most widely used classical techniques in multiobjective optimization analysis. Based on this method, all objective functions are combined together and converted into a single objective function using different coefficient for each function as weight factor. In other words, a new objective function is constructed, and then, the Pareto set is built to find the optimal solution. Consider $f_1(x)$, $f_2(x)$, …, $f_n(x)$ as a set of objective functions that should be minimized or maximized. The new objective function for the optimization process can be written as a scalar optimization form;

$$F(x) = w_1 \cdot f_1(x) + w_2 \cdot f_2(x) + \cdots + w_n \cdot f_n(x) = \sum_{k=1}^{n} w_k \cdot f_k(x) \qquad (4.2a)$$

with

$$w_k \in [0, 1]; \quad \sum_{k=1}^{n} w_k = 1 \tag{4.2b}$$

in which, w_k are the scalar-valued weights that show the relation between the objective functions.

The most difficult part of this method is finding the appropriate weights for all objectives based on the relative importance of each objective function when there is insufficient information about the desired problem. In other words, the problem can be solved repeatedly with various values for weight coefficients, then; the decision makers select the proper solutions regarding the importance of the each objective function. The advantages of using this method are its efficiency, simplicity to apply, and finding the optimal solutions on the entire Pareto set. However, there are a number of disadvantages regarding the weighted method as;

1. Decision-makers need to determine the weights by their intuition/experience or judgment,
2. Changing the weights maybe cause a big or small changes in the objectives,
3. The method cannot find the appropriate solutions on the non-convex part of Pareto set,
4. There is possibility to have a number of minimum solutions for a particular set of weights in which it generates various solutions in the Pareto set.

Example 4.1 Consider the functions $f_1(x)$ and $f_2(x)$ as follows;

$$f_1(x) = 6(x-7)^2 + 2x + 17$$
$$f_2(x) = (x-2)^3 + 23$$

Subject to;

$$2 \le x \le 10$$

Minimize $F(x) = g[f_1(x), f_2(x)]$ using the weighted method.

Solution: Figure 4.3 shows the objective functions and the obtained constraint of the problem. In order to minimize the $f_1(x)$ and $f_2(x)$ based on the weighted method, $F(x)$ should be written as;

$$F(x) = w_1 f_1(x) + w_2 f_2(x)$$

where w_1 and w_2 are the weights of each objective function.

The simplest way to solve this problem is considering different values for weights and then solving the single objective equation to find the optimal solution. However, without prior information about the weights, choosing the appropriate values for them is difficult while, the quality of solution highly depends on the selection of the weights.

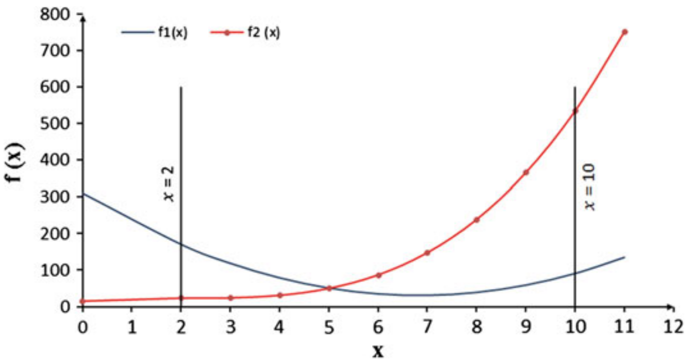

Fig. 4.3 The objective functions and constraints of the problem

Table 4.2 Minimum of $F(x)$ in different weights

x	$\mathrm{f}_1(x)$	$\mathrm{f}_2(x)$	$F_{w_1=0,\,w_2=1}$	$F_{w_1=0.3,\,w_2=0.7}$	$F_{w_1=0.5,\,w_2=0.5}$	$F_{w_1=0.8,\,w_2=0.2}$	$F_{w_1=0,\,w_2=1}$
2	171.00	23.00	**23.00**	67.40	97.00	141.40	171.00
2.5	143.50	23.13	23.13	59.24	83.31	119.43	143.50
3	119.00	24.00	24.00	52.50	71.50	100.00	119.00
3.5	97.50	26.38	26.38	47.71	61.94	83.28	97.50
4	79.00	31.00	31.00	**45.40**	55.00	69.40	79.00
4.5	63.50	38.63	38.63	46.09	51.06	58.53	63.50
5	51.00	50.00	50.00	50.30	**50.50**	50.80	51.00
5.5	41.50	65.88	65.88	58.56	53.69	46.38	41.50
6	35.00	87.00	87.00	71.40	61.00	**45.40**	35.00
6.5	31.50	114.13	114.13	89.34	72.81	48.03	31.50
7	31.00	148.00	148.00	112.90	89.50	54.40	**31.00**
7.5	33.50	189.38	189.38	142.61	111.44	64.68	33.50
8	39.00	239.00	239.00	179.00	139.00	79.00	39.00
8.5	47.50	297.63	297.63	222.59	172.56	97.53	47.50
9	59.00	366.00	366.00	273.90	212.50	120.40	59.00
9.5	73.50	444.88	444.88	333.46	259.19	147.78	73.50
10	91.00	535.00	535.00	401.80	313.00	179.80	91.00

In this problem, different values from 0 to 1 (with 0.1 increments) are assigned and the problem is solved in each value. The weights are used to compare the results and to contribute to the relative importance of each objective function. First, we have tried to find the minimum of $F(x)$ without using Excel or any other computer program and the results are presented for five combinations of w_1 and w_2 in Table 4.2. The bold values in this table show the minimum value of $F(x)$ based on the allocated weights to $\mathrm{f}_1(\mathrm{x})$ and $\mathrm{f}_2(\mathrm{x})$. For example, $F_{w_1 0.3,\, w_2=0.7}$ means the value of $F(x)$ as;

$$F(x) = 0.3 \times \mathrm{f}_1(x) + 0.7 \times \mathrm{f}_2(x)$$

Table 4.3 The minimums of $F(x)$ for each combination of w_1 and w_2

x	w_1	w_2	$f_1(x)$	$f_2(x)$	min $F(x)$
2.00	0.0	1.0	171.00	23.00	23.00
3.26	0.1	0.9	107.44	25.00	33.24
3.75	0.2	0.8	87.70	28.40	40.26
4.15	0.3	0.7	74.16	32.89	45.27
4.50	0.4	0.6	63.60	38.56	48.57
4.83	0.5	0.5	54.90	45.68	50.29
5.16	0.6	0.4	47.55	54.69	50.40
5.51	0.7	0.3	41.31	66.31	48.81
5.89	0.8	0.2	36.19	81.79	45.31
6.32	0.9	0.1	32.44	103.39	39.54
6.83	1.0	0.0	30.83	135.91	30.83

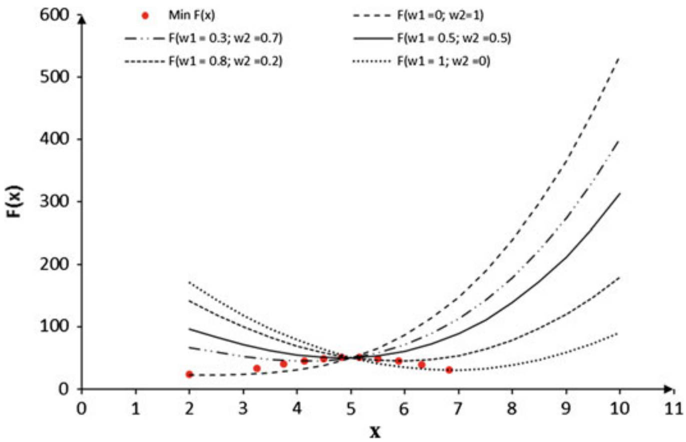

Fig. 4.4 Values of $F(x)$ in different weights in conjunction with its minimums

However, there is a problem with the results and we cannot trust them readily. For instance, the minimum values in the fifth and seventh columns are 45.40, but how can we be assured that the minimum happened at $x = 4$, and not at $x = 3.8$ or $x = 4.1$? The minimum of $F(x)$ which are calculated using the Solver Tools in Excel for each new combination of w_1 and w_2, are presented in Table 4.3.

It is important to note that the GRG nonlinear solver method of Excel is applied to compute the minimum values of $F(x)$. In addition, Figs. 4.4 and 4.5 show the value of $F(x)$ with different weights in conjunction with its minimums; and the Pareto points in the objective space, respectively.

Figure 4.5 illustrates the Pareto optimal solutions in which each solution corresponds to a particular decision and it helps decision-makers to make trade-off decisions between all objective functions.

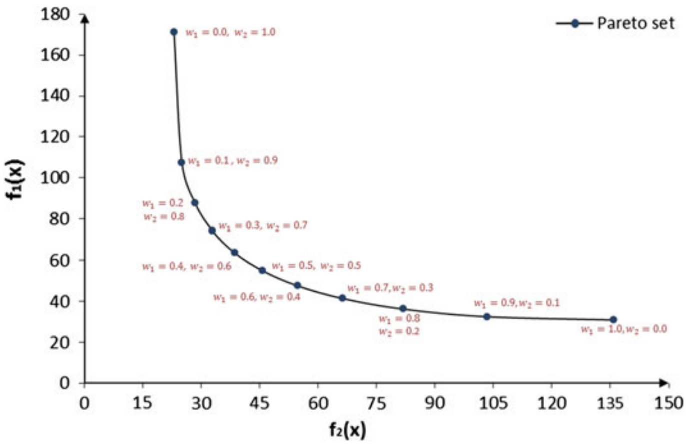

Fig. 4.5 The Pareto frontier for the objectives functions $f_1(x)$ and $f_2(x)$

It is important to note that the weighted method is not a robust approach as the solutions are sensitive to the selected weights. In other words, the applied increment of weights is 0.1 in this problem, while the minimum of $F(x)$ may happen in some other weights such as $w_1 = 0.25$ and $w_2 = 0.75$ that needs lower increment to be found. Therefore, decision-makers need to perform several optimization analyses using various weights to make an appropriate decision.

Example 4.2 Find the optimal solution of the following two-objective optimization problems using the weighted method.

$$\min\ f_1(x_1, x_2) = 1.5(x_1 - 1)^2 + (x_2 + 1)^2$$
$$\min\ f_2(x_1, x_2) = 0.35(x_1 + x_2 - 1)^2 + (2x_2 - x_1)^2 + 4$$

Subject to:

$$0 \le x_1 \le 5$$
$$0 \le x_2 \le 6$$
$$2x_1 - x_2 \le 6$$
$$x_1 - 4x_2 \le 0$$

Solution: All necessary steps to solve this problem are presented in the following section.

1. The first step to minimize both $f_1(x_1, x_2)$ and $f_2(x_1, x_2)$ is building the function $F(x_1, x_2)$ as, $F(x_1, x_2) = w_1 f_1(x_1, x_2) + w_2 f_2(x_1, x_2)$. Then, different values of w_i should be considered to find the Pareto set and optimal solutions. Figure 4.6 shows the linear constraints in this problem and the shaded area as the feasible region.

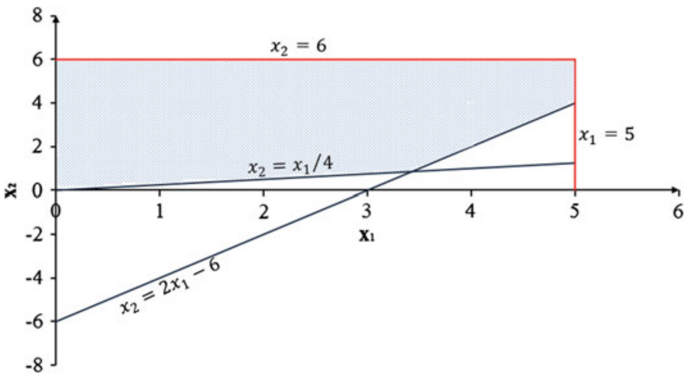

Fig. 4.6 Linear constraints of the problem

Table 4.4 The minimums of $F(x_1, x_2)$ when both f_1 and f_2 should be minimized

w_1	w_2	x_1	x_2	$f_1(x_1, x_2)$	$f_2(x_1, x_2)$	min $F(x_1, x_2)$
0	1	0.67	0.33	1.94	4.00	4.00
0.1	0.9	0.67	0.30	1.86	4.00	3.79
0.2	0.8	0.67	0.26	1.76	4.02	3.57
0.3	0.7	0.68	0.22	1.64	4.06	3.34
0.4	0.6	0.69	0.17	1.52	4.13	3.08
0.5	0.5	0.72	0.18	1.51	4.13	2.82
0.6	0.4	0.74	0.18	1.51	4.14	2.56
0.7	0.3	0.75	0.19	1.50	4.14	2.30
0.8	0.2	0.77	0.19	1.50	4.15	2.03
0.9	0.1	0.79	0.20	1.50	4.15	1.77
1	0	0.80	0.20	1.50	4.16	1.50

The problem is solved using the weighted method by applying the Solver tool in Excel and results are listed in Table 4.4. As can be seen in this table, the values of x_1 and x_2 are placed in the feasible region and also by reducing f_1 the values of f_2 increased simultaneously.

In addition to the table, Fig. 4.7 shows the Pareto optimal solutions for all used weights in this example. Based on the Pareto curve, the decision-maker can make a trade-off decision for the problems, however, the optimum value cannot be simply determined at a single point in the design space.

Based on Fig. 4.7, the least value of f_2 occurs when the weight of f_1 or w_1 is zero, while by moving on the curve the weights of f_1 will be increased, that means the order of importance of f_1 is increased. However, from the point $w_1 = 0.4$ to $w_1 = 1.0$ any changes of f_2 causes little change in f_1. In other words, the sensitivity of f_2 with respect to f_1 is not high in this part of curve. It is important to note that this sensitivity information can be a useful source for decision makers to make trade-off decision in this problem. For instance, point $w_1 = 0.4$, $w_2 = 0.6$ is an

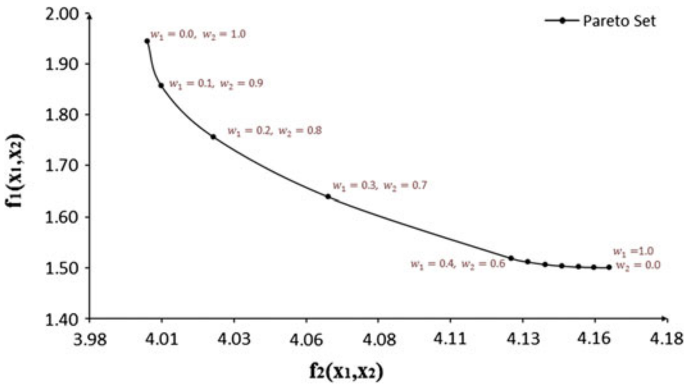

Fig. 4.7 The Pareto frontier for the objectives functions $f_1(x_1, x_2)$ and $f_2(x_1, x_2)$

appropriate choice when the minimum value of f_2 is more acceptable and also we don't like to totally forgo the objective function f_1. In this case the GRG nonlinear solution method of Excel is applied to compute the minimum value of MOO function $F(x)$.

4.2.1 Optimization of Hydropower and Turbine

Hydropower is the power extracted from the natural potential of falling or flowing water to generate electricity. It is the most widely-used renewable sources of energy and generating power based on this method is one of the oldest methods for centuries. Water turbine is a turbo-machine that takes kinetic energy from moving water and converts it into a mechanical energy. A hydropower plant generally includes three main parts as; (1) dam to control water flow, (2) reservoir to store water, and (3) power plant to generate electricity. Firstly, water goes through an intake screen and then continues in a large pipe which is called penstock. Afterward, the kinematic energy of water spins the turbine to generate electricity, and then, the water goes out of the penstock flowing downstream of dam (Fig. 4.8). The amount of power that a hydroplant generates is function of two factors;

1. The effective or net head (H_e) which is the difference between water level in the reservoir and the water level in the tail race minus losses in the conveying system of the plant.
2. Flow (Q) which is the amount of stream passes through the turbine to spin.

The key equation to estimate the available power from falling water is;

$$P(w) = \eta \rho g Q H_e \tag{4.4}$$

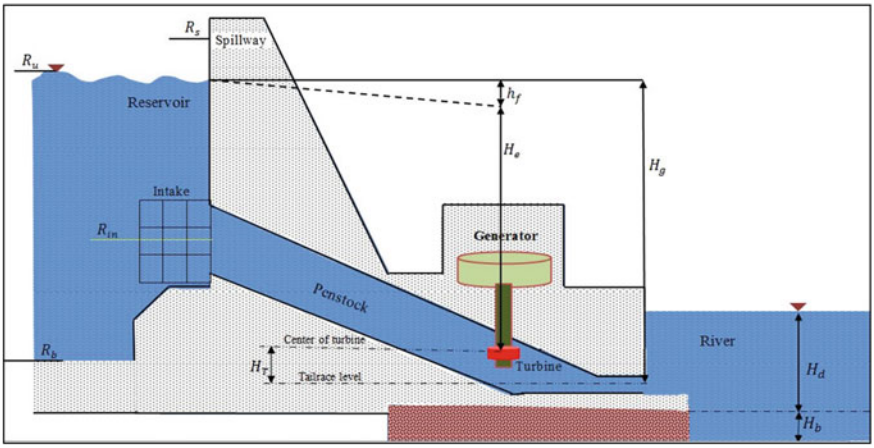

Fig. 4.8 Schematic view of a hydropower dam

Table 4.5 Basic information of dam

R_{in} (msl)	R_s (msl)	R_c (msl)	R_T (msl)	R_{tail} (msl)	R_b (msl)	H_b (m)	D (m)	L (m)	η
1,040.0	1,057.0	1,060.0	997.0	994.0	1,000.0	3.5	3.0	50.0	0.9

where, η is the turbine efficiency, ρ is density of water (1,000 kg/m^3), g is acceleration of gravity (9.81 m/s^2), Q is the flow rate (m^3/s), and H_e is the effective head.

Making trade-off between hydropower generation and control flood in downstream area are two of the highest priorities for water resources engineers. On the one hand, reducing the water level at reservoirs to prevent downstream flood damages is an important issue, while providing necessary electricity for downstream areas which is main priority, needs high water water level at the reservoir. It is clear that these two objectives are in conflict with each other, and so, we have to optimize both objectives simultaneously to improve regulation operations. The main objective functions in this problem are;

1. Minimizing downstream flood peak to minimize flood damages,
2. Maximizing the hydropower generation to supply downstream electricity needs.

Example 4.3 Consider a dam with a crest width of 10 m, a crest length of 350 m, and structural height of 60 m. The elevation of spillway is 3.0 m below the dam crest and it is placed at 1,057 (m) above sea-level.

The other basic information for desired hydropower dam including elevations of penstock entrance (R_{in}), spillway (R_s), dam crest (R_c), turbine (R_T), tailrace (R_{tail}), reservoir bed level (R_b), downstream base flow (H_b), penstock diameter (D), penstock length (L), and turbine efficiency (η) are shown in Fig. 4.8 and Table 4.5.

In this problem, downstream water depth (H_d) is considered as a function of water elevation in the reservoir (R_u) and it is computed based on one of the following equations.

1. If $R_u \leq R_{in}$, there is no release and so,

$$H_d = 0 \tag{4.5}$$

where, R_u is water elevation in the reservoir. In this case, the flow that passes through the turbine is zero ($Q = 0$).

2. If $R_{in} \leq R_u \leq R_{in} + 0.75(R_s - R_{in})$, then;

$$H_d = 0.74(R_u - R_{in})^{0.65} \tag{4.6}$$

The flow rate in this condition is $Q = 100\,\text{m}^3/\text{s}$.

3. If $R_{in} + 0.75(R_s - R_{in}) \leq R_u \leq R_s$, the amount of flow that passes through the turbine is $Q = 200\,\text{m}^3/\text{s}$, and so;

$$H_d = 0.94(R_u - R_{in})^{0.71} \tag{4.7}$$

4. If $R_s \leq R_u \leq R_c$, the amount of water that spills from spillway also is added to the downstream river, and hence, the depth of water will be increased.

$$H_d = 1.2(R_u - R_{in})^{0.83} \tag{4.8}$$

The flow rate in this case is same as previous condition and it is equal to $200\,\text{m}^3/\text{s}$.

In addition, the damage cost $D(\$)$ due to downstream flooding is considered as direct function of downstream water elevation and follows the equations below:

$$\begin{cases} H_d > H_b & D(\$) = 825\exp[0.35(H_d - H_b] \\ H_d \leq H_b & D(\$) = 0.0 \end{cases} \tag{4.9}$$

where, H_b is base flow depth in the river in which its value is 3.5 m in this problem.

Set up and solve the optimization model to find the optimum values of flood peak (H_d) and hydropower generation (P). It is important to note that the downstream water depth should not be less than 3.5 m (or 993.5 msl) regarding some environmental issues like keeping fish, plants and wildlife alive, and also water elevation in the reservoir must not be less than 1,040 msl. Consider the penstock pipe as a steel—smooth pipe with 50 m length.

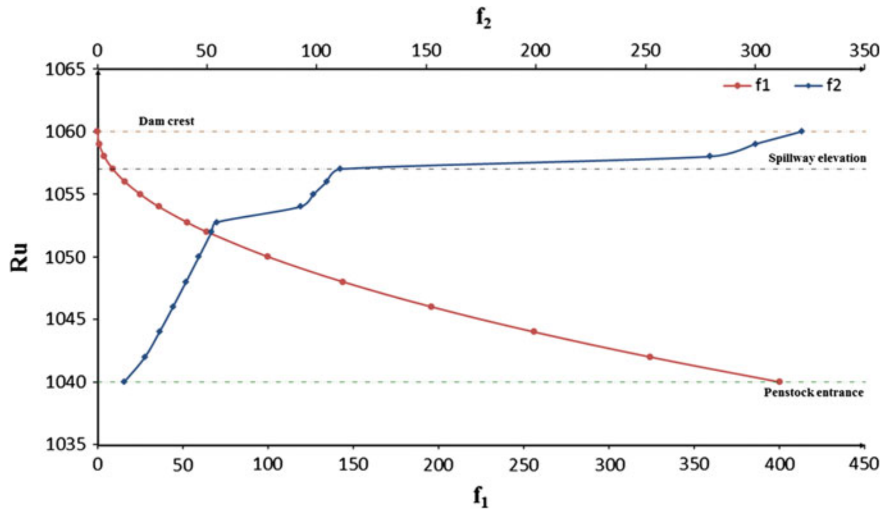

Fig. 4.9 Objective functions f_1 and f_2 versus water level in the reservoir for $m = 2$

Solution: The objective functions in this case can be expressed as (Ngo et al. 2007);

$$\min \quad \begin{array}{l} f_1(R_u) = (R_c - R_u)^m \\ f_2(H_d) = (H_d + H_b)^m \end{array}$$

This problem is solved for two different forms of objective functions by considering $m = 2$, and $m = 1$ with the following constraints;

$$H_d \geq 0$$
$$1{,}040 \leq R_u \leq 1{,}060$$

1. For $m = 2$

Figure 4.9 shows how both objective functions vary by changing the water level in the reservoir. As it can be seen from this figure, there are conflicts between two objectives and increasing one of them due to raising water level in the reservoir results in decreasing another one simultaneously.

Therefore, the aggregated objective function F which should be minimized using the weighted method, can be written as (Fig. 4.10);

$$F(R_u, H_d) = w_1 f_1(R_u) + w_2 f_2(H_d) = w_1 (R_c - R_u)^2 + w_2 (H_d + H_b)^2$$

In order to estimate the Pareto front and find the trade-off between desired objectives, the GRG nonlinear solution method of Excel is used in different combinations of weights to find the optimum points (Fig. 4.11).

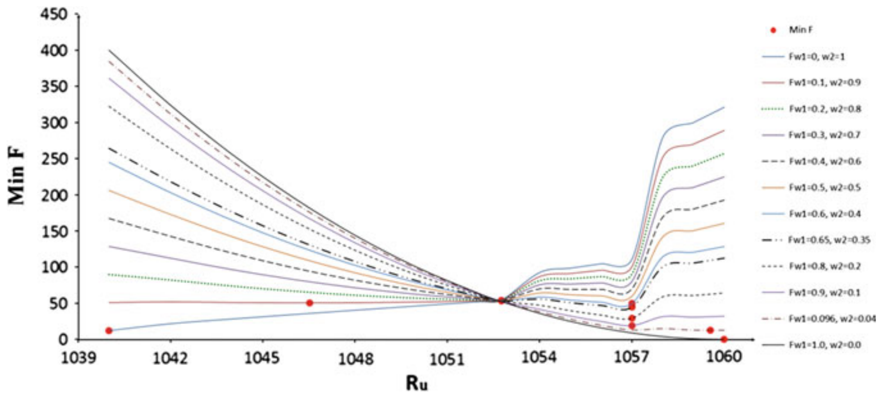

Fig. 4.10 Values of F and its minimums in different weights for $m = 2$

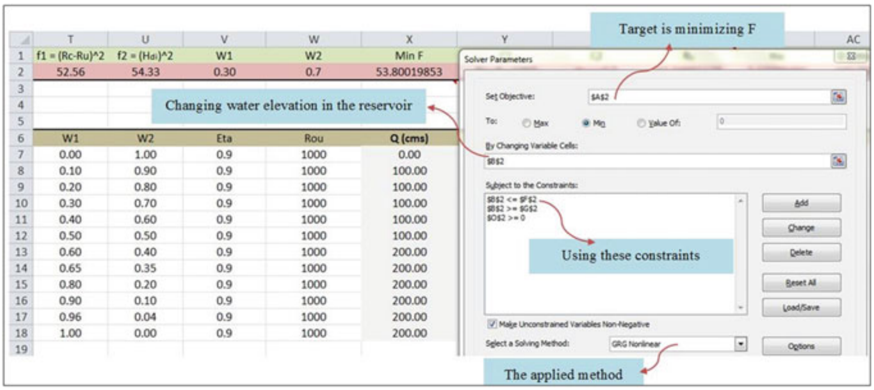

Fig. 4.11 Using Excel's Solver tool for optimization analysis

The Pareto optimum points, minimum values of F and the Pareto curve are shown in Table 4.6 and Fig. 4.12, respectively.

The Pareto curve which provides useful information to verify all possibly conflicting and quantitative properties of an optimization system is shown in the Fig. 4.12. As it can be seen in this figure, by increasing f_2 the first objective function (f_1) is decreasing and this reduction is more considerable for f_1 when w_1 varies from 0.1 to 0.6. However, increasing w_1 from 0.6 to 1.0 resulted in small changes of f_1 while f_2 varies significantly. Choosing the optimum point on the Pareto curve depends on the priority of dam administrative, and the operator can choose a single point among all the available Pareto solutions.

Table 4.6 The minimums of F with generated hydropower and damage costs

w_1	w_2	$R_u - R_b$ (m)	Q (m^3/s)	H_d (m)	*min F*
0.00	1.00	40.00	0.00	3.50	12.25
0.10	0.90	46.52	100.00	6.00	50.61
0.20	0.80	52.75	100.00	7.37	53.97
0.30	0.70	52.75	100.00	7.37	53.80
0.40	0.60	52.75	100.00	7.37	53.62
0.50	0.50	52.75	100.00	7.37	53.45
0.60	0.40	57.00	200.00	10.53	49.72
0.65	0.35	57.00	200.00	10.53	44.63
0.80	0.20	57.00	200.00	10.53	29.36
0.90	0.10	57.00	200.00	10.53	19.18
0.96	0.04	59.55	200.00	17.66	12.66
1.00	0.00	60.00	200.00	17.92	0.00

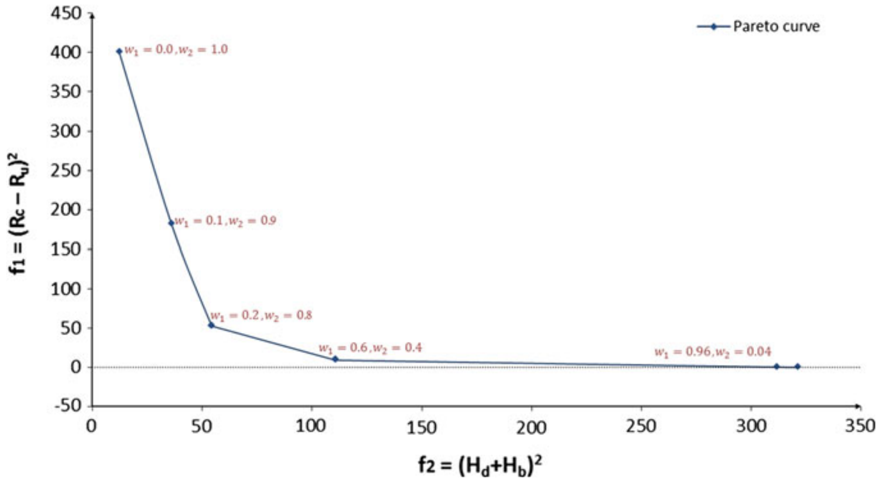

Fig. 4.12 The Pareto frontier for the objectives functions f_1 and f_2 for $m = 2$

To compute the value of power in all desired steps, we need to calculate the effective head as follows;

$$H_e = H_g - h_f - H_T \tag{4.9}$$

where, H_g is the gross head, h_f is the head losses due to loss at penstock, and H_T is the differential level between center of turbine and tailrace level. The variables H_g and H_T can be computed as follow, respectively;

$$H_g = R_u - R_{tail} = R_u - 994$$

and

Table 4.7 Manning's roughness coefficient (*n*) for different materials

Material	Manning's roughness coefficient (*n*)
Earth asphalt	0.025
Steel—smooth	0.012
Wood	0.012
Asphalt	0.016
Concrete—steel forms	0.011
Plastic	0.009

$$H_T = R_T - R_{tail} = 997 - 994 = 3.0$$

The friction loss at penstock (h_f) for a circular pipe can be computed using the Darcy-Weisbach equation as;

$$h_{\mathrm{f}} = \mathrm{f}\frac{L}{D}\frac{V^2}{2g} \tag{4.10}$$

where, f is friction factor that can be found from Moody diagram, L is length of penstock (m), V is velocity at penstock (m/s), D is diameter of penstock (m), and g is gravitational acceleration (m/s^2). The friction factor also can be calculated as function of pipe diameter and coefficient of roughness (n) using the following equation;

$$\begin{aligned} \mathrm{f} &= 124.5\frac{n^2}{D^3} \quad \text{in SI unit} \\ \mathrm{f} &= 185\frac{n^2}{D^3} \quad \text{in English unit} \end{aligned} \tag{4.11}$$

The coefficient of roughness or Manning's roughness coefficient for different materials can be found in various references. However, the values of this coefficient for several commonly used materials are presented in Table 4.7.

According to the presented information above, the value of f can be estimated as;

$$\mathrm{f} = 124.5\frac{n^2}{D^3} = 124.5\frac{0.012^2}{3^3} = 6.64 \times 10^{-4}$$

The next important parameter in Eq. (4.10) is velocity at penstock. In general, this parameter is expressed in terms of flow rate in the pipe as;

$$V^2 = \frac{Q^2}{A_w^2} \tag{4.12}$$

where, A_w is the cross-sectional wetted area (m^2) that is an implicit function of pipe flow, cross-sectional area of flow, pipe slope, etc. For simplicity and considering the maximum losses, it is assumed pipe is flowing full and so, the wetted area is calculated as;

Table 4.8 The generated powers and damage costs based on optimized water elevations for $m = 2$

w_1	w_2	R_u	$R_u - R_{tail}$ (m)	H_e (m)	P (MW)	D(1,000 \$)
0.00	1.00	1,040.00	46.00	42.55	0.00	0.00
0.10	0.90	1,046.52	52.52	49.07	43.32	1.98
0.20	0.80	1,052.75	58.75	55.30	48.82	3.20
0.30	0.70	1,052.75	58.75	55.30	48.82	3.20
0.40	0.60	1,052.75	58.75	55.30	48.82	3.20
0.50	0.50	1,052.75	58.75	55.30	48.82	3.20
0.60	0.40	1,057.00	63.00	59.55	105.15	9.65
0.65	0.35	1,057.00	63.00	59.55	105.15	9.65
0.80	0.20	1,057.00	63.00	59.55	105.15	9.65
0.90	0.10	1,057.00	63.00	59.55	105.15	9.65
0.96	0.04	1,059.55	65.55	62.10	109.65	117.05
1.00	0.00	1,060.00	66.00	62.55	110.45	128.42

$$A_w^2 = \left(\frac{\pi D^2}{4}\right)^2 = \frac{\pi^2 D^4}{16} \tag{4.13}$$

and hence, the Darcy-Weisbach equation can be written as;

$$h_{f_1} = \frac{8\mathrm{f}LQ^2}{g\pi^2 D^5} \tag{4.14}$$

Therefore, the value of h_{f_1} in this problem is;

$$h_{f_1} = \frac{8\mathrm{f}LQ^2}{g\pi^2 D^5} = \frac{8 \times (6.64 \times 10^{-4}) \times 50 \times 200^2}{9.81 \times \pi^2 \times 3^5} = 0.452\,\mathrm{m}$$

The values of the effective heads, generated powers, and damage costs based on the optimized water elevations are presented in Table 4.8.

For example, the values of generated hydropower and damage cost at $w_1 = 0.5$ and $w_2 = 0.5$ in Table 4.8 are calculated as;

$$\begin{cases} H_g = R_u - R_{tail} = 1{,}052.75 - 994 = 58.75\,(\mathrm{m}) \\ H_T = R_T - R_{tail} = 997 - 994 = 3.0\,(\mathrm{m}) \\ H_e = H_g - h_f - H_T = 58.75 - 0.452 - 3.0 = 55.30\,(\mathrm{m}) \\ P(\mathrm{MW}) = \eta\rho g Q H_e = \dfrac{0.9 \times 1{,}000 \times 9.81 \times 100 \times 55.30}{10^6} = 48.82\,(\mathrm{MW}) \\ D(1{,}000\,\$) = 825\ \exp[0.35(H_d - H_b)] = \dfrac{825\exp[0.35(7.37 - 3.5]}{1{,}000} = 3.20(1{,}000\,\$) \end{cases}$$

Figure 4.13 shows the effect of increasing water elevation in the reservoir on the generated powers and downstream flood damage costs. Based on this figure, both functions are increasing by raising water elevations in the reservoir, while the

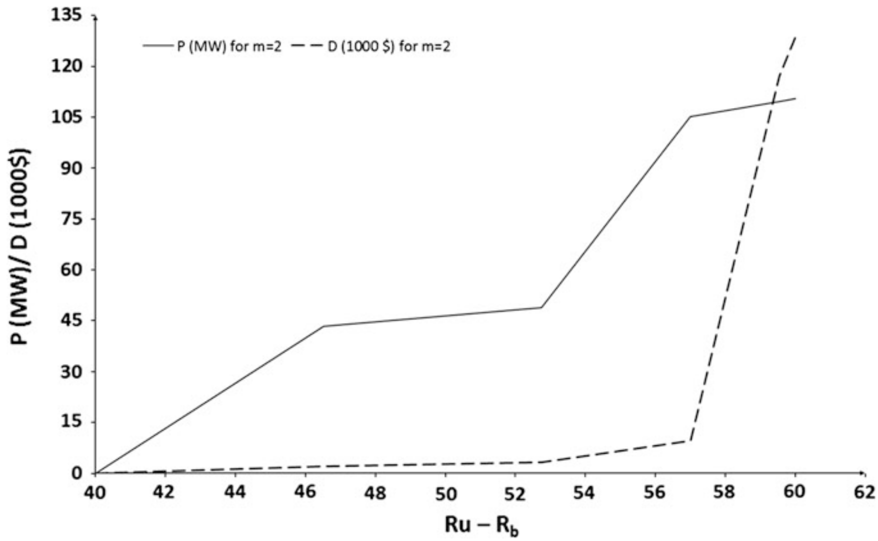

Fig. 4.13 The generated powers and downstream flood damages for $m = 2$

main purpose of optimization analysis in this problem is increasing the generated power and decreasing downstream flood damage simultaneously. Therefore, the results of optimization analysis can be applied to make trade-off between generated power and flood damage cost in this problem.

2. For $m = 1$

As it can be seen in Fig. 4.14 the first objective function is a linear function of water elevation in the reservoir for $m = 1$ and still there is conflict between two objectives in which increasing one of them results in decreasing the other simultaneously.

In this case, the aggregated objective function F can be written as;

$$F(R_u, H_d) = w_1 f_1(R_u) + w_2 f_2(H_d) = w_1(R_c - R_u) + w_2(H_d + H_b)$$

The constraints are the same as the previous condition, and the GRG nonlinear solution method of Excel is used in different combinations of weights to find the optimum points. The Pareto optimum points, minimum values of F, and the Pareto curve are shown in Table 4.9 and Figs. 4.15 and 4.16, respectively.

As noted above, the Pareto curve provides valuable information to verify all possibly conflicting and quantitative properties of an optimization system and it is shown in the Fig. 4.16.

The values of effective heads, generated powers, and damage costs based on the optimized water elevations in the case of $m = 1$ are presented in Table 4.10.

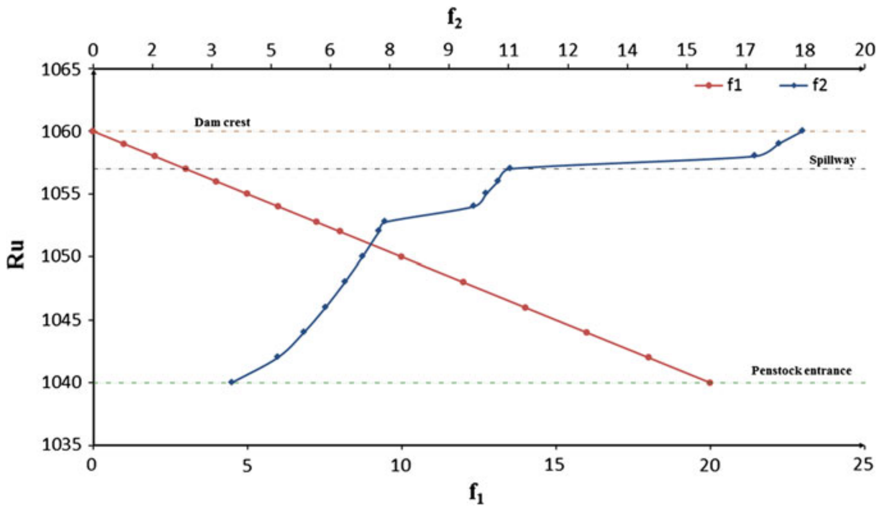

Fig. 4.14 The objective functions f_1 and f_2 versus water level in the reservoir for $m = 1$

Table 4.9 The minimums of F with generated hydropower and damage costs for $m = 1$

w_1	w_2	$R_u - R_b$ (m)	Q (m^3/s)	H_d (m)	$min\ F$
0.00	1.00	40.00	0.00	3.50	3.50
0.10	0.90	40.00	0.00	3.50	5.15
0.20	0.80	52.75	100.00	7.37	7.34
0.30	0.70	52.75	100.00	7.37	7.34
0.40	0.60	52.75	100.00	7.37	7.32
0.50	0.50	57.00	200.00	10.53	6.76
0.60	0.40	57.00	200.00	10.53	6.01
0.65	0.35	57.00	200.00	10.53	5.63
0.80	0.20	60.00	200.00	17.92	3.58
0.90	0.10	60.00	200.00	17.92	1.79
0.96	0.04	60.00	200.00	17.92	0.72
1.00	0.00	60.00	200.00	17.92	0.00

Figure 4.17 illustrates how the generated powers and downstream flood damage costs are changing by increasing water elevation in the reservoir. As it can be seen in this figure, the generated power and damage costs are increasing a little more smoothly in the case of $m = 2$ in comparison to $m = 1$. For example, P changes from 0 to 48.82 by changing w_1 from 0 to 0.2 for $m = 1$, while it varies from 0 to 43.32 and then to 48.82 by increasing w_1 from 0 to 0.2 in the case of $m = 2$. However, the overall trend is same and both achieved values are acceptable as results of optimization analysis.

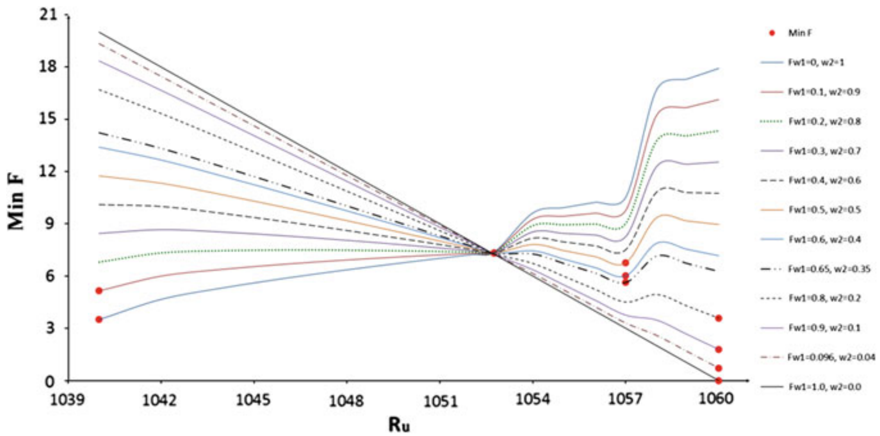

Fig. 4.15 Values of *F* and its minimums in different weights for $m = 1$

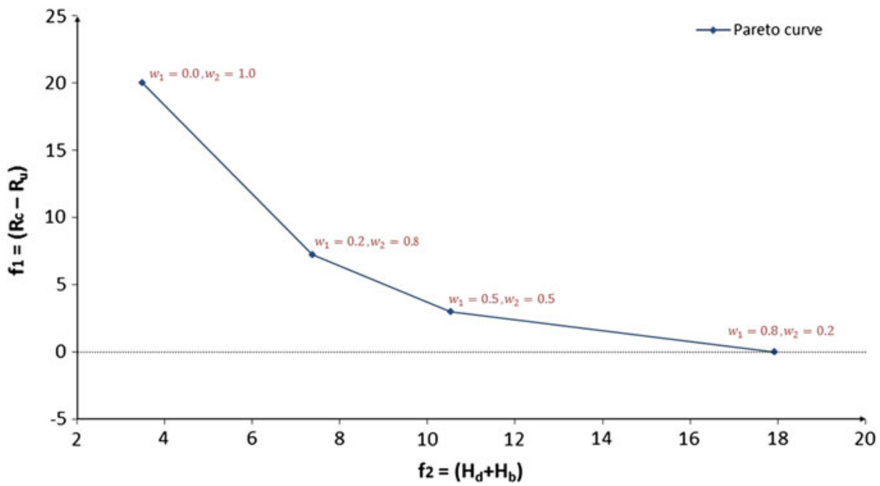

Fig. 4.16 The Pareto frontier for the objectives functions f_1 and f_2 for $m = 1$

4.2.2 Optimization of Broad-Crested Weir

Flow measurement is the quantification of fluid movement parameters. Since the early days of hydraulics, various flow measuring devices such as hydraulic structures have been developed and used in waterways such as open channels or rivers to estimate rate of flow based on the measured upstream water level (Boiten 1993). For instance, weirs as a type of hydraulic structure consist of some obstruction to increase water level and are commonly used to measure discharge. Among different

Table 4.10 The generated powers and damage costs based on optimized water elevations for $m = 1$

w_1	w_2	R_u	$R_u - R_{tail}$ (m)	H_e (m)	P (MW)	D (1,000 $)
0.00	1.00	1,040.00	46.00	42.55	0.00	0.00
0.10	0.90	1,046.52	46.00	42.55	0.00	0.83
0.20	0.80	1,052.75	58.75	55.30	48.82	3.20
0.30	0.70	1,052.75	58.75	55.30	48.82	3.20
0.40	0.60	1,052.75	58.75	55.30	48.82	3.20
0.50	0.50	1,052.75	63.00	59.55	105.15	9.65
0.60	0.40	1,057.00	63.00	59.55	105.15	9.65
0.65	0.35	1,057.00	63.00	59.55	105.15	9.65
0.80	0.20	1,057.00	66.00	62.55	110.45	128.42
0.90	0.10	1,057.00	66.00	62.55	110.45	128.42
0.96	0.04	1,059.55	66.00	62.55	110.45	128.42
1.00	0.00	1,060.00	66.00	62.55	110.45	128.42

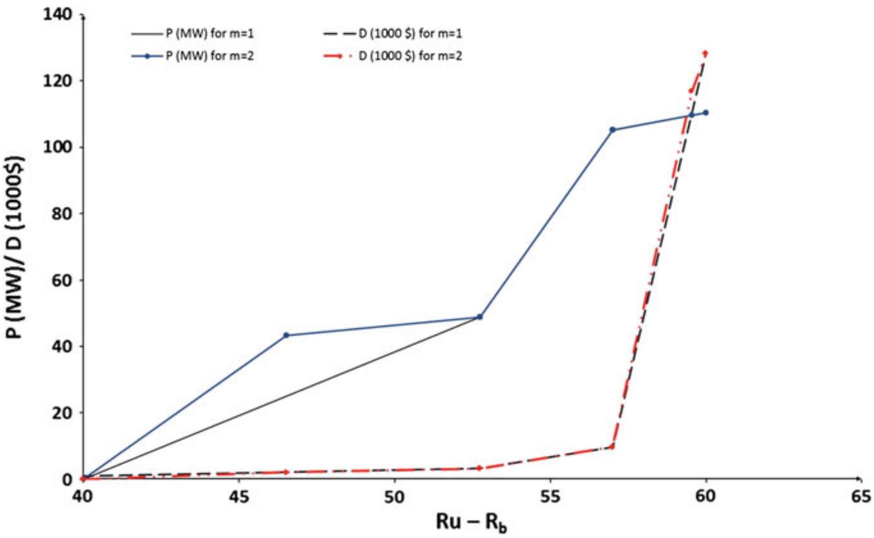

Fig. 4.17 The generated powers and downstream flood damages for $m = 1$ and $m = 2$

types of weirs, rectangular broad-crested weir is one of the most widely used hydraulic structures in open channels and rivers. It is a common engineering structure with a horizontal crest above which the streamlines are practically straight and parallel. This simple structure has often been used in irrigation systems, hydroelectric schemes, and highways. The rectangular broad-crested weirs with 90° upstream face slope which is known as square-edged weir (edge refers to the entrance from the approach channel) is a simple design structure with the following advantages;

1. Constant discharge coefficient in optimum flow condition,
2. Less sensitivity to downstream submergence,
3. Simple design and construction,
4. Low construction and utility costs (Goodarzi et al. 2012a).

In particular situations, the weir's structural design can present some flexibility for modifying the upstream face slopes to provide better hydraulic characteristics and measure discharge efficiency at higher precision. Goodarzi et al. (2012a) experimentally studied different models of broad-crested weirs with a rectangular compound cross section. In their study, the upstream slope of weir was changed from 90° to 75°, 60°, 45°, 30°, 22.5°, 15°, and 10° and a new correction factor to estimate the discharge coefficient over weirs with various upstream slopes were introduced. The results of this study showed decreasing upstream slopes from 90° to 10° leading to increasing discharge coefficient. In general, the rate of flow and the upstream water level over the crest can be related as;

$$Q = C_{d_\alpha}\left[\frac{2}{3}\left(\frac{2}{3}g\right)^{1/2}\right]B \cdot h_1^{3/2} \tag{4.15}$$

where Q is flow discharge $(\mathrm{m^3/s})$, B is the weir's breadth which spans the full channel width, g is gravitational acceleration $(\mathrm{m/s^2})$, and C_{d_α} is discharge coefficient for the weir with upstream face slope α. This coefficient can be calculated using the parameter C_r which is the ratio of C_{d_α} to the discharge coefficient of a standard broad-crested weir $(C_{d_{90}})$ as follow;

$$C_r = 1.0 + \frac{4.63\, Cos^{3/2}(\alpha)}{g(2.33 + \xi^4)} \tag{4.16a}$$

$$C_r = \frac{C_{d_\alpha}}{C_{d_{90}}} \tag{4.16b}$$

where P is weir height, L is weir length, h_1 is depth of flow upstream of the standard broad-crested weir, $\xi = h_1/L$.

Example 4.4 Consider a broad crested weir with the crest length, $L = 1.75$ (m), weir's breadth $B = 3.1$ (m) which spans the full channel width, weir height $P = 1.25$ (m), and $C_{d_{90}} = 0.9$ in a channel with the average flow of $Q_u = 1.5\,(\mathrm{m^3/s})$ and the standard deviation of $\sigma_Q = 0.2\,(\mathrm{m^3/s})$. The schematic view of the weir is shown in the Fig. 4.18. The cost of building the weir including the construction cost and the expense of materials is estimated using the following equation;

$$\mathrm{Cost}(\$) = 115 + 365.156\, V_T^{0.0775}$$

where, $V_T(\mathrm{m^3})$ is the total volume of the weir.

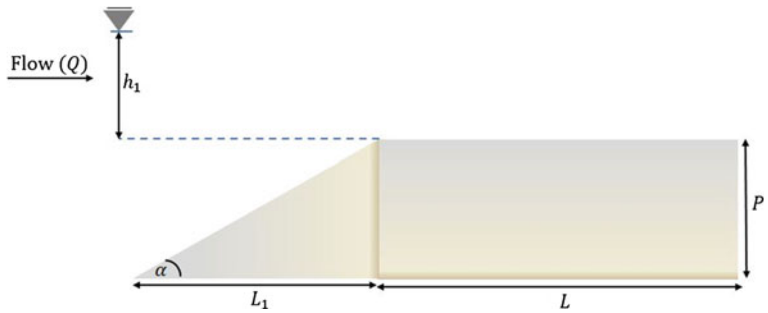

Fig. 4.18 Schematic view of broad-crested weir

It is important to note that the volume of weir is computed based on the following relation;

$$V_T = V_r + V_p$$

in which, V_r is the volume of cubic central part of the weir with a constant value and it can be calculated as;

$$V_r(\mathrm{m}^3) = B \times P \times L$$

and V_p is the volume of prism section which is function of upstream face slope, as;

$$V_p\left(\mathrm{m}^3\right) = \frac{P \times L_1}{2} \times B = \frac{P^2 \times B}{2} Tan(\alpha)$$

Based on the aforementioned information, design a weir with the maximize discharge coefficient (C_{d_α}) and minimum cost of construction. Determine the appropriate objective functions and obtain the Pareto front for this problem based on the weighted method.

Solution: The objective functions in this problem can be defined as;

$$\max \quad C_{d_\alpha} = C_r \times C_{d_{90}} = \left[1.0 + \frac{4.63\, Cos^{3/2}(\alpha)}{g(2.33 + \xi^4)}\right] \times C_{d_{90}}$$

and

$$\min \quad \mathrm{Cost}(1{,}000\ \$) = \frac{\left[115 + 365.156 V_T^{0.0775}\right]}{1{,}000}$$

As the average discharge in the channel is $Q_u = 1.5\,(\mathrm{m}^3/\mathrm{s})$ with the standard deviation of $\sigma_Q = 0.2\,(\mathrm{m}^3/\mathrm{s})$, and the upstream face slopes should be varied between 90° to 10°, the following constraints should be considered in the design of the weir;

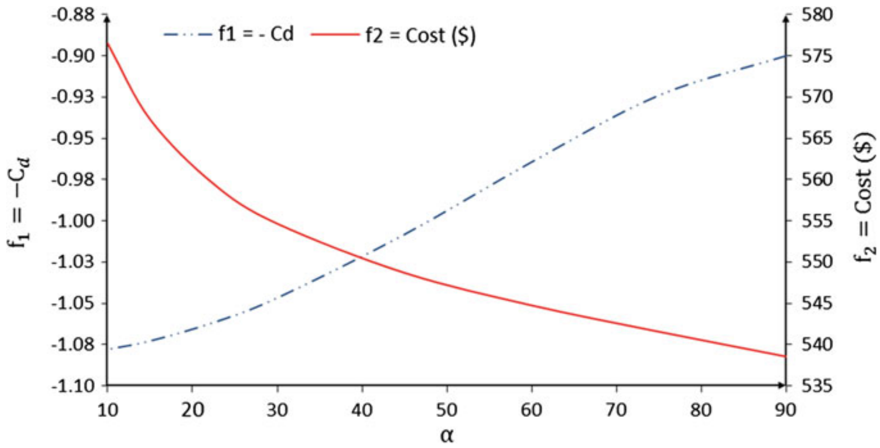

Fig. 4.19 The objective functions versus upstream face slopes

$$\underbrace{Q_u - \sigma_Q}_{1.5-0.2=1.3} \leq Q \leq \underbrace{Q_u + \sigma_Q}_{1.5+0.2=1.7}$$

$$10^\circ \leq \alpha \leq 90^\circ$$

If the upstream face slope approaches to 90°, the cost of weir construction decreases since the volume reduces, whereas the discharge coefficient also decreases. In this problem, on the one hand we need to decrease the upstream face slopes (approach to 10°) to get higher value for discharge coefficient and on the other hand, the values of α should be increased (approach to 90°) to decrease the cost of building. To convert the problem into a single objective function and solve it using the weighted method, the objective functions should be written as;

$$\min \quad \mathrm{f}_1(\alpha, \xi) = -C_{d_\alpha} = -\left[1.0 + \frac{4.63\, Cos^{3/2}(\alpha)}{g(2.33 + \xi^4)}\right] \times C_{d_{90}}$$

$$\min \quad \mathrm{f}_2(\mathrm{V}_T) = \mathrm{Cost}(1{,}000\,\$) = \frac{\left[115 + 365.156\, V_T^{0.0775}\right]}{1{,}000}$$

Based on the weighting method, we have;

$$F = w_1 \mathrm{f}_1 + w_2 \mathrm{f}_2 = w_1 C_{d_\alpha} + w_2 \mathrm{Cost}$$

As it can be seen in the Fig. 4.19, there is conflict between two defined objective functions in which by increasing the upstream face slopes (α), the first objective f_1 is increasing, while, the second objective function f_2 is decreasing.

The GRG nonlinear method of Excel is applied to find the optimal solution and estimate the values of discharge coefficient, upstream depth, total volume of weir, and cost of building in different weights and the results are presented in the Table 4.11.

Table 4.11 The optimized values of decision variables

w_1	w_2	F	α	$-C_d$	C_d	h (m)	Volume (m^3)	Cost ($)
1	0	−1.078	10.00	−1.078	1.078	0.533	12.320	558.609
0.9	0.1	−0.914	10.00	−1.078	1.078	0.533	12.320	558.609
0.8	0.2	−0.750	10.00	−1.078	1.078	0.533	12.320	558.609
0.7	0.3	−0.587	10.00	−1.078	1.078	0.533	12.320	558.609
0.6	0.4	−0.423	11.03	−1.077	1.077	0.533	11.793	557.110
0.5	0.5	−0.260	12.93	−1.075	1.075	0.534	11.036	554.842
0.4	0.6	−0.097	15.15	−1.072	1.072	0.535	10.388	552.786
0.3	0.7	0.065	18.01	−1.068	1.068	0.536	9.785	550.761
0.2	0.8	0.227	22.29	−1.062	1.062	0.538	9.163	548.549
0.1	0.9	0.387	31.15	−1.044	1.044	0.544	8.397	545.624
0	1	0.539	90.00	−0.900	0.900	0.601	6.781	538.551

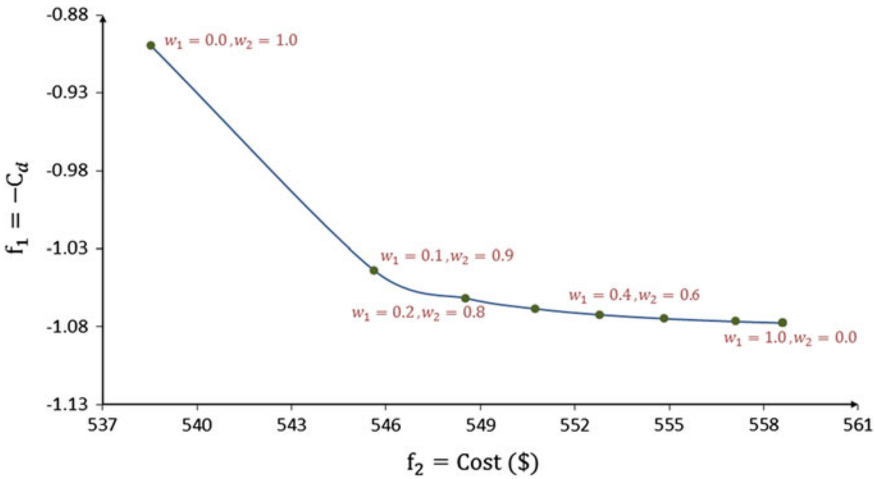

Fig. 4.20 The Pareto front for desired broad crested weir

Figure 4.20 shows the Pareto front for two objective functions f_1 and f_2. As it can be seen from this figure, any optimal solution for discharge coefficient cannot be better off without making the other objective functions worse off. In other words, the higher values of discharge coefficient impose much more costs of construction. Therefore, the decision maker needs to make the appropriate decision based on the hydraulic requirement efficiency of weir and also the available financial resources of the project.

Figure 4.21 shows how the discharge coefficient and the cost of building of weir are changed by increasing the upstream face slope. Based on this figure, both of those decision variables are increasing by approaching the upstream slopes from 90° to 10°. The presented results in Table 4.11 are only for 11 different weights

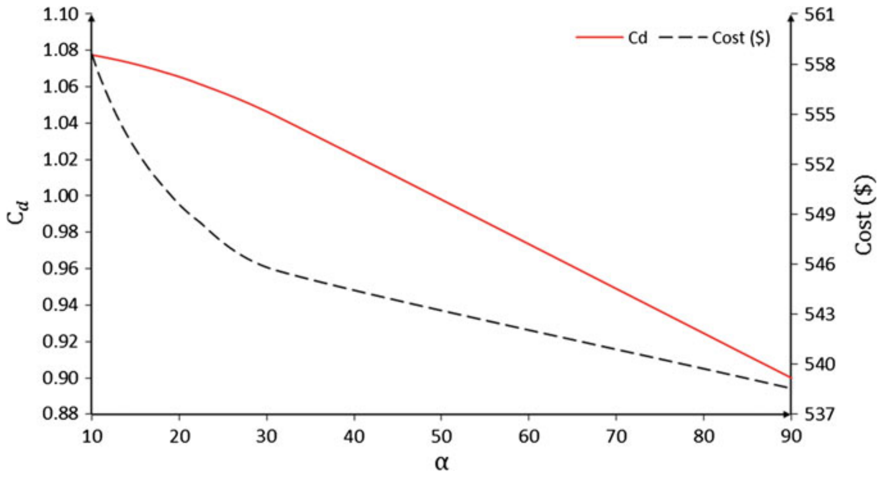

Fig. 4.21 Changing discharge coefficient and the cost of building versus upstream face slope

with the step 0.1, while, many in-between values can be used by choosing smaller step size. An efficient way to produce much more weights is applying random generator techniques such as Monte Carlo technique.

Monte Carlo is one of the most famous and widely used statistical methods from the early 1940s. With the remarkable increase in computer capabilities and the development of variance reduction schemes in recent years, application of this method has increased in different scientific fields. The basic part of this method is iteration and generation of random variables from a specific range. In other words, it is a numerical simulation which replicates stochastic input random variables from desired probability distribution (Goodarzi et al. 2012b). In the case of this problem, 200 random numbers which are uniformly distributed between [0, 1] are generated for w_1 by using Excel Data Analysis Tools. The estimated Pareto curve in conjunction with values of discharge coefficient, upstream depth, total volume of weir, and cost of building are presented in the Fig. 4.22 and Table 4.12, respectively.

4.3 ε-Constraint Method

In addition to the weighted method, the ε-constraint approach can also be used to build the Pareto set for a multi-objective optimization problem. This method was first proposed by Chankong and Haimes in 1983 to transform a multiobjective problem into a traditional single objective problem by choosing one objective as main objective function and considering the remaining objectives as constraints in

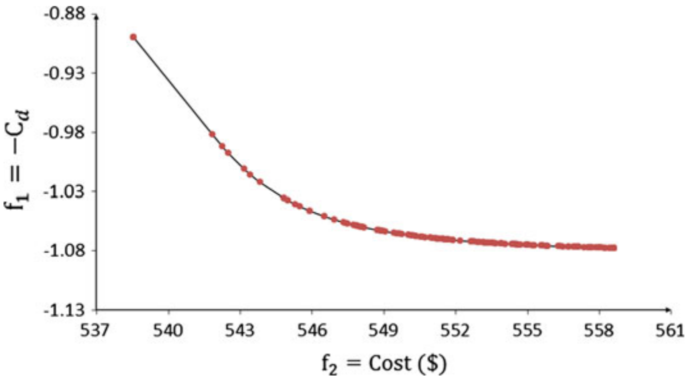

Fig. 4.22 Pareto curve based on generated random data

the desired optimization problem. Based on this method, the optimization problem mathematically can be defined as;

$$\min \quad f_i(X); \quad X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \tag{4.15}$$

Subject to;

$$g_j(X) \leq 0, \quad j = 1, 2, \ldots m$$
$$f_k(X) \leq \varepsilon_k \quad k = 1, \ldots n; \; k \neq i$$

where, $\varepsilon = \varepsilon_1, \varepsilon_2, \ldots, \varepsilon_{i-1}, \varepsilon_{i+1}, \ldots, \varepsilon_n$.

Therefore, the Pareto set is generated by solving the optimization problem repeatedly for different values of ε. It is important to note that this method is not very efficient for problems with more than two objective functions, and also finding the Pareto solution strongly depends on the choice of ε. Example 4.5 is solved using the ε-constraint method to become more familiar with the concept of this technique in solving optimization problems, and also compare the achieved results based on this method with the outcomes of weighted approaches.

Example 4.5 Minimize the following functions $f_1(x)$ and $f_2(x)$;

$$f_1(x) = 6(x - 7)^2 + 2x + 17$$
$$f_2(x) = (x - 2)^3 + 23$$

Subject to

$$2 \leq x \leq 10$$

Table 4.12 The optimized values based on the random weights

i	w_1	w_2	C_d	h (m)	α	Cost ($)
1	0.9963	0.0037	1.0775	0.5331	10.00	558.60
2	0.9948	0.0052	1.0775	0.5331	10.00	558.60
3	0.9948	0.0052	1.0775	0.5331	10.00	558.60
.	.	.	.	.	.	.
.	.	.	.	.	.	.
64	0.6575	0.3425	1.0775	0.5331	10.00	558.60
65	0.6536	0.3464	1.0775	0.5331	10.07	558.49
66	0.6509	0.3491	1.0774	0.5331	10.119	558.42
.	.	.	.	.	.	.
.	.	.	.	.	.	.
80	0.6005	0.3995	1.0766	0.5333	11.01	557.12
81	0.5983	0.4017	1.0766	0.5334	11.05	557.06
82	0.5947	0.4053	1.0765	0.5334	11.12	556.98
.	.	.	.	.	.	.
.	.	.	.	.	.	.
102	0.4852	0.5148	1.0744	0.5341	13.23	554.53
103	0.4823	0.5177	1.0744	0.5341	13.29	554.47
104	0.4799	0.5201	1.0743	0.5341	13.34	554.42
.	.	.	.	.	.	.
.	.	.	.	.	.	.
123	0.3948	0.6052	1.0721	0.5349	15.28	552.68
124	0.3933	0.6067	1.0720	0.5349	15.32	552.65
125	0.3919	0.6081	1.0720	0.5349	15.35	552.62
.	.	.	.	.	.	.
.	.	.	.	.	.	.
167	0.1827	0.8173	1.0598	0.5390	23.31	548.12
168	0.1797	0.8203	1.0595	0.5391	23.50	548.05
169	0.1790	0.8210	1.0594	0.5391	23.55	548.03
.	.	.	.	.	.	.
.	.	.	.	.	.	.
187	0.0806	0.9194	1.0356	0.5473	34.59	544.82
188	0.0612	0.9388	1.0223	0.5521	39.77	543.82
189	0.0549	0.9451	1.0159	0.5544	42.16	543.42
.	.	.	.	.	.	.
.	.	.	.	.	.	.
198	0.0096	0.9904	0.9000	0.6010	90.00	538.55
199	0.0080	0.9920	0.9000	0.6010	90.00	538.55
200	0.0038	0.0249	0.9000	0.6010	90.00	538.55

Solution: As mentioned above, one of the objective functions would be considered as the main objective and the second one will be added to the constraints. It should be noted that it really doesn't matter which function is kept as the main objective and which one is considered as a constraint of the problem. Here the objective function $f_1(x)$ is chosen to be minimized and hence, the problem can be written as;

Table 4.13 Different values of x, $f_1(x)$, and $f_2(x)$ for the values of selected ε_2

x	min $f_1(x)$	$f_2(x)$	ε	x	min $f_1(x)$	$f_2(x)$	ε
–	–	–	15	5.476	41.887	65.000	65
–	–	–	22	5.733	38.104	75.000	75
2.019	169.915	23.000	23	5.958	35.432	85.000	85
3.000	119.000	24.000	24	6.254	32.845	100.000	100
3.913	82.006	30.000	30	6.514	31.444	115.000	115
4.289	69.662	35.000	35	6.747	30.878	130.000	130
4.571	61.535	40.000	40	6.820	30.834	135.000	135
4.802	55.590	45.000	45	6.833	30.833	135.912	136
5.000	51.000	50.000	50	6.833	30.833	135.912	150
5.175	47.338	55.000	55	6.833	30.833	135.912	190

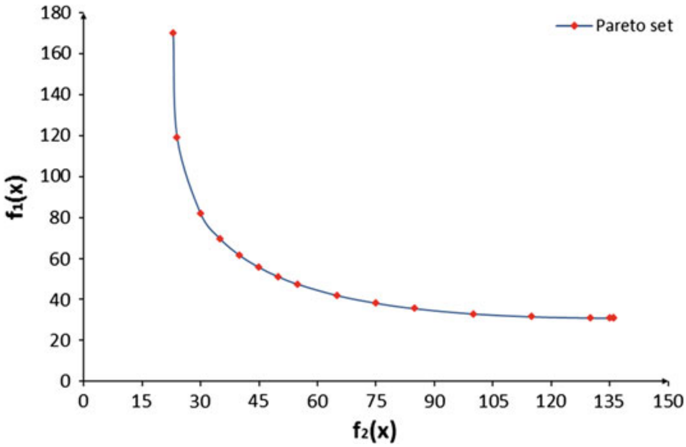

Fig. 4.23 The Pareto set based on ε-constraints method

$$\min \quad f_1(x) = 6(x-7)^2 + 2x + 17$$

subject to;

$$2 \le x \le 10$$
$$f_2(x) = (x-2)^3 + 23 \le \varepsilon$$

Now, we need to solve the problem for different values of ε_2 to find the Pareto set in this problem. Table 4.13 shows the values of x, $f_1(x)$, and $f_2(x)$ for the selected values of ε. Based on the results, there is no feasible solution for $\varepsilon < 23$, shown by the dash line in the table, while the values of x, $f_1(x)$, and $f_2(x)$ approaches to a constant value for $\varepsilon > 135$.

In this problem, the GRG nonlinear method of Excel is applied to solve the optimization problem. Figure 4.23 demonstrates the Pareto optimal solutions based on ε-constraint technique for desired two-objective minimization problem.

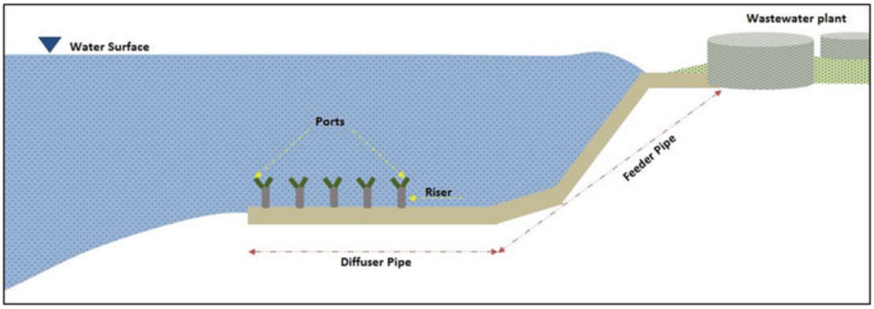

Fig. 4.24 A marine wastewater disposal system

4.3.1 Optimization of Marine Outfalls

Wastewater can be defined as combination of liquid and solid wastes and it can be generated from many resources like domestic (e.g., household wastes like toilets, shower, cooking, laundry, and the floor drains) and industrial sources (e.g., food waste, mining, oil waste, fertilizers, and toxic chemicals). Three important types of wastewater can be named as sewage, influent, and effluent. In general, the domestic wastewater is known as *sewage*, the wastewater which is flowing into a treatment plant is known as *influent*, and the treated wastewater from treatment plant which can be discharged into a stream, river, lagoon, lake or ocean is called *effluent*. The main purpose of wastewater treatment before disposal is removing industrial and human wastes to protect the health of our community and preventing the potential diseases as well as supplying clean water. The treatment process typically includes three main steps as;

1. Preliminary treatment; to remove large particles and materials in raw wastewater,
2. Primary treatment; to remove suspended solids by sedimentation, and
3. Secondary treatment; to remove or reduce residual organics that are left from the previous steps.

After completion of the treatment procedures, the wastewater flows through a marine outfall to the sea. The marine outfall is an important part of costal infrastructure to transform sewage or effluent from desired sources to an undersea disposal point far from coast. The main objectives of marine outfall systems are disposing of wastewater in a safe point to reduce the health risk of coastal communities, swimmers, and temporary coastal visitors. A marine outfall system generally consists of three main parts as follows;

1. An outfall pipe or feeder pipe,
2. Diffuser pipe, and
3. Risers and ports (Fig. 4.24).

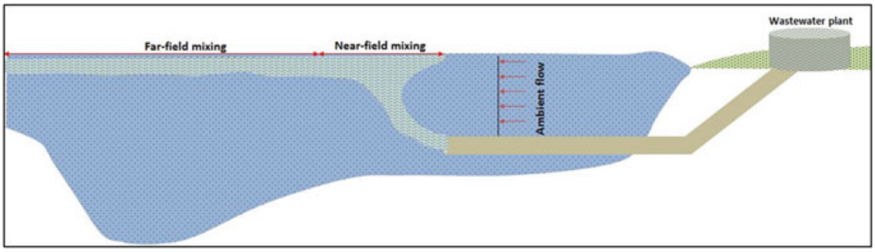

Fig. 4.25 Near-field and far-field locations for a single outfall pipe

It is important to note that mixing zone is defined as a limited area around diffusers where an effluent discharges into ocean, and mixing the effluent with the water body results in dilution of effluent in the vicinity of the plume. To estimate dilution the following relation can be applied;

$$S_n = \frac{C_0 - C_a}{C - C_a} \tag{4.16}$$

where, C, C_0, and C_a are conductivity at sampling point, effluent source, and ambient water, respectively (Abessi et al. 2012). It is important to note that although mixing changes the quality of ambient, the standards of water quality in this zone should be limited to critical toxicity condition.

Near-field (or initial mixing region) and far-field are two major phases of mixing processes of an effluent. In the near-field region, rapid mixing takes place and the shape of this area is affected by the source characteristics, outfalls geometry and interaction of buoyancy flux with the ambient current (Roberts et al. 1989). On the other hand in the far-field region, the effect of source is reduced and also the rate of increasing dilution is slower than near-field area (Kang et al. 1999). Figure 4.25 shows the near-field and far-field locations for a single outfall pipe that discharges effluent into the ocean. Based on Tian et al. (2004), the edge of near-field region is approximately placed within 10 % of whole mixing zone.

In the following section optimization analysis of an outfall system as a practical example in designing marine outfalls is presented.

Example 4.6 Assume, a city needs to discharge an average municipal wastewater of $Q_T = 1.6\,\mathrm{m}^3/\mathrm{s}$ into the ocean though a marine outfall. In preliminary design, outfall considered as a long pipe on the floor at the shallow coast of the study area. The outfall terminates to a diffuser with T-shape nozzles spaced equal to S from each other (Fig. 4.26). The discharge needs to be released at least 200 m (or $L \geq 200$ m) far from the shore due to local environmental regulations. To diminish environmental impacts of sewage in local marine ecosystem reaching minimum dilution 100 at the end of near-field suggested in receiving ambient water quality standards.

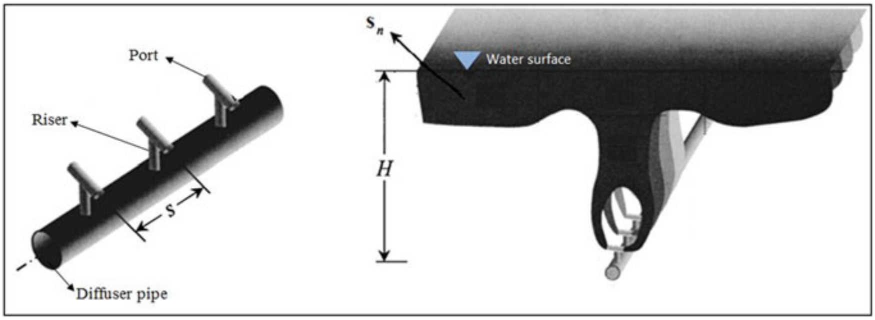

Fig. 4.26 Sketch of outfall in primarily design (Tian et al. 2004)

It is assumed that the receiving water is unstratified (homogeneous) for the sake of design procedure, and the diffusers are shortly spaced and flow behave as line plume;

$$\frac{S_n\, q}{b^{1/3} H} = 0.49 \quad \text{for } \frac{S}{H} \le 0.3 \tag{4.17}$$

and when the diffuser in long spaced and flow behave as point plume, we have;

$$\frac{S_n\, q}{b^{1/3} H} = 0.41 \left(\frac{S}{H}\right)^{-2/3} \quad \text{for } \frac{S}{H} \ge 1 \tag{4.18}$$

where, $H(m)$ is the water depth, S_n is dilution, $S(m)$ space between nozzles, $L_d(m)$ diffuser length, n total number of ports, q is volume flux, b is buoyancy flux and they are defined as;

$$\begin{aligned} q &= \frac{Q_T}{L_d} \\ b &= g_0' q \end{aligned} \tag{4.19}$$

in which, g_0' is modified acceleration due to gravity and can be calculated as;

$$g_0' = \frac{g(\rho_a - \rho_0)}{\rho_0} \tag{4.20}$$

in which, $\rho_0(\mathrm{kg/m^3})$ is effluent density and $\rho_a(\mathrm{kg/m^3})$ is ambient density (Tian et al. 2004).

As the nozzles are T-shaped, each riser has two ports on it in which $L_d = \frac{n}{2} S$. In addition, it is assumed that the effluent and ambient densities are 998 and $1{,}021\ \mathrm{kg/m^3}$, respectively; and the ambient depth in the area changes linearly from the shore as follow;

$$H = 0.03 \times (L + L_d)$$

where, L and L_d are the feeder pipe and diffuser pipe lengths.

Determine the optimal design of outfall to meet environmental required standards with the minimum cost when the construction cost for each meter of outfall initial pipe in 25,000 $ and for each meter of diffuser is 45,000 $. In this problem for the sake of simplicity, it is assumed that diffusers are shortly spaced and flow behaves as line plume, and hence, only Eq. (4.17) should be applied. In addition, the maximum allocated budget for this project is $25 M.

Solution: In this problem, we are going to maximize dilution while minimizing the cost of building project. Based on Eq. (4.17), we have;

$$\frac{S_n q}{b^{1/3}H} = 0.49 \Rightarrow S_n = \frac{0.49 \times b^{1/3}H}{q}$$

$$S_n = \frac{\left[0.49 \times \left(g_0' \frac{Q_T}{L_d}\right)^{1/3}\right] \times [0.03 \times (L + L_d)]}{\frac{Q_T}{L_d}}$$

$$= \frac{\left[0.49 \times \left(\frac{g(\rho_a - \rho_0)}{\rho_0}\right)^{1/3}\right] \times [0.03 \times (L + L_d)]}{\left(\frac{Q_T}{L_d}\right)^{2/3}}$$

By applying the values of g, ρ_a, ρ_0, and Q_T, dilution can be calculated as;

$$S_n = \frac{\left[0.49 \times \left(\frac{9.81 \times (1{,}021 - 998)}{998}\right)^{1/3}\right] \times [0.03 \times (L + L_d)]}{\left(\frac{1.6}{L_d}\right)^{2/3}} \qquad (4.21)$$

$$\Rightarrow S_n = 0.0065 \times L_d^{2/3}(L + L_d)$$

On the other hand, it is assumed that there are two ports for each risers or $L_d = \frac{n}{2}S$, and so, Eq. (4.21) can be written in the following form;

$$S_n = 0.0065 \times \left(\frac{n}{2}S\right)^{2/3}\left(L + \frac{n}{2}S\right) \qquad (4.22)$$

Therefore, the first objective function that should be maximized is Eq. (4.22). The constraints of this problem are;

$$L \geq 200\,(\mathrm{m})$$
$$\frac{S}{H} \leq 0.3$$

The second objective is the cost function that needs be minimized. This function is sum of the cost of feeder pipe and diffuser package (including diffuser pipe, risers, and ports) and can be computed as;

$$C(\$) = C_1 + C_2$$

in which,

$$C_1 = 25{,}000 \times L$$
$$C_2 = 45{,}000 \times L_d$$

Therefore, the total cost is;

$$C(\$) = 25{,}000 \times L + 45{,}000 \times (\frac{n}{2} S)$$

In brief, the objective functions and constraints of this problem are;

$$\min \; -S_n = -\left[0.0065 \times \left(\frac{n}{2} S\right)^{2/3} \left(L + \frac{n}{2} S\right)\right]$$
$$\min \; C(\$) = 25{,}000 \times L + 45{,}000 \times \left(\frac{n}{2} S\right)$$

Subject to the following constraints;

$$L \geq 200\,(\text{m})$$
$$\frac{S}{H} \leq 0.3$$

It is important to note the values of dilution higher than 100 are acceptable in receiving ambient water quality. By applying the ε-constraint method, one of objective functions can be considered as main objective and the second one will be added to the constraints, and so, the problem will be changed into a single objective function. In this example, dilution function is kept to be maximized and the cost function is considered as constraints. Hence, the objective function and constraints can be written as follows;

$$\min \; -S_n = -\left[0.0065 \times \left(\frac{n}{2} S\right)^{2/3} \left(L + \frac{n}{2} S\right)\right]$$

Subject to;

$$25{,}000 \times L + 45{,}000 \times \left(\frac{n}{2} S\right) \leq \varepsilon$$
$$L \geq 200\,(\text{m})$$
$$\frac{S}{H} \leq 0.3$$

As the maximum allocated budget for this project is $25 M, the value of ε varies between 0 and 25 to find the optimum values of dilution, numbers of ports, length of feeder pipe, and length of diffuser pipe. This problem is solved for different values of ε, and the results of optimization analysis as well as Pareto set are presented in the Table 4.14 and Fig. 4.27, respectively.

Table 4.14 The optimized values of dilution and cost

n	S	L	S_n	Cost	ε	L_d	H	S/H
–	–	–	–	–	5	–	–	–
79.36	2.80	200	46.74	10	10	111.11	9.33	0.3
97.22	3.20	200	66.84	12	12	155.55	10.66	0.3
116.96	**3.80**	**200**	**100.68**	**15**	**15**	**222.22**	**12.66**	**0.3**
122.22	**4.00**	**200**	**112.94**	**16**	**16**	**244.44**	**13.33**	**0.3**
126.98	**4.20**	**200**	**125.67**	**17**	**17**	**266.67**	**14.00**	**0.3**
131.31	**4.40**	**200**	**138.87**	**18**	**18**	**288.88**	**14.66**	**0.3**
135.26	**4.60**	**200**	**152.53**	**19**	**19**	**311.11**	**15.33**	**0.3**
138.88	**4.80**	**200**	**166.66**	**20**	**20**	**333.33**	**16.00**	**0.3**
142.22	**5.00**	**200**	**181.23**	**21**	**21**	**355.55**	**16.66**	**0.3**
145.30	**5.20**	**200**	**196.26**	**22**	**22**	**3.77.77**	**17.33**	**0.3**
148.14	**5.40**	**200**	**211.72**	**23**	**23**	**400.00**	**18.00**	**0.3**
150.79	**5.60**	**200**	**227.62**	**24**	**24**	**422.22**	**18.66**	**0.3**
153.26	**5.80**	**200**	**243.96**	**25**	**25**	**444.44**	**19.33**	**0.3**
155.55	6.00	200	260.71	26	26	466.66	20.00	0.3
157.70	6.20	200	277.88	27	27	488.88	20.66	0.3
159.72	6.40	200	295.47	28	28	511.11	21.33	0.3

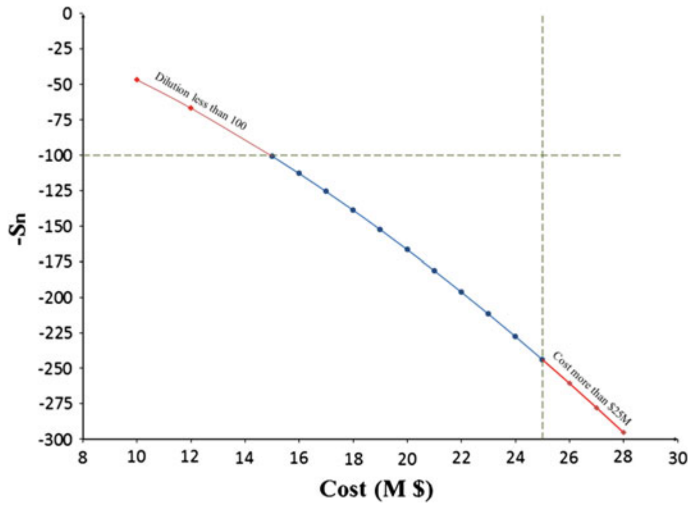

Fig. 4.27 The Pareto set solutions based on ε-constraints method

4.4 Problems

Problem 4.1 Minimize $\mathrm{F(x)} = \mathrm{g}[\mathrm{f}_1(\mathrm{x}), \mathrm{f}_2(\mathrm{x})]$ using the weighted method

$$\mathrm{f}_1(x) = (x-1)^3$$
$$\mathrm{f}_2(x) = -x^2 + 5x$$

Subject to the following constraint and plot the Pareto set;

$$2 \leq x \leq 4$$

Problem 4.2 Find the optimal solution of the following optimization problem using the weighted method and plot the Pareto front.

$$\min \mathrm{f}_1(x_1, x_2) = (x_1 - 1)^2 + (x_2 - 2)^2$$
$$\min \mathrm{f}_2(x_1, x_2) = -x_1^2 - x_2^2 + 13$$

Subject to:

$$5x_1 - 3x_2 \leq 9$$
$$6x_1 + 3x_2 \leq 5$$
$$-2 \leq x_1 \leq 4$$
$$-2 \leq x_2 \leq 6$$

Problem 4.3 Solve Example 4.3 by considering the turbine efficiencies as 65, 75, 85, and 95 %, and then, compare your results to see the effect of different efficiencies values on the optimization results.

Problem 4.4 Solve Example 4.3 by considering the penstock diameter $D = 2.5$, 2.5, and 3.5 m and then, compare your results to see the effect of different penstock diameters on the optimization results.

Problem 4.5 Assume two different values for discharge coefficients as $C_{d_{90}} = 0.8$ and $C_{d_{90}} = 0.95$ in Example 4.4, and find the Pareto front based on the weighted method.

Problem 4.6 Solve Example 4.4 by applying ε-constraint approach and compare the achieved results with the outcomes of weighted method.

Problem 4.7 Solve Example 4.6 if desired city needs to discharge an average municipal wastewater of $Q_T = 1.0$ and $2.2\,\mathrm{m}^3/\mathrm{s}$ into the ocean though a marine outfall. Compare your results to see the effect of different discharges on the optimization results.

Problem 4.8 Solve Example 4.6 if diffusers are long spaced.

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Chapter 5
Optimization Analysis Using LINGO and MATLAB

Abstract LINGO and MATLAB are two of the well-known computer programs and powerful languages used today for expressing optimization models. In this chapter, the process of solving both single and multiobjective optimization problems using those programs is presented in details. Furthermore, a number of useful examples are provided and solved step by step to better understand the application of LINGO and MATLAB in solving linear and nonlinear optimization problems.

5.1 LINGO

The procedure of solving optimization problems to find the optimal solutions usually needs a large number of mathematical computations in particular when there are numerous objective functions and constraints. With the remarkable increase in the computation power in the past decade, using custom designed computer programs and commercial software provide a smart way to solve different types of optimization problems more efficiently and in a reasonable time. Among all optimization models, LINGO is the most comprehensive optimization tool in solving a wide range of optimization problems and it is capable to model any large or small systems efficiently. The word LINGO is the abbreviation of Linear, Integer, Nonlinear, and Global Optimization that implies the ability of this software in finding optimal solution of linear, integer, and nonlinear problems. The main characteristics of LINGO are;

1. Availability of different optimizer functions including general integer and fast linear,
2. Useful features for editing input data,
3. Linked to the libraries of FORTRAN as a strong programming language in engineering and science,
4. Using the subroutines to run a given program in LINGO, and
5. Availability of a comprehensive on-line help for all LINGO users.

E. Goodarzi et al., *Introduction to Optimization Analysis in Hydrosystem Engineering*, Topics in Safety, Risk, Reliability and Quality 25, DOI: 10.1007/978-3-319-04400-2_5,

5.1.1 Creating an Input Model

As noted in the previous chapters, the structure of any optimization model includes three main elements as; objective function, decision variables, and constraints. Before building a model in LINGO, we need to precisely define and determine those elements to see what model should be optimized, what variables would be changed to find the optimal solution, and what restrictions must be applied on the decision variables to obtain valid solutions in a feasible region. Some of the main operators in LINGO are shown in Table 5.1.

The objective function should be written in the first line and the phrases Min and Max are used to demonstrate the objective as a minimization or maximization problem, respectively. Furthermore, the following points should be considered in writing desired model;

1. All comments in the model must be initialized with the exclamation point (!) and the text will be appeared in green color.
2. Each line should be ended with a semicolon (;), otherwise the LINGO cannot solve the problem.
3. As noted above, the first line involves the objective function and the constraints are immediately after the objective line without any word such as “subject to”, “st”, and etc.
4. The operators “<” and “>” also denote “≤” and “≥”, respectively.
5. To solve the model, we can use the Solve button or using the CTRL+S shortcut.

To become more familiar with the LINGO in solving an optimization problem, consider the following simple example.

Example 5.1 Maximize the function f(x) as;

$$\mathrm{f}(x) = 40x + 25y$$

Subject to;

$$x + y \leq 15$$
$$5 \leq y \leq 35$$

Solution: First we need to define the objective function as following;

$$\mathrm{Max} = 40 * \mathrm{x} + 25 * \mathrm{y};$$

Then, the constraints should be written immediately below the objective function as;

$$\mathrm{x} + \mathrm{y} < = 15;$$
$$\mathrm{y} < = 35;$$
$$\mathrm{y} > \ = 5;$$

Table 5.1 The main LINGO operators

Operator	Sign	Operator	Sign
Addition	+	Equals	=
Multiplication	*	Greater than	>
Subtraction	−	Less than	<
Division	/	Logics	NOT, AND, OR
Exponent	^	Strings	& and &&

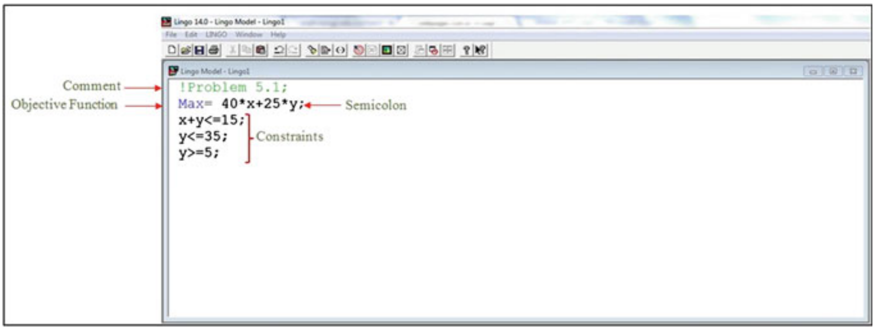


Fig. 5.1 Created model in LINGO

Figure 5.1 shows all elements of created model for this simple problem. As it can be seen from Fig. 5.1, the text between exclamation point and semicolon appears in green color and LINGO ignore this line when running the model.

It is important to note that comments can be written anywhere and it is not necessary to input them in the first line and before all commands. For example, Problem 5.1 is solved while each expression in the model includes a comment (Fig. 5.2).

5.1.2 *Solving Linear Models*

As already noted, the model can be solved using the *Solve* button on the toolbar or applying the CTRL+S shortcut. If an error occurs, LINGO shows an error message to notify user. For example, when a syntax error happens, LINGO provides the following error message as *a syntax error has occurred* (Fig. 5.3).

If no error occurs, LINGO runs the model to find the optimal solution using a solver module, and then, the *Solver Status* and *Solution Report* windows will appear (Figs. 5.4, 5.5). The solver status window includes lots of useful information such as;

- The class of model i.e., LP, NLP, or IP. In the case of this problem, the model is classified as LP or linear programming.

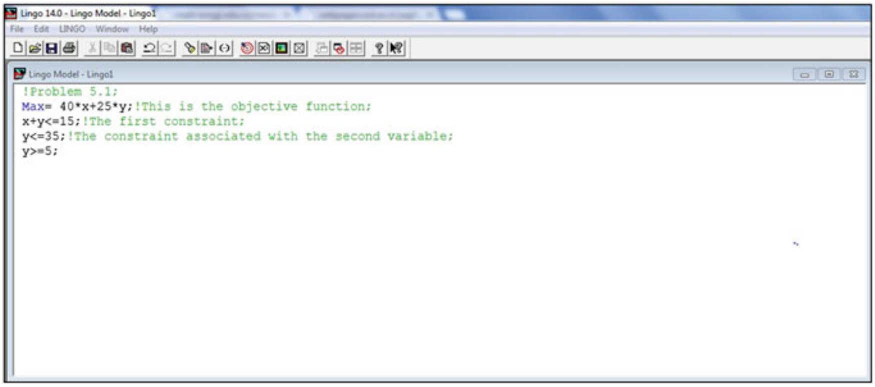


Fig. 5.2 The LINGO model with a number of comments on each line

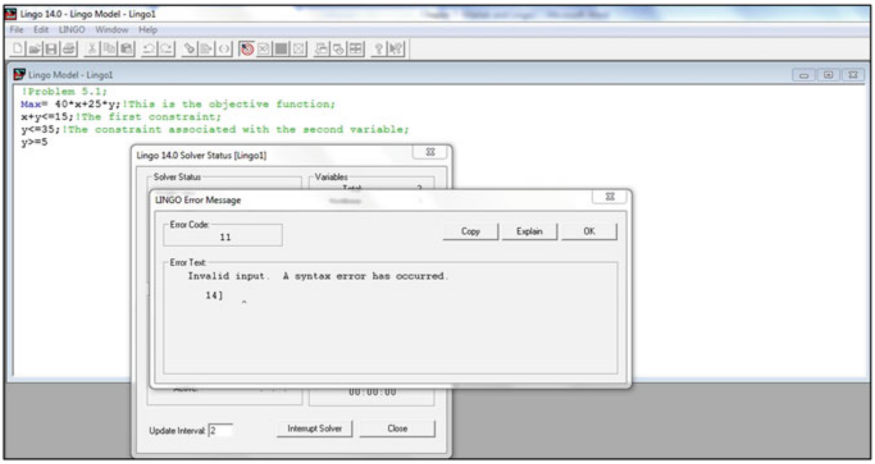


Fig. 5.3 The syntax error in LINGO

- The state of estimated solution which can be global optimum, local optimum, feasible or infeasible solution is presented. Based on Fig. 5.4, the model found a global optimum solution.
- The number of iteration that is used to find the optimal solution.
- The total number of variables as well as the nonlinear and integer variables also is shown by Solver Status. As this problem includes only two linear variables, the total number of variables equals two, while, the number of nonlinear is zero.

The next window that includes the optimal solution and the value of each variable associated with the optimal solution is the Solution Report Window, shown in Fig. 5.5. Based on the presented results in this window, the objective value which shows the maximum of function f(x) is 525.0 and variables *x* and *y* are

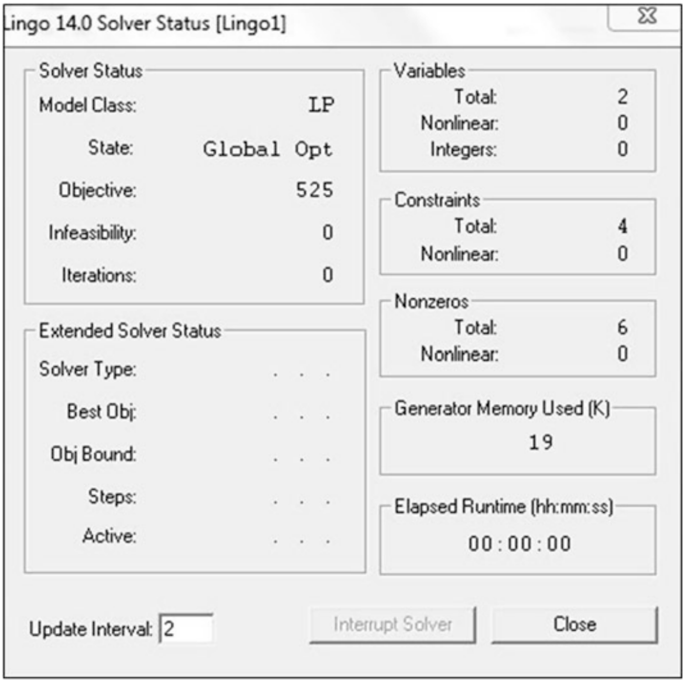


Fig. 5.4 Solver status window

10 and 5, respectively. The Slack or Surplus column can accept the following values;

- Zero; when desired constraint is satisfied as an equality,
- Positive; the number of variable units that should be added to the optimal value before desired constraint becomes an equality,
- Negative; shows the constraint is violated.

Example 5.2 Solve Problem 2.3 using LINGO. The objective function associated with this problem is;

$$\max R(\$) = \left(n_{p_1} \times 20\,\$\right) + \left(n_{p_2} \times 25\,\$\right)$$

and the constraints are;

$$n_{p_1} \leq 25; \quad n_{p_2} \leq 35; \quad 2n_{p_1} + 3n_{p_2} \leq 140$$

Solution: The input model in LINGO and the results of analysis are shown in Fig. 5.6.

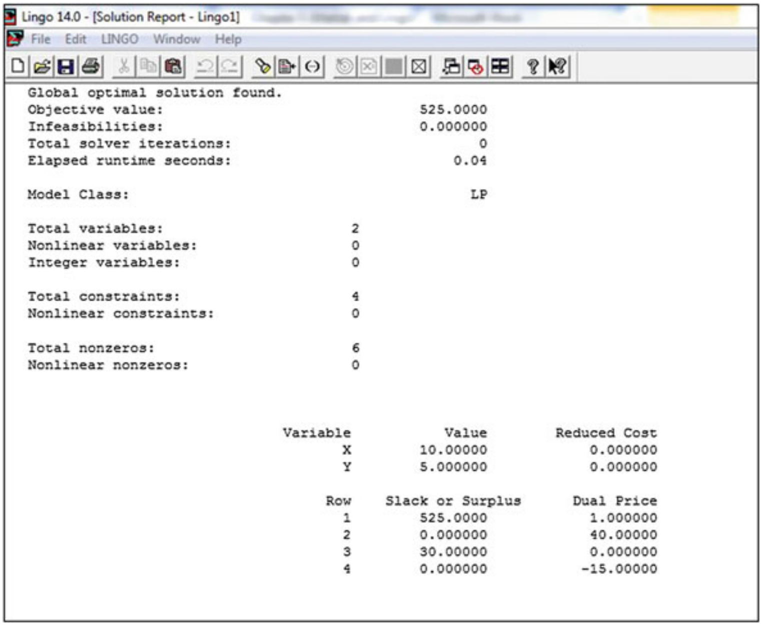


Fig. 5.5 Solution report window

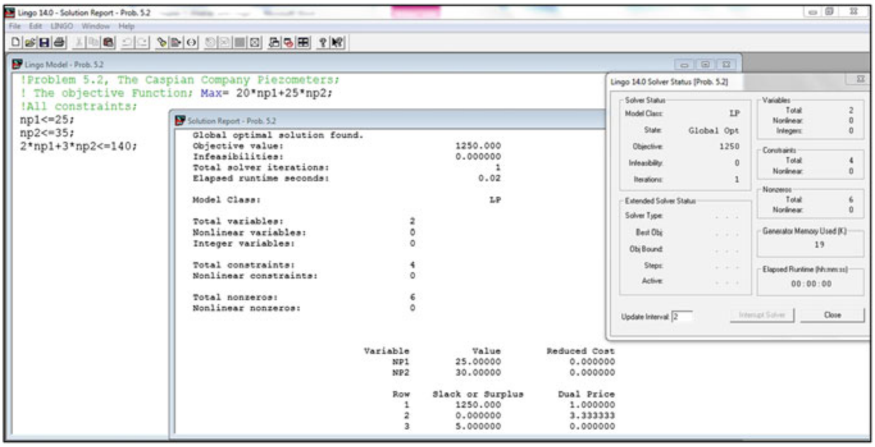


Fig. 5.6 The model and achieved results for Example 5.2

Based on the results, the model class is LP and the solution is a global optimum solution with the maximum profit of $1,250. The values of variables n_{p_1} and n_{p_2} that will produce the optimal solution are 25 and 30, respectively.

Example 5.3 Apply LINGO to solve the Problem 2.4 only for demand discharge Q_2 ($Q_A = 14$, $Q_B = 18$, and $Q_C = 20$) when there is no pump station. The objective function in that problem is defined as;

$$\begin{aligned}\min Z &= \left(C_{0,1,1}l_{0,1,1} + C_{0,1,2}l_{0,1,2}\right) + \left(C_{1,2,1}l_{1,2,1} + C_{1,2,2}l_{1,2,2}\right) + \left(C_{2,3,1}l_{2,3,1} + C_{2,3,2}l_{2,3,2}\right)\\ &\quad + \left(C_{2,4,1}l_{2,4,1} + C_{2,4,2}l_{2,4,2}\right) + \left(C_{1,5,1}l_{1,5,1} + C_{1,5,2}l_{1,5,2}\right)\\ &= \left(10 \times l_{0,1,1} + 15 \times l_{0,1,2}\right) + \left(10 \times l_{1,2,1} + 15 \times l_{1,2,2}\right) + \left(10 \times l_{2,3,1} + 15 \times l_{2,3,2}\right)\\ &\quad + \left(10 \times l_{2,4,1} + 15 \times l_{2,4,2}\right) + \left(10 \times l_{1,5,1} + 15 \times l_{1,5,2}\right)\end{aligned}$$

And, the constraints are;

(a) The length constraints as;

$$\begin{aligned} l_{0,1,1} + l_{0,1,2} &= 1{,}000\,\text{ft}\\ l_{1,2,1} + l_{1,2,2} &= 1{,}000\,\text{ft}\\ l_{2,3,1} + l_{2,3,2} &= 1{,}000\,\text{ft}\\ l_{2,4,1} + l_{2,4,2} &= 1{,}000\,\text{ft}\\ l_{1,5,1} + l_{1,5,2} &= 1{,}000\,\text{ft}\end{aligned}$$

(b) The hydraulic constraint for user A in Q_2 will be calculated as;

$$\begin{aligned}650 &- \left(0.0830 \times l_{0,1,1} + 0.0426 \times l_{0,1,2}\right) - \left(0.0314 \times l_{1,2,1} + 0.0161 \times l_{1,2,2}\right)\\ &- \left(0.0060 \times l_{2,3,1} + 0.0031 \times l_{2,3,2}\right) \geq 550\end{aligned}$$

And, for user B in Q_2 is;

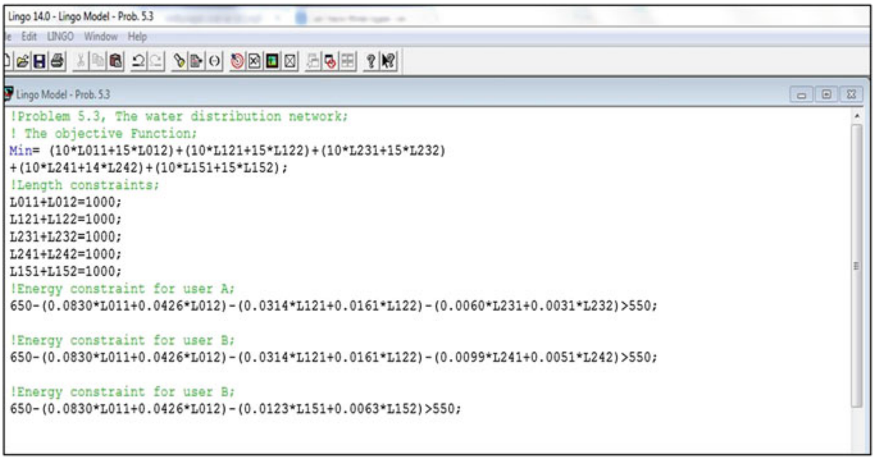
$$\begin{aligned}650 &- \left(I_{0,1,1} \times l_{0,1,1} + I_{0,1,2}l_{0,1,2}\right) - \left(I_{1,2,1}l_{1,2,1} + I_{1,2,2}l_{1,2,2}\right) - \left(I_{2,4,1}l_{2,4,1} + I_{2,4,2}l_{2,4,2}\right)\\ &= 650 - \left(0.0830 \times l_{0,1,1} + 0.0426 \times l_{0,1,2}\right) - \left(0.0314 \times l_{1,2,1} + 0.0161 \times l_{1,2,2}\right)\\ &\quad - \left(0.0099 \times l_{2,4,1} + 0.0051 \times l_{2,4,2}\right) \geq 550\end{aligned}$$

And, finally for user C in Q_2 is;

$$\begin{aligned}650 &- \left(I_{0,1,1} \times l_{0,1,1} + I_{0,1,2}l_{0,1,2}\right) - \left(I_{1,5,1}l_{1,5,1} + I_{1,5,2}l_{1,5,2}\right)\\ &= 650 - \left(0.0830 \times l_{0,1,1} + 0.0426 \times l_{0,1,2}\right)\\ &\quad - \left(0.0123 \times l_{1,5,1} + 0.0063 \times l_{1,5,2}\right) \geq 550\end{aligned}$$

Solution: The model for water distribution network under demand discharge Q_2 is shown in Fig. 5.7. As it can be seen from this figure, you can simply input the objective function and constraints and also put desired comments for each line as well.

Based on the results, the model class is LP and the optimal solution is a global optimum with the value of $53,007.4. The estimated minimum value here is close

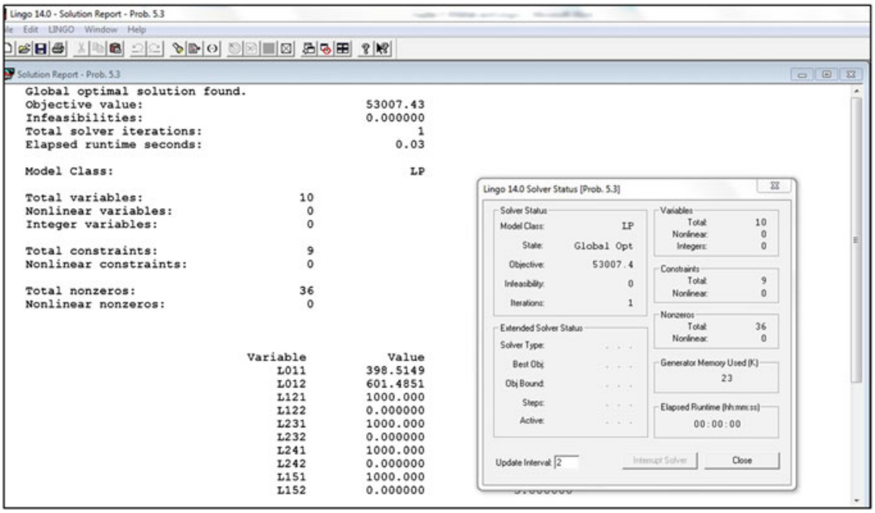


```
!Problem 5.3, The water distribution network;
! The objective Function;
Min= (10*L011+15*L012)+(10*L121+15*L122)+(10*L231+15*L232)
+(10*L241+14*L242)+(10*L151+15*L152);
!Length constraints;
L011+L012=1000;
L121+L122=1000;
L231+L232=1000;
L241+L242=1000;
L151+L152=1000;
!Energy constraint for user A;
650-(0.0830*L011+0.0426*L012)-(0.0314*L121+0.0161*L122)-(0.0060*L231+0.0031*L232)>550;

!Energy constraint for user B;
650-(0.0830*L011+0.0426*L012)-(0.0314*L121+0.0161*L122)-(0.0099*L241+0.0051*L242)>550;

!Energy constraint for user B;
650-(0.0830*L011+0.0426*L012)-(0.0123*L151+0.0063*L152)>550;
```

Fig. 5.7 Created model in LINGO



```
Global optimal solution found.
Objective value:                        53007.43
Infeasibilities:                        0.000000
Total solver iterations:                       1
Elapsed runtime seconds:                    0.03

Model Class:                                  LP

Total variables:              10
Nonlinear variables:           0
Integer variables:             0

Total constraints:             9
Nonlinear constraints:         0

Total nonzeros:               36
Nonlinear nonzeros:            0

                        Variable           Value
                            L011        398.5149
                            L012        601.4851
                            L121        1000.000
                            L122        0.000000
                            L231        1000.000
                            L232        0.000000
                            L241        1000.000
                            L242        0.000000
                            L151        1000.000
                            L152        0.000000
```


Fig. 5.8 The optimal values of pipe length

to the archived result in Chap. 2 (min $Z = 53{,}018.95$) using the Solver tool of Excel (Fig. 5.8).

Table 5.2 shows the results of optimization analysis using LINGO and Excel for the length of pipes and minimum value of desired objective function

Example 5.4 Solve Example 2.5 by applying LINGO for the following conditions; (1) the minimum value of the total desired discharge (W_{min}) from all wells equals 4 ft/day, and (2) the minimum value of the desired discharge (W_{min}) from each

Table 5.2 The optimized results

Pipe segment	LINGO	Excel
$l_{0,1,1}$	398.51	396.21
$l_{0,1,2}$	601.48	603.79
$l_{1,2,1}$	1,000	1,000
$l_{1,2,2}$	0.00	0.00
$l_{2,3,1}$	1,000	1,000
$l_{2,3,2}$	0.00	0.00
$l_{2,4,1}$	1,000	1,000
$l_{2,4,2}$	0.00	0.00
$l_{1,5,1}$	1,000	1,000
$l_{1,5,2}$	0.00	0.00
min Z ($)	53,007.43	53,018.95

well equals 4 ft/day. As noted in Chap. 2, the necessary information to solve this problem are $W_{min} = 4$ ft/day, $\Delta x = 100$ ft, $T = 10{,}000$ ft^2/day, $h_0 = 125$ ft, and $h_4 = 100$ ft.

Solution: The objective function to maximize the hydraulic heads for various pumping rates is;

$$\max Z = \sum_{i=1}^{n=3} h_i = h_1 + h_2 + h_3$$

And, the constraints of this problem are;

$$(2h_1 - h_2) + \left(\frac{W_1 \times \Delta x^2}{T_x}\right) = 125$$
$$(2h_2 - h_1 - h_3) + \left(\frac{W_2 \times \Delta x^2}{T_x}\right) = 0$$
$$(2h_3 - h_2) + \left(\frac{W_3 \times \Delta x^2}{T_x}\right) = 100$$

For the first part of question, the other constraints are;

$$W_1 + W_2 + W_3 \geq W_{min}$$
$$h_1, h_2, h_3 \geq 0$$
$$W_1, W_2, W_3 \geq 0$$
$$h_0 \geq h_1 \geq h_2 \geq h_3 \geq h_4$$

Figures 5.9 and 5.10 show the model of one-dimensional confined aquifer in LINGO and the optimized values of hydraulic heads and discharge rates, respectively.

Table 5.3 shows the optimized hydraulic heads and discharge rates using two different computer programs. As it can be seen from this table, there is only a little difference between Excel and LINGO outcomes.

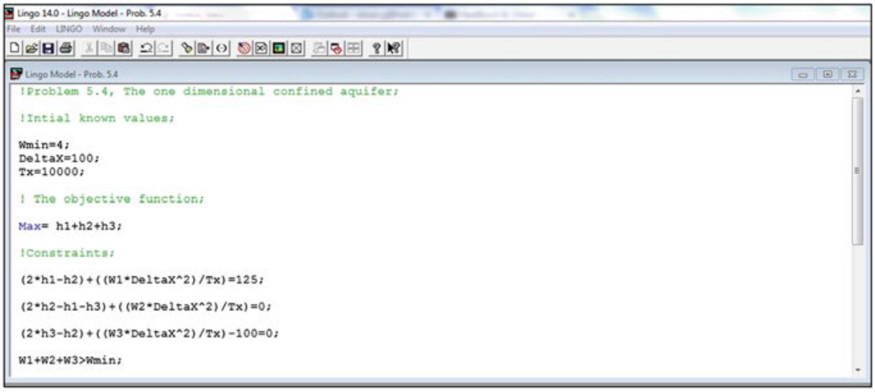


Fig. 5.9 Input model in LINGO for one-dimensional confined aquifer

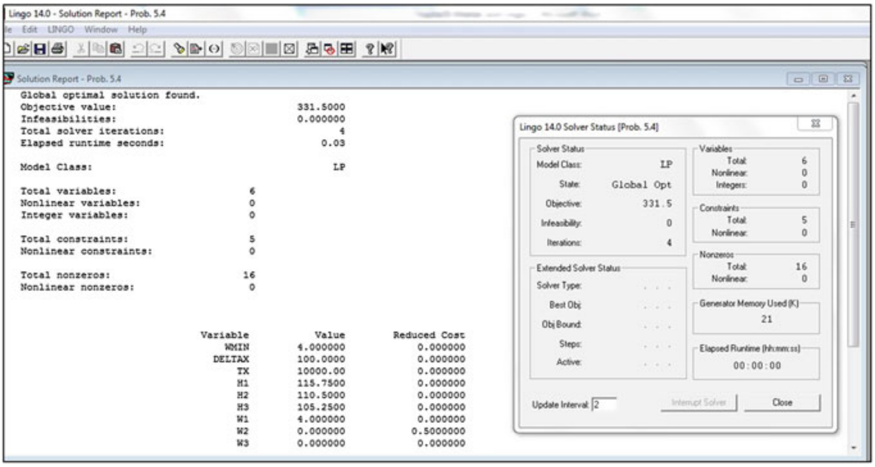


Fig. 5.10 The optimized values of hydraulic heads and discharge rates

If the minimum value of the desired discharge (W_{min}) from each well is considered 4 ft/day, the model is almost same as previous section except the following condition as $W_i \geq W_{min}$. The applied model and optimized results using LINGO is shown in Fig. 5.11.

The outcomes of LINGO are compared with the achieved results using Excel in Table 5.4. In this case, both programs resulted in the same values of hydraulic heads and discharge rates.

Table 5.3 The optimized hydraulic heads and discharge rates in LINGO and Excel

	Z (ft)	Hydraulic head (ft)					Discharge rate (ft/day)		
		h_0	h_1	h_2	h_3	h_4	W_1	W_2	W_3
Excel	331.5	125.0	117.75	110.5	103.25	100.0	0.0	0.0	4.0
LINGO	331.5	125.0	115.75	110	105.25	100.0	4.0	0.0	0.0

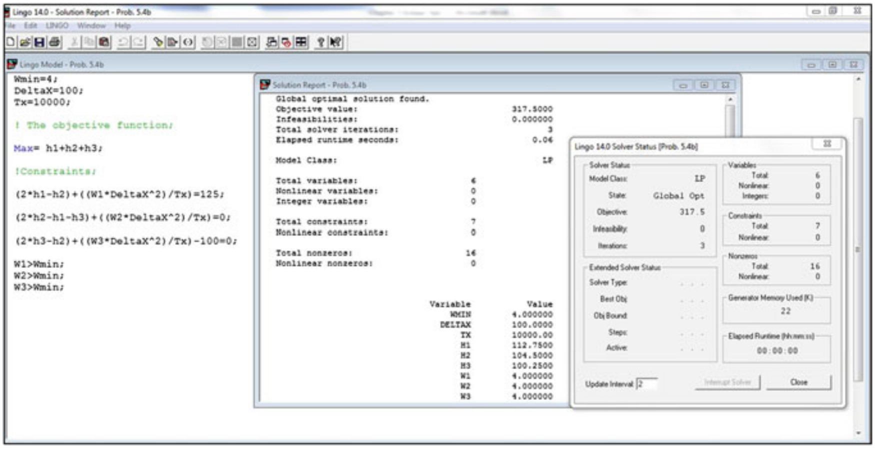


Fig. 5.11 The optimized values of hydraulic heads and discharge rates when $W_i \geq W_{min}$

5.1.3 Solving Nonlinear Models

As mentioned previously, LINGO is capable to find the local or global optimum solution of nonlinear models as well as linear models. To solve a nonlinear optimization problem, LINGO uses the Generalized Reduced Gradient (GRG) technique which is explained comprehensively in Chap. 3. In general, LINGO includes a set of built-in solvers to find the optimal solution of linear and nonlinear optimization problems in which some of the most important solvers in LINGO are;

1. Simplex solvers for linear models,
2. Barrier solver for linear models,
3. Integer solver that works with both linear and nonlinear models,
4. General nonlinear solver for nonlinear problems,
5. Global solver to find the global optimum of non-convex problems.

In the following section, several nonlinear optimization problems that are already presented in previous chapters are solved using LINGO and the achieved results have been compared with the outcomes of Excel program.

Table 5.4 The optimized hydraulic heads and discharge rates in LINGO and Excel when $W_i \geq W_{min}$

	Z (ft)	Hydraulic head (ft)					Discharge rate (ft/day)		
		h_0	h_1	h_2	h_3	h_4	W_1	W_2	W_3
Excel	331.5	125.0	117.75	110.5	103.25	100.0	4.0	4.0	4.0
LINGO	317.5	125.0	112.7	104.5	100.25	100.0	4.0	4.0	4.0

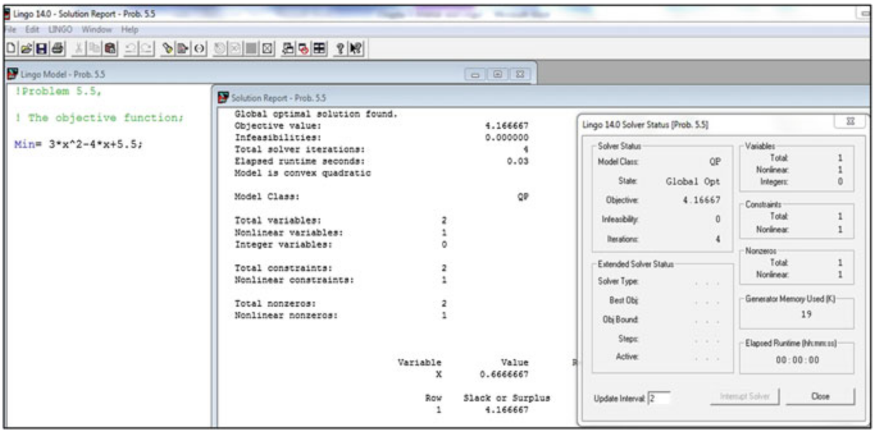


Fig. 5.12 The optimal solution

Example 5.5 Solve Example 3.4 using LINGO and compare the results with the outcomes of Fibonacci and golden section methods.

$$\min \mathrm{f}(x) = 3x^2 - 4x + 5.5$$

Solution: The simple nonlinear model in conjunction with the final solution for this problem is shown in Fig. 5.12. The result shows that the optimal solution is 4.166 and it happens at the point $x = 0.666$. According to Chap. 3, the estimated result using the Fibonacci method is 4.167 at $x = 0.647$, while, the minimum value based on the golden section method is 4.166 at $x = 0.66$.

Example 5.6 Solve the Example 3.7 using LINGO to find the minimum dimensions of cylindrical water tank.

$$\mathrm{f}(r) = \pi r^2 + \frac{400}{r}$$

Solution: The problem can be solved simply like the previous problems shown in Fig. 5.13. As it can be seen in the Solver Status window, the optimal solution is obtained as local solution as f(r) = 150.239 at the point $r = 3.993$. Now, we need to find an appropriate answer for this question; is the local estimated optimum

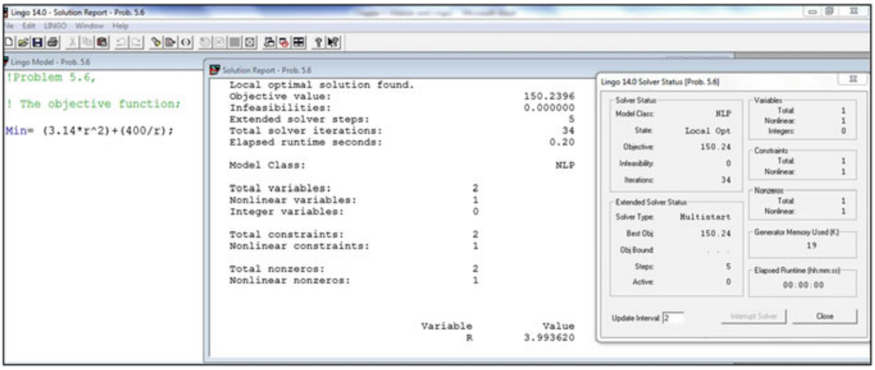


Fig. 5.13 The minimum dimensions of cylindrical water tank

solution the same as the global solution? Or is there another solution for this problem. As noted above, LINGO has a Global Solver engine to find the global optimum of non-convex problems. This engine is a strong tool that searches over the desired problem until the global optimum is found.

To run the model and find the global optimum solution, choose *options* from toolbar and a window like Fig. 5.14 will appear as *Lingo Options*. In this window, click the *Global Solver* button and check mark the *Use Global Solver* and then click OK.

The model is solved again using Global Solver and the results are shown in Fig. 5.15. As it can be seen from the Solver Status window, the solution is global optimum and the results are the same in both conditions.

Example 5.7 Apply LINGO to solve Example 3.8 and compare the results with the univariate method. The objective function is;

$$\mathrm{f}(x) = 1.25x_1 - 0.45x_2 + x_1^4 + x_1x_2 + x_2^2$$

Solution: The optimal solution using LINGO is -0.0506 at the points $x_1 = 0.0$ and $x_2 = 0.225$ (see Fig. 5.16). Comparing these results with the outcomes of univariate method shows there is a big difference between them. As the default assumption in LINGO is considering lower bound equals zero for all variables, the results are restricted to the positive value.

To remove this restriction, we need to apply the function *@Free (variable)* for each decision variable to accept any value. The results based on using this new function are shown in Fig. 5.17. The optimal solution here is -0.9831 and at the points $x_1 = -0.775$ and $x_2 = 0.612$.

A number of the variable domain functions of LINGO that restrict variables in different ways in conjunction with their applications are presented in Table 5.5.

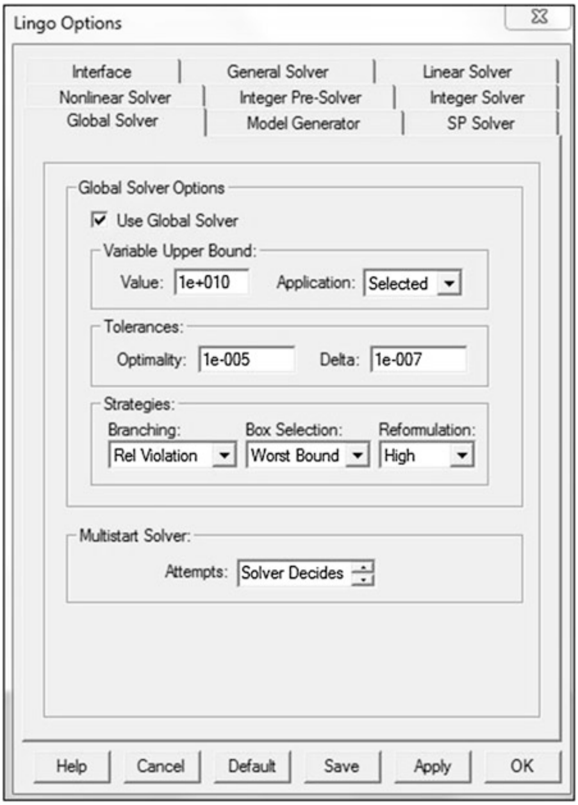


Fig. 5.14 Use the global solver in LINGO

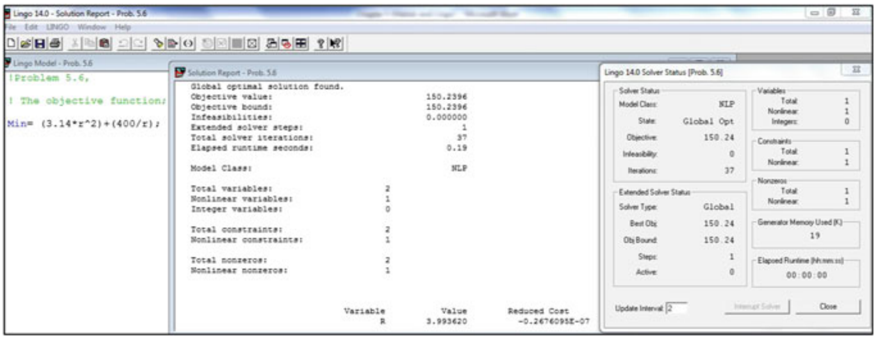


Fig. 5.15 The global optimum solution for cylindrical water tank

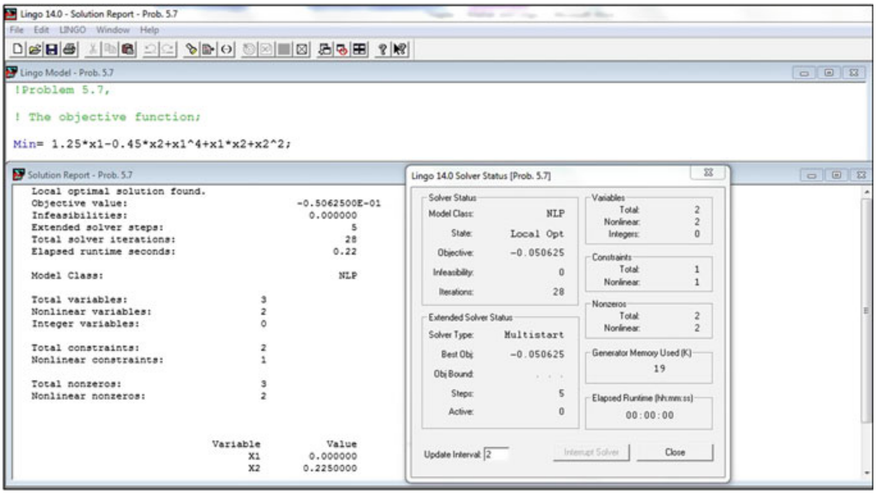


Fig. 5.16 The optimal solution

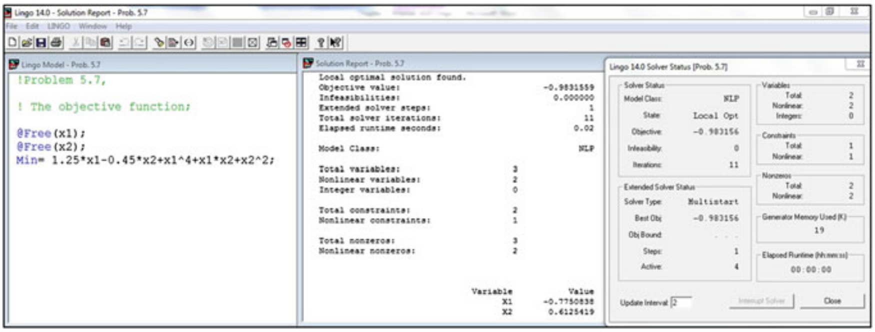


Fig. 5.17 The optimal solution

Table 5.5 The LINGO variable domain functions

Function	Description
@BIN(variable)	To being a binary integer value
@BND(lower bound, variable, upper bound)	To being greater-than-or-equal-to lower bound and less-than-or-equal-to upper bound
@FREE(variable)	To being either positive or negative
@GIN(variable)	To being integer value

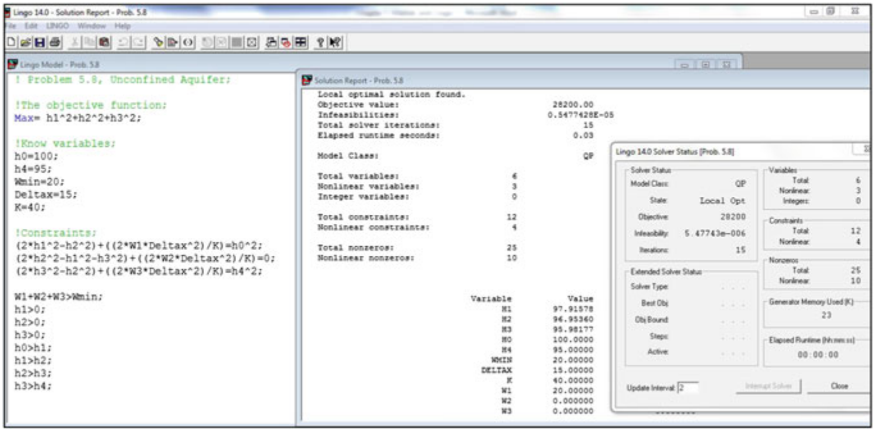


Fig. 5.18 The unconfined aquifer model and local optimum solutions

Example 5.8 Apply LINGO to determine the optimum head in each well for the minimum discharge of 20 (m/day) in Example 3.14. The optimization model in this problem is defined as;

$$\max Z = h_1^2 + h_2^2 + h_3^2$$

Subject to the following constraints;

$$\left(2h_1^2 - h_2^2\right) + \left(\frac{2\times\Delta x^2}{K} W_1\right) = h_0^2$$
$$\left(2h_2^2 - h_1^2 - h_3^2\right) + \left(\frac{2\times\Delta x^2}{K} W_2\right) = 0$$
$$\left(2h_3^2 - h_2^2\right) + \left(\frac{2\times\Delta x^2}{K} W_3\right) = h_4^2$$

where $h_0 = 100$ m and $h_4 = 95$ m. The other constraints are;

$$W_1 + W_2 + W_3 \geq W_{min}$$
$$h_1, h_2, h_3 \geq 0$$
$$W_1, W_2, W_3 \geq 0$$
$$h_0 \geq h_1 \geq h_2 \geq h_3 \geq h_4$$

Solution: The model used in LINGO and all optimal values of head in each well are shown in Fig. 5.18. It is important to note that the Solver Status shows the estimated results are local optimum solution. The model is re-run again using the global solver engine in LINGO and the outcomes are shown in Fig. 5.19. As it can be seen from these figures, the maximum of function Z are the same in both analyses, while there is a difference between the values of h and W.

Table 5.6 shows the estimated values of hydraulic heads h and the sink term W using both LINGO and Excel.

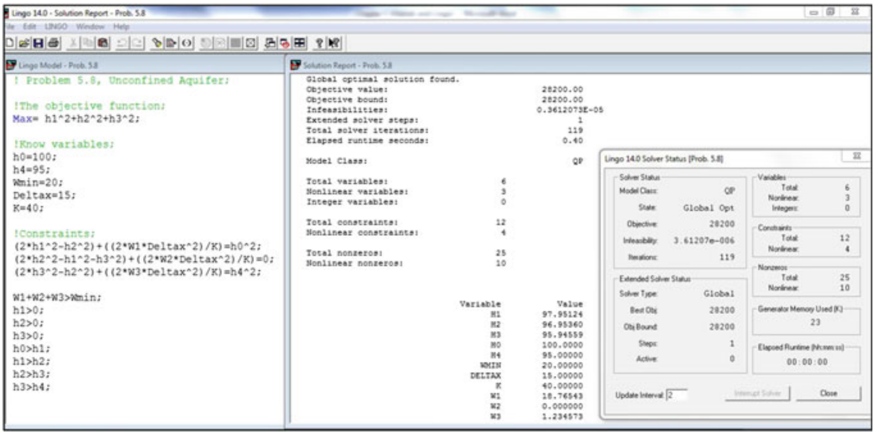


Fig. 5.19 The unconfined aquifer model and global optimum solutions

Table 5.6 The values of hydraulic heads h and the sink term W in Excel and LINGO

Variables	Excel	Local optimum solution	Global optimum solution
h_0	100.00	100.00	100.00
h_1	98.20	97.91	97.95
h_2	96.95	96.95	96.95
h_3	95.70	95.98	95.94
h_4	95.00	95	95.00
W_1	10.26	20.00	18.76
W_2	0.00	0.00	0.00
W_3	9.74	0.00	1.23
Z	28,200	28,200	28,200

5.2 MATLAB

MATLAB or Matrix Laboratory is a powerful and high-performance language for numerical computations, programming, data analysis and visualizations with a friendly user interface. The syntax of this high-level language is similar to the *C* programming language and it can be applied to create models and develop different algorithms to solve simple or complex problems. The applications of MATLAB cover a vast range of areas including; civil engineering, mechanical engineering, signal processing and communications, modeling and simulation, physics, biology, chemistry, economic, etc. This program includes a number of interfaces to import and export data as well as managing the available decision variables. In addition, it has extensive facilities for displaying two and three-dimensional data visualization as graphs with high quality and good resolution.

Table 5.7 Arithmetic operators

Operator	Sign	Operator	Sign
Plus	+	Matrix left-division	\
Minus	–	Matrix right-division	/
Matrix multiplication	*	Array left-division	.\
Multiplication	.*	Array right-division	./
Matrix exponentiation	^	Colon operator	:
Array exponentiation	.^	Transpose	.'

Table 5.8 Rational and logical operators

Operator	Sign	Operator	Sign
$<$	Less than	$\leq$	Less than or equal to
$>$	Greater than	$\geq$	Greater than or equal to
=	Equal to	$\neq$	Not equal to
&	AND	$\vert$	OR

Furthermore, the MATLAB library includes lots of simple and sophisticates mathematical functions for linear algebra, numerical integration, Fourier analysis, and optimization (for more information see the online Documentation Center of MATLAB). Tables 5.7 and 5.8 show a numbers of arithmetic, rational and logical operators which are used in MATLAB.

Furthermore, a numbers of commands for managing a session in MATLAB are presented in Table 5.9.

The Optimization Toolbox of MATLAB is a powerful optimization platform that uses well known algorithms to solve wide ranges of constrained and unconstrained optimization problems. This toolbox includes a numbers of mathematical functions to find the optimal solutions of linear, nonlinear, and multiobjective problems as well as binary integer programming, nonlinear least squares, and nonlinear system of equations (Coleman et al. 1999). The Optimization Toolbox in MATLAB can be opened by using the command *Optimtool* in the Command Window and the toolbox should be as shown in Fig. 5.20.

The Optimization Tool includes two main windows as *Problem Setup and Results* (left side window) and *Options* (right side window), as shown in Fig. 5.21. The *Problem Setup and Results* window is used to choose the Solver method, determine the appropriate algorithm, and define the objective function and constraints of desired optimization problem.

The existing Solver methods in MATLAB can be categorized in six main groups as;

1. Linear and quadratic minimization problems; this part involves linear programming (linprog) and quadratic programming (quadprog),
2. Linear least squares; this option includes linear least squares with linear constraints (lsqlin) and linear least squares with nonnegative constraints (lsqnonneg),

Table 5.9 A numbers of commands for managing a session in MATLAB

Command	Sign	Operator	Sign
Clear command window	clc	Search for a help topic	help
Check for existence of file or variable	exit	Declare variables to be global	global
Remove variables from memory	clear	Searches help entries for a keyword	Look for
List current variables	who	Stops MATLAB	quit
List all files in current directory	dir	Displays contents of a file	type
Display current date	date	Deletes a file	delete

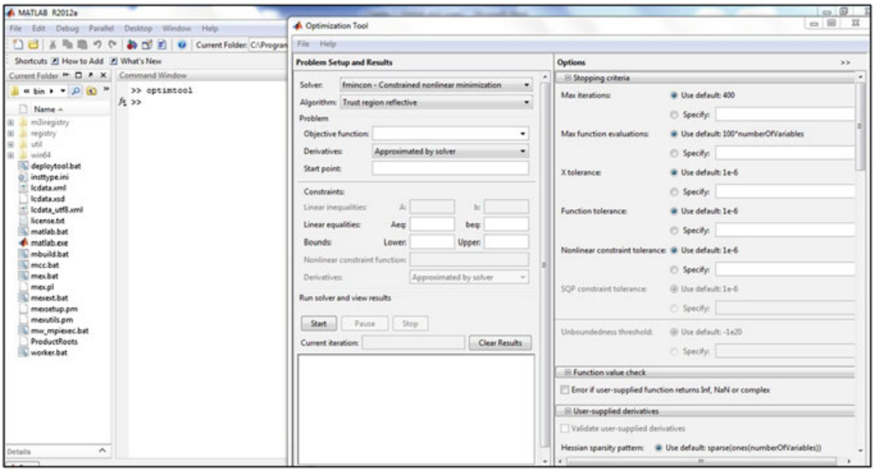


Fig. 5.20 Command window and optimization toolbox

3. Nonlinear zero finding; this section contains single-variable nonlinear equation solving (fzero) and nonlinear equation solving (fsolve),
4. Nonlinear minimization of functions; this option involves unconstrained nonlinear minimization (fminsearch), single-variable fixed interval minimization (fminbnd), constrained nonlinear multivariable minimization (fmincon), minimization of unconstrained multivariable function (fminunc), and minimization of semi-infinitely constrained multivariable nonlinear function (fseminf),
5. Nonlinear least squares; this section includes solver for nonlinear curve-fitting (data-fitting) via least-squares (lsqcurvefit) and nonlinear least squares problems,
6. Nonlinear minimization of multi-objective optimization problems; this option has two solvers to find the optimal solution for multiobjective goal attainment (fgoalattain) and minimax (fminimax) problems.

The next tab is *Algorithm* that includes four parts as; interior point, SQP, Active set (or medium scale), and trust region reflective or (large scale). All necessary information about all of these options and their applications in different optimization analysis are available in the Help button of MATLAB.

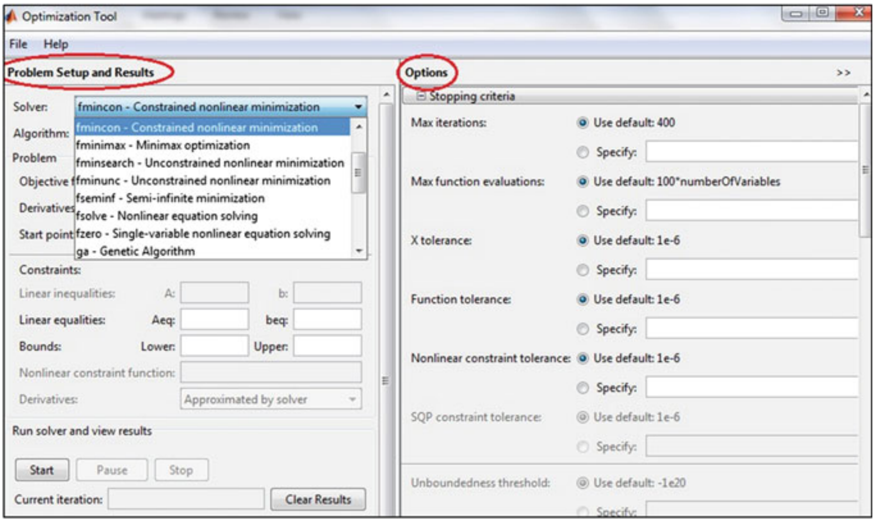


Fig. 5.21 Main options of optimization toolbox

5.2.1 Solving Linear Optimization Problems

The *linprog* function can be used to solve linear programming problems in the following format;

$$\min_{x} Z = \mathrm{f}^{t}\mathrm{x} \tag{5.1a}$$

Subject to;

$$\begin{aligned} Ax &\le \mathrm{b} \\ A_{eq}\mathrm{x} &= b_{eq} \\ \mathrm{x} &\ge lb \\ \mathrm{x} &\le ub \end{aligned} \tag{5.1b}$$

where, f is any vector, the matrices A, A_{eq} and the vectors b and b_{eq} defines the linear constraints, and lb and ub are lower and upper bounds, respectively.

A linear program in the format of Eq. 5.1a can be solved using the following command in MATLAB;

1. x = linprog(f, A, b, Aeq, beq, lb, ub): solve the minimization problem while satisfying the inequality constraint $Ax \le \mathrm{b}$ and equality constraint $A_{eq}\mathrm{x} = b_{eq}$ with the lower and upper bounds lb and ub on decision variable x.
2. x = linprog(f, A, b, Aeq, beq, lb, ub, x0); solve the minimization problem same as above while setting x0 as the starting point.
3. [x, fval] = linprog(f, A, b, Aeq, beq, lb, ub, x0); return the minimum of function f at solution point x.

4. [x, fval, exitflag] = linprog(f, A, b, Aeq, beq, lb, ub, x0); determine *exitflag* value that shows the exit condition.
5. [x, fval, exitflag, output] = linprog(f, A, b, Aeq, beq, lb, ub, x0); return useful information about the output of optimization.

A number of linear examples are presented in the following section that shows how to apply the linprog function in MATLAB for linear programming optimization.

Example 5.9 Find the maximum of following linear program using MATLAB.

$$\mathrm{f}(x) = 2x_1 + x_2 + 5x_3$$

Subject to the following constraints;

$$\begin{aligned}
&x_1 + 2x_2 \le 4 \\
&2x_1 + 2x_3 \le 5 \\
&x_1 + x_2 + x_3 = 2.5 \\
&1 \le x_1 \le 2.5 \\
&0.5 \le x_2 \le 2 \\
&0 \le x_3 \le 3
\end{aligned}$$

Solution: The first step is converting the above linear equation and its constraints in the MATLAB format as following;

$$\mathrm{f}(x) = -[2 \quad 1 \quad 5]\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

As the linprog is a minimizer we need to use a negative of function f(x) to convert problem into a minimization problem. The constraints in the form of matrix can be written as;

$$\text{Inequalities:} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \le \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$\text{Equalities:} [\,1 \quad 1 \quad 1\,]\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 2.5$$

It important to note that all comments in the model must be initialized with the sign "%", and so, the text will appear in a green color.

Figure 5.22 shows the whole procedure of solving this linear optimization problem and the solution points x_1, x_2, and x_3. As it can be seen in the workspace window of Fig. 5.22, the values of decision variables x_1, x_2, and x_3 are 1.0, 0.5, and 1.0, respectively. It should be noted that in addition to the optimal solution of

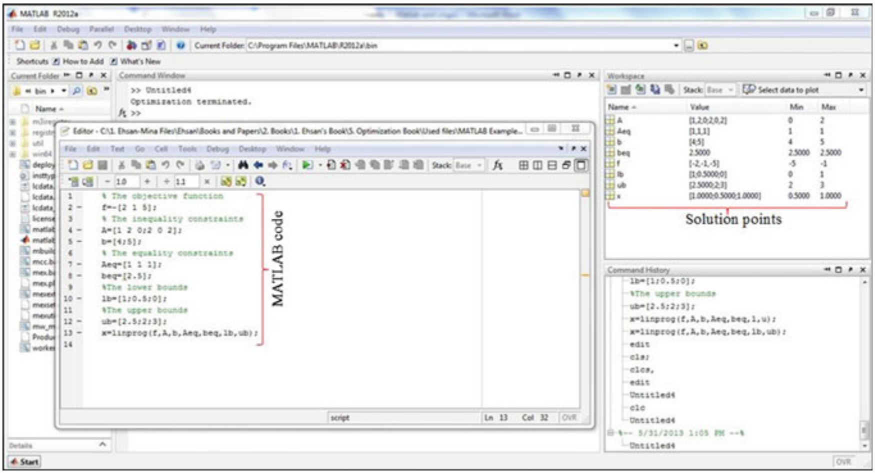


Fig. 5.22 The MATLAB code and the solution points

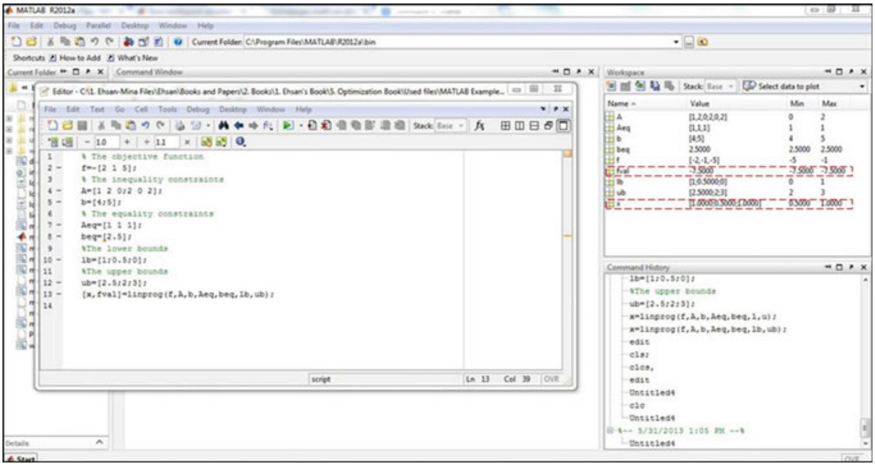


Fig. 5.23 The optimal solution of function f(x)

decision variables, we need the maximum value of objective function f(x). Hence, the command $x = \text{linprog}(f, A, b, Aeq, beq, lb, ub)$ should be changed to $[x, fval] = \text{linprog}(f, A, b, Aeq, beq, lb, ub)$.

As it can be seen in Fig. 5.23 the minimum of $-f(x)$ is -7.5, and so, the maximum of this function will be 7.5.

If we don't use the semicolon (;) at the end of the last command line, the results will appear immediately after the commands at the same page (Fig. 5.24). In general, if a command line terminates with semicolon, the output associated with that statement will not be displayed.

```
Command Window
>> % The objective function
f=-[2 1 5];
% The inequality constraints
A=[1 2 0;2 0 2];
b=[4;5];
% The equality constraints
Aeq=[1 1 1];
beq=[2.5];
%The lower bounds
lb=[1;0.5;0];
%The upper bounds
ub=[2.5;2;3];
>> [x,fval]=linprog(f,A,b,Aeq,beq,lb,ub)
Optimization terminated.

x =

    1.0000
    0.5000
    1.0000


fval =

   -7.5000

fx >> +
```

Fig. 5.24 The optimal solution in the command window

The Optimization Toolbox also can be simply applied to solve the given linear problem. In this case, open the Optimization Tool by writing the Optimtool in the command window, and then, choose *linprog-Linear programming* from the *Solver* section. Then, input the objective function and its constraints in the associated sections and apply *Start* button to solve the problem (Fig. 5.25).

If there are no lower or upper bounds on the desired decision variables, an empty set as $lb = []$; or $ub = []$; should be used in MATLAB.

Example 5.10 Solve Example 2.3 using MATLAB. The objective function is;

$$\max R(\$) = \left(n_{p_1} \times 20\,\$\right) + \left(n_{p_2} \times 25\,\$\right)$$

And, the constraints are;

$$n_{p_1} \leq 25; \quad n_{p_2} \leq 35; \quad 2n_{p_1} + 3n_{p_2} \leq 140$$

Solution: The objective function in the matrix format can be written as;

$$\mathrm{f}(x) = -[20 \quad 25]\begin{bmatrix} n_{p1} \\ n_{p2} \end{bmatrix}$$

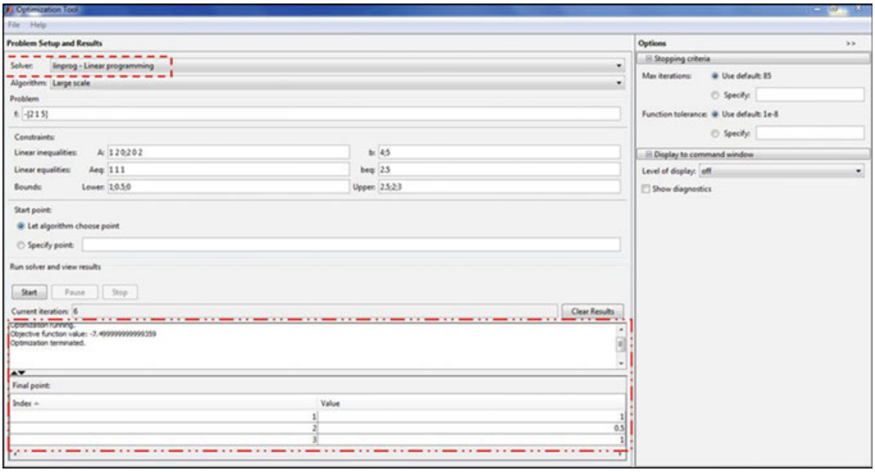


Fig. 5.25 Solving problem using optimization tool

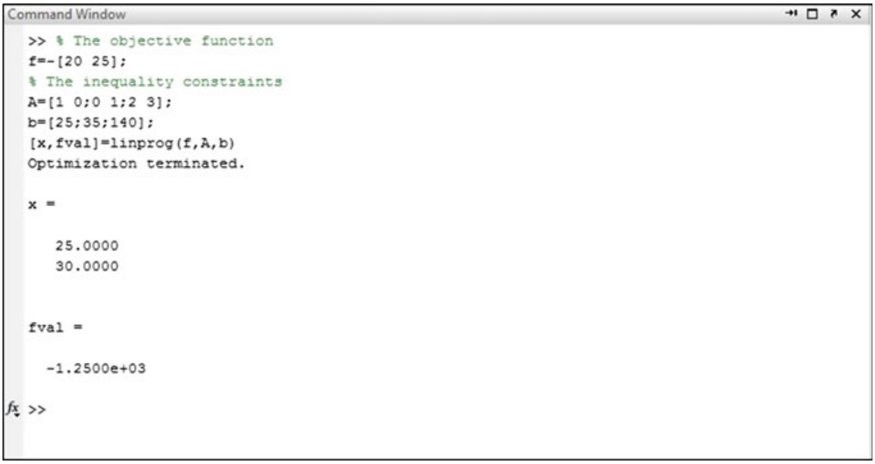


Fig. 5.26 The maximum profit $R(\$)$ and values of n_{p_1} and n_{p_2}

Subject to;

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} n_{p1} \\ n_{p2} \end{bmatrix} \leq \begin{bmatrix} 25 \\ 35 \\ 140 \end{bmatrix}$$

The MATLAB codes for this linear optimization problem are shown in Fig. 5.26. Based on the results, the maximum profit of \$1,250 and the values of variables n_{p_1} and n_{p_2} at the optimal points are 25 and 30, respectively. Figures 5.26 and 5.27

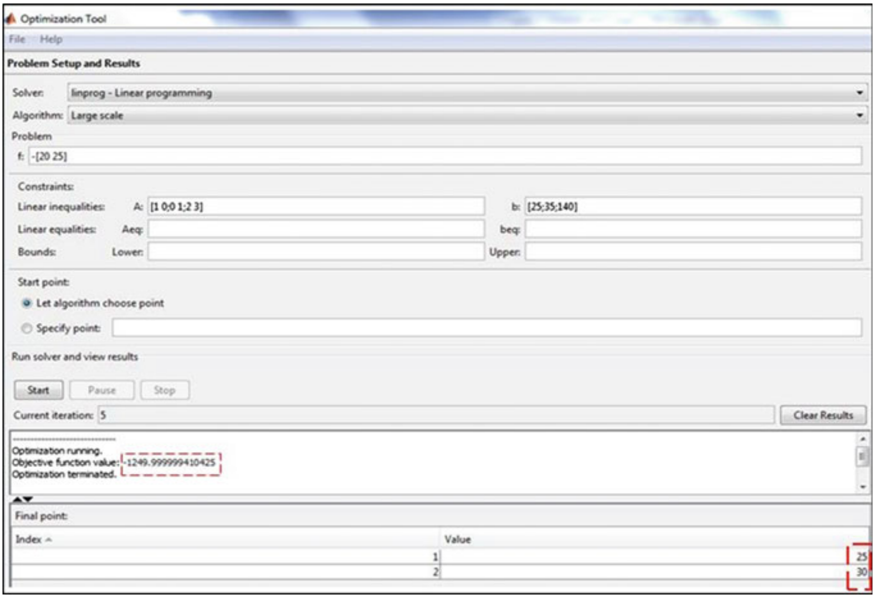


Fig. 5.27 The optimal solutions using optimization tool

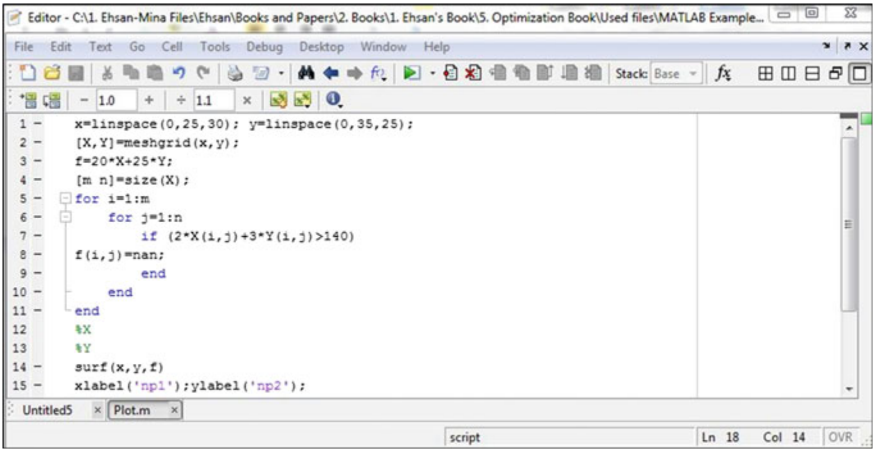
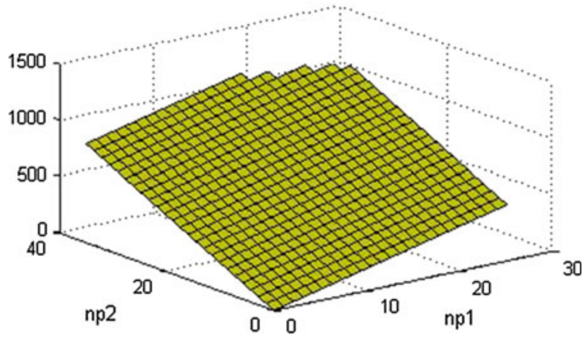


Fig. 5.28 MATLAB codes to plot feasible region

show the optimization procedure using Command Window and Optimization Tool, respectively.

Figure 5.28 shows the MATLAB codes that is used to plot the feasible area and the feasible region which is presented in Fig. 5.29.

Fig. 5.29 The feasible area



The first line in Fig. 5.28 includes the linspace function which generates linearly spaced vectors. In the case of this example, the command $x =$ linspace(0,25,30) generates a row vector x in which 30 points are linearly spaced between 0 and 25. As constraints of this problem are restricted to $n_{p_1} \leq 25$ and $n_{p_2} \leq 35$, the values of 25 and 35 are chosen in the linspace functions. The second line includes the meshgrid function that transforms the vector x and y into a set of arrays X and Y. The rest of codes are used to apply the last constraints in this problem and then plot the feasible area.

Example 5.11 Apply MATLAB to solve the Example 5.3 for demand discharge Q_2 ($Q_A = 14$, $Q_B = 18$, and $Q_C = 20$) when there is no pump station.

Solution: As the objective function in the matrix format is;

$$Z = \begin{bmatrix} 10 & 15 & 10 & 15 & 10 & 15 & 10 & 15 & 10 & 15 \end{bmatrix} \begin{bmatrix} l_{0,1,1} \\ l_{0,1,2} \\ l_{1,2,1} \\ l_{1,2,2} \\ l_{2,3,1} \\ l_{2,3,2} \\ l_{2,4,1} \\ l_{2,4,2} \\ l_{1,5,1} \\ l_{1,5,2} \end{bmatrix}$$

The hydraulic constraint for users A, B and C in the demand discharge Q_2 are considered as inequality constraints and can be written as;

$$\begin{bmatrix} 0.0830 & 0.0426 & 0.0314 & 0.0161 & 0.0060 & 0.0031 & 0 & 0 & 0 & 0 \\ 0.0830 & 0.0426 & 0.0314 & 0.0161 & 0 & 0 & 0.0099 & 0.0051 & 0 & 0 \\ 0.0830 & 0.0426 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0123 & 0.0063 \end{bmatrix}$$

$$\times \begin{bmatrix} l_{0,1,1} \\ l_{0,1,2} \\ l_{1,2,1} \\ l_{1,2,2} \\ l_{2,3,1} \\ l_{2,3,2} \\ l_{2,4,1} \\ l_{2,4,2} \\ l_{1,5,1} \\ l_{1,5,2} \end{bmatrix} \leq \begin{bmatrix} 100 \\ 100 \\ 100 \end{bmatrix}$$

And, the length constraints which are considered as equality constraint are;

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} l_{0,1,1} \\ l_{0,1,2} \\ l_{1,2,1} \\ l_{1,2,2} \\ l_{2,3,1} \\ l_{2,3,2} \\ l_{2,4,1} \\ l_{2,4,2} \\ l_{1,5,1} \\ l_{1,5,2} \end{bmatrix} \leq \begin{bmatrix} 1000 \\ 1000 \\ 1000 \\ 1000 \\ 1000 \end{bmatrix}$$

The lower bounds (*lb*) and upper bounds (*ub*) in this problem can be defined as;

$$lb = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad ub = \begin{bmatrix} 1000 \\ 1000 \\ 1000 \\ 1000 \\ 1000 \\ 1000 \\ 1000 \\ 1000 \\ 1000 \\ 1000 \end{bmatrix}$$

The MATLAB codes and the estimated results for this linear optimization problem are shown in Fig. 5.30. In addition, this problem is solved using the Optimization Tool of MATLAB and the input values as well as the achieved results are shown in Fig. 5.31.

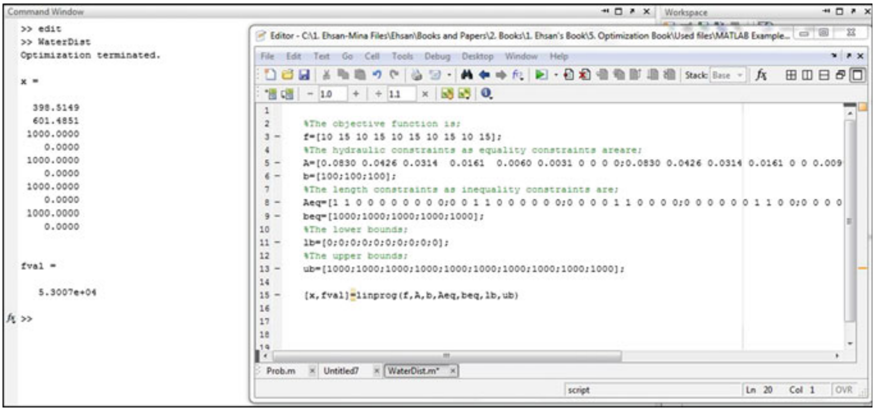


Fig. 5.30 MATLAB codes and achieved results

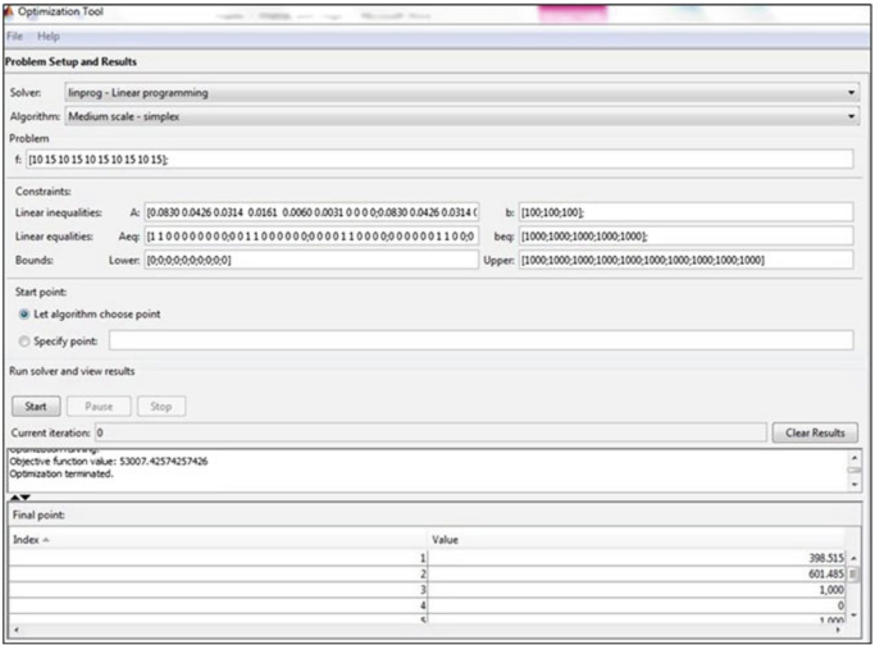


Fig. 5.31 Apply optimization tool

Table 5.10 compares the estimated results of LINGO, Excel, and MATLAB for water distribution network Example. These results include the pipe lengths and minimum values of piping cost *Z*.

Table 5.10 The optimized results

Pipe segment	LINGO	Excel	MATLAB
$l_{0,1,1}$	398.51	396.21	398.51
$l_{0,1,2}$	601.48	603.79	601.48
$l_{1,2,1}$	1,000	1,000	1,000
$l_{1,2,2}$	0.00	0.00	0.00
$l_{2,3,1}$	1,000	1,000	1,000
$l_{2,3,2}$	0.00	0.00	0.00
$l_{2,4,1}$	1,000	1,000	1,000
$l_{2,4,2}$	0.00	0.00	0.00
$l_{1,5,1}$	1,000	1,000	1,000
$l_{1,5,2}$	0.00	0.00	0.00
min Z ($)	53,007.43	53,018.95	53,007.42

5.2.2 *Solving Unconstrained Nonlinear Optimization Problems*

To find the minimum of an unconstrained nonlinear optimization problem, the function fminsearch can be applied as follows;

1. x = fminsearch(fun, x0); solve the minimization problem by considering the starting point x0.
2. [x, fval] = fminsearch(fun, x0); find the minimum of function f at the solution point x.

The following problem shows the application of above function in MATLAB to find the minimum value of an unconstrained nonlinear equation.

Example 5.12 Solve Example 3.4 using MATLAB and compare the results with the outcomes of LINGO, Fibonacci and golden section methods.

$$\min \mathrm{f}(x) = 3x^2 - 4x + 5.5$$

Solution: The MATLAB code and result are presented in Figs. 5.32 and 5.33, respectively. The result shows that the optimal solution is 4.1667 and at the point $x = 0.6667$, which is the same as the outcomes of LINGO. In addition, the achieved result from Fibonacci and golden section methods are (0.647,4.167) and (0.66,4.166) respectively.

It is important to note that this problem also can be simply solved using the Optimization Tool. Figure 5.34 shows the process of input data and the solution to the problem using this toolbox.

Example 5.13 Solve Example 3.8 using MATLAB and compare the results with the univariate method. The objective function in this problem is;

$$\mathrm{f}(x) = 1.25x_1 - 0.45x_2 + x_1^4 + x_1x_2 + x_2^2$$

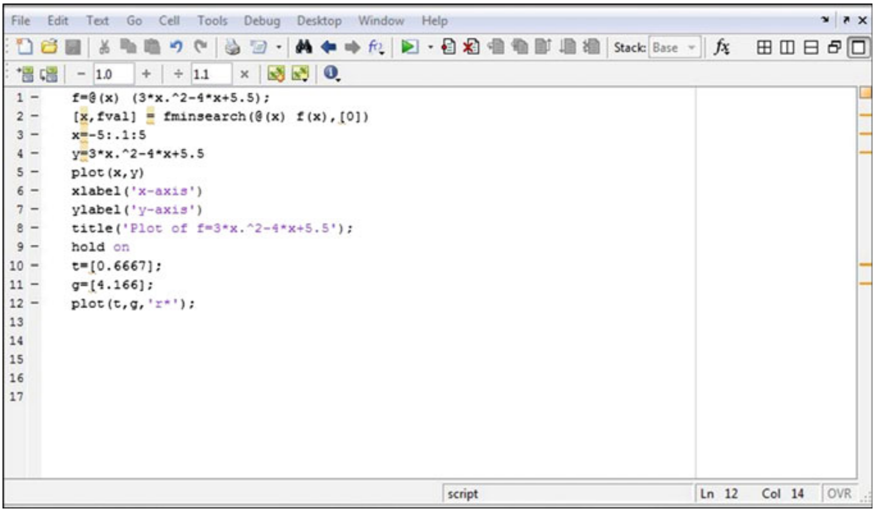


Fig. 5.32 MATLAB codes

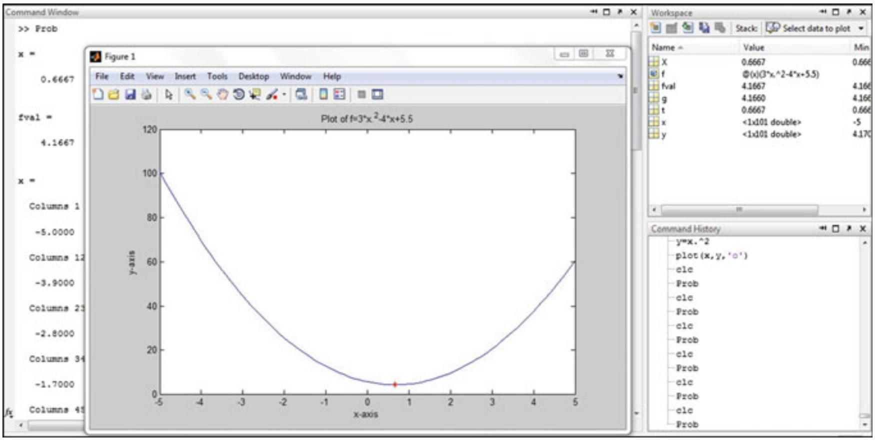


Fig. 5.33 The results of optimization analysis

Solution: Figures 5.35 and 5.36 show the process of solving this unconstrained nonlinear optimization problem and its solution, respectively. As it can be seen in Fig. 5.35, the fminsearch function with the starting points (0,0) is applied and the result shows the minimum value is −0.9832 at the solution point −0.7751, 0.6126. The optimal solution from LINGO is −0.9831 at point (−0.775,0.612), and the univariate method resulted in minimum of −0.9831 at (−0.7745,0.6084).

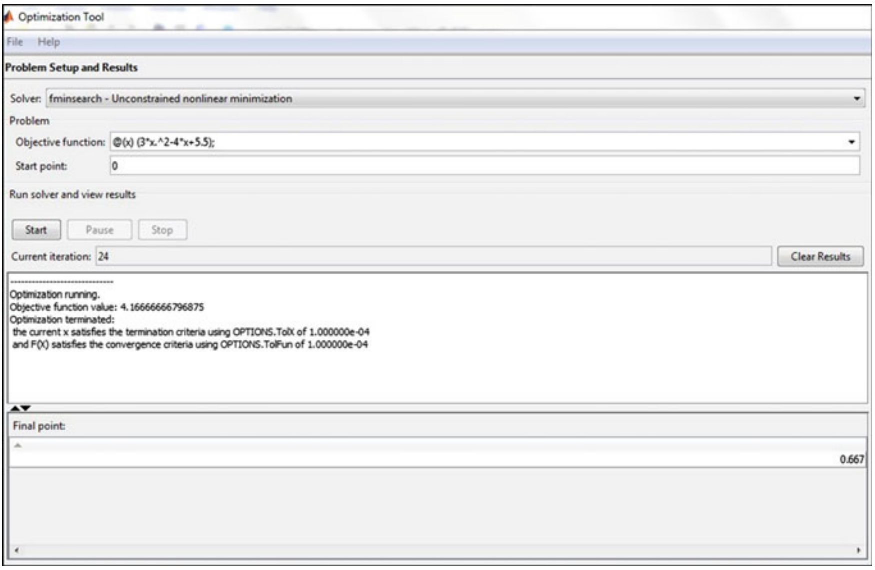


Fig. 5.34 Solve unconstrained nonlinear optimization problem using optimization tool

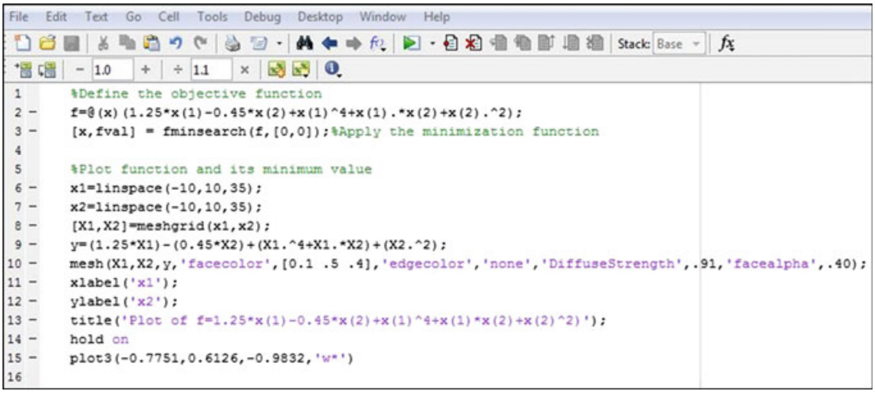


Fig. 5.35 MATLAB codes

It is important to note that the *hold on* command can be used to add another plot to the existing graph. In other words, hold on keeps the present plot and its axis properties and then adds the new graph to the existing graph. In the case of this problem, plot3 also is applied to add the minimum of function f(x) as a single point (red point) on the graph (Fig. 5.36).

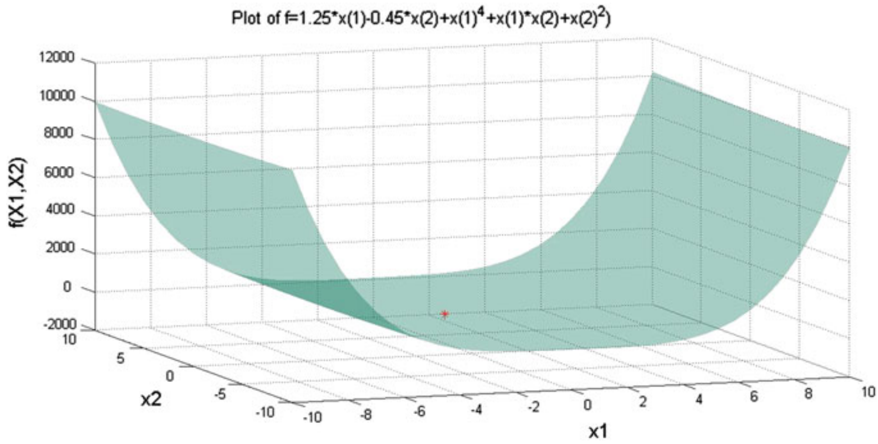


Fig. 5.36 Function f(x) and the minimum point

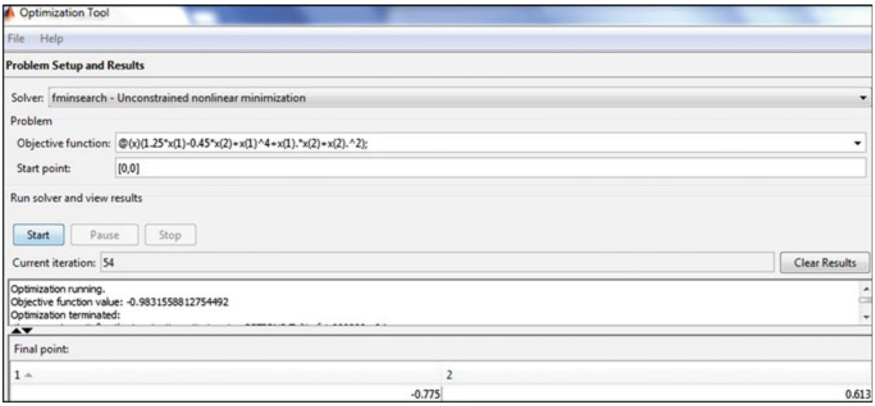


Fig. 5.37 Apply optimization tool to find the optimal solutions

This problem also can be solved simply by using the Optimization Tool in MATLAB. Figure 5.37 shows the solver function, objective function, starting point, and the results of optimization analysis.

5.2.3 Solving Constrained Nonlinear Optimization Problems

To find the minimum value of a constrained nonlinear problem, the "fmincon" solver of MATLAB can be used in the following forms;

1. $x = \text{fmincon}(\text{fun}, x0, A, b)$, find the minimum of function f(x) which is described by the term “fun” at starting point $x0$ and subject to linear inequality $Ax \leq$ b.
2. $x = \text{fmincon}(\text{fun}, x0, A, b, Aeq, beq)$, find the minimum of function f(x), described by term “fun”, at starting point $x0$ and subject to linear inequality $Ax \leq$ b and linear equality $Aeq\ x = beq$.
3. $x = \text{fmincon}(\text{fun}, x0, A, b, Aeq, beq, lb, ub)$, find the minimum of function f(x) at starting point $x0$ and subject to linear inequality $Ax \leq$ b and linear equality $Aeq\ x = beq$ with the lower and upper bounds lb and ub, repectively.
4. $[x, \text{fval}] = \text{fmincon}(\text{fun}, x0, A, b)$, return the minimum of function f(x) at the solution point x subject to linear inequality $Ax \leq$ b.
5. $[x, \text{fval}] = \text{fmincon}(\text{fun}, x0, A, b, Aeq, beq)$, return the minimum of function f(x) at the solution point x subject to linear inequality $Ax \leq$ b and linear equality $Aeq\ x = beq$.
6. $[x, \text{fval}] = \text{fmincon}(\text{fun}, x0, A, b, Aeq, beq, lb, ub)$, return the minimum of function f(x) at the solution point x subject to linear inequality $Ax \leq$ b and linear equality $Aeq\ x = beq$ with the lower and upper bounds lb and ub, respectively.

It is important to note that the function “fun” accepts a scalar like x and returns a scalar f(x). The fmincon function can be used as $[x, \text{fval}] = \text{fmincon}(@\text{myfun}, x0, A, b, Aeq, beq, lb, ub)$, in which @myfun is defined as;

Function f = myfun (x)
f = f(x)

The following examples represent the process of using fmincon function in solving nonlinear optimization problems.

Example 5.14 Apply MATLAB to determine the maximum value of function f(x) as;

$$\max\ f(x) = \exp(-x_1^3 + x_1 - 2x_2^2)$$

Subject to the following constraints;

$$x_1 + x_2 \leq 0.15$$
$$2x_1 - x_2 \leq 0.85$$

Solution: Figure 5.38 shows the behavior of function f(x) against both decision variables x_1 and x_2 in the range of $x_1, x_2 \in [-1, 1]$.

The required steps to solve this nonlinear optimization problem using MATLAB are;

1. Write an *m*-file as shown in Fig. 5.39, and save it with the name of “objfunc.m”;
2. Write a new *m*-file for the constraints, starting points, and fmincon function as is shown in Fig. 5.40.

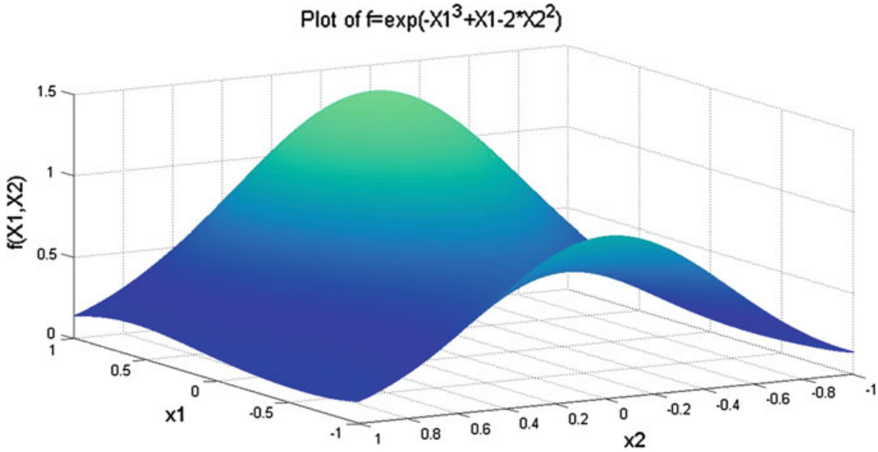


Fig. 5.38 Function f(x) against two variables x_1 and x_2

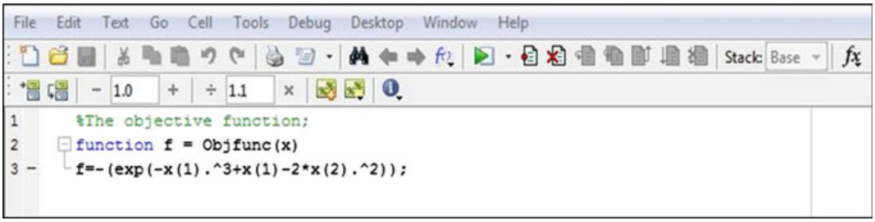


Fig. 5.39 The objective function *m*-file

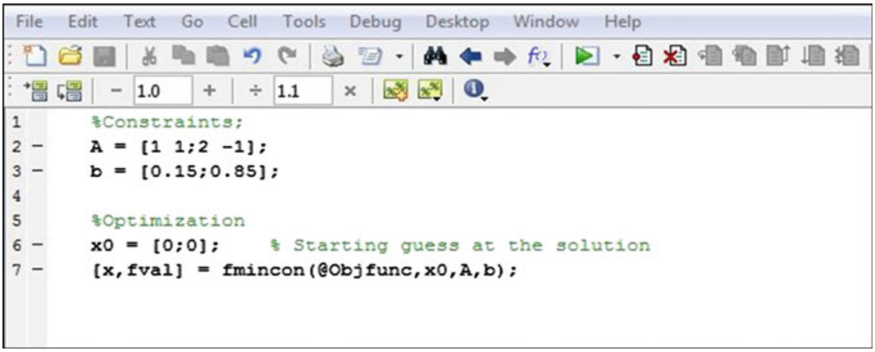


Fig. 5.40 The constraint *m*-file

It should be noted that as both constraints are linear, they are formulated as matrix inequality in the form of $Ax \leq$ b. The optimal solution for this nonlinear problem occurs in $x_1 = 0.3221$, $x_2 = -0.1721$, and f(x_1,x_2) $= -1.2579$. Hence,

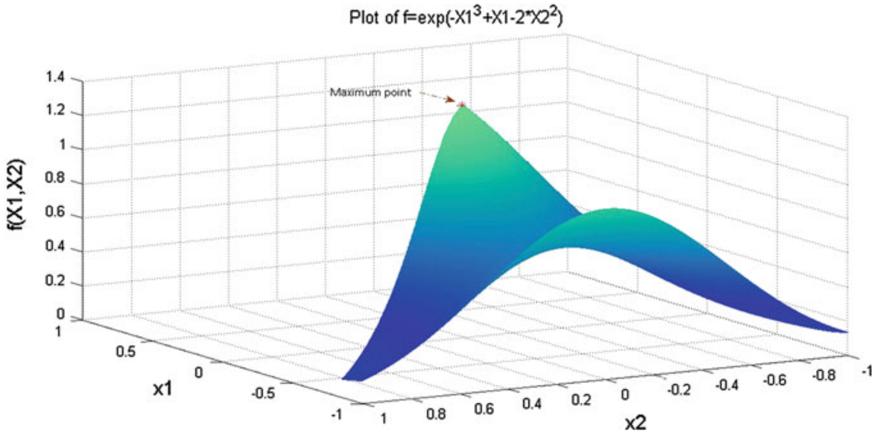


Fig. 5.41 The function f(x) and its maximum value after applying constraints

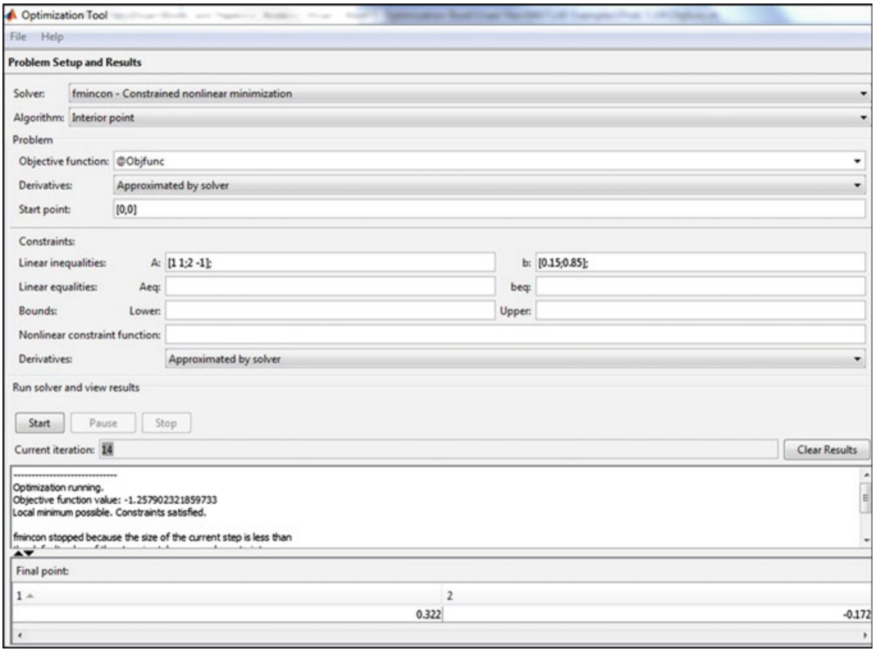


Fig. 5.42 Apply Optimtool in solving nonlinear optimization problem

the maximum of function f(x) is 1.2579. Figure 5.41 shows the function f(x) and its maximum value after applying desired constraints.

This problem also can be solved using Optimtool in MATLAB. In this case, the procedure of solving desired nonlinear optimization problem by applying Optimtool is shown in Fig. 5.42.

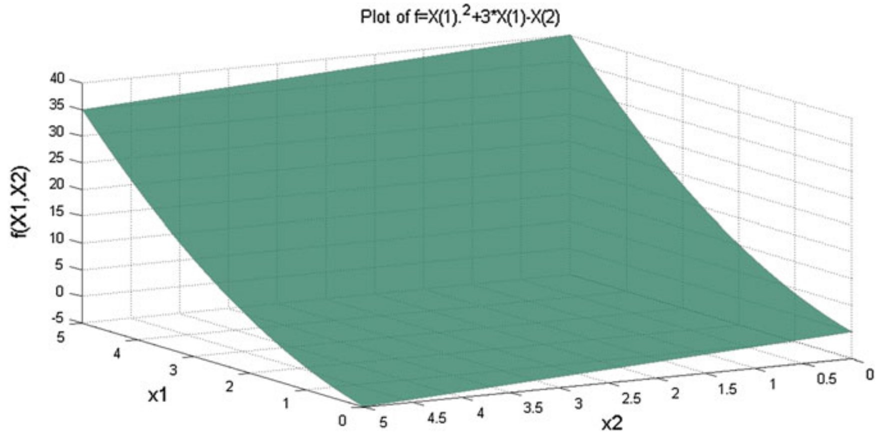


Fig. 5.43 Function f(x) in a specific range

Example 5.15 Apply MATLAB to solve Problem 3.13 and compare results with the GRG method. The objective function and constraints are;

$$\max \mathrm{f}(x) = x_1^2 + 3x_1 - x_2$$

Subject to the below constraints;

$$\begin{cases} x_1^2 + 4x_2 \leq 15 \\ 2x_1^2 - 3x_2 \leq 20 \\ x_1 \geq 0, x_2 \geq 0 \end{cases}$$

Solution: As f(x) and its constraints are nonlinear, we have nonlinear programming and the nonlinear solver like fmincon should be applied to find the optimum solution. In the case of this problem, the negative function of f(x) should be used to convert problem into a minimization problem. Afterward, two different m-files must be written for the objective function and constraints and saved under the work path of MATLAB. Figure 5.43 shows the behavior of function f(x) versus two decision variables x_1 and x_2.

The necessary steps to solve this problem in MATLAB briefly are;

1. Write an *m*-file under the name of "objfunc.m" for the objective function as is shown in Fig. 5.44. It is important to note that the name of m-file should be exactly same as "Objfunc". To become more familiar with the command "function", please see the help feature of MATLAB.
2. Write an m-file as Confun.m for the constraints (Fig. 5.45). The current m-file also should be saved under the name of "confun".
3. To solve this optimization problem, we can either use the "fmincon" function in command window (Fig. 5.46) or apply the Optimtool of MATLAB.

```
% f is the objective function
function f = Objfunc(x)
f = -(x(1).^2+3*x(1)-x(2));
```

Fig. 5.44 The objective function m-file

```
function [c, ceq] = Confunc (x)
%Nonlinear inequality constraints
c = [x(1).^2+4*x(2)-15;2*x(1).^2-3*x(2)-20];
%Linear inequality constraints
ceq=[];
```

Fig. 5.45 The constraint m-file

```
[x,fval] =fmincon(@Objfunc,[0 0],[],[],[],[],[],[],@Confunc);

%Plot

x=linspace(0,5,500);
[X1 X2]=meshgrid(x,x);

f=X1.^2+3*X1-X2;

I1=find(X1.^2+4*X2>15);
f(I1)=nan;

I2=find(2*X1.^2-3*X2>20);
f(I2)=nan;

figure;
mesh(X1,X2,f);
view([-120,22]);
%box on;
xlabel('x1');
ylabel('x2');
title('Plot of f=X(1).^2+3*X(1)-X(2)');
hold on
plot3(3.3710,0.9091,20.5675,'r*')
```

Fig. 5.46 MATLAB codes to solve desired constrained nonlinear problem

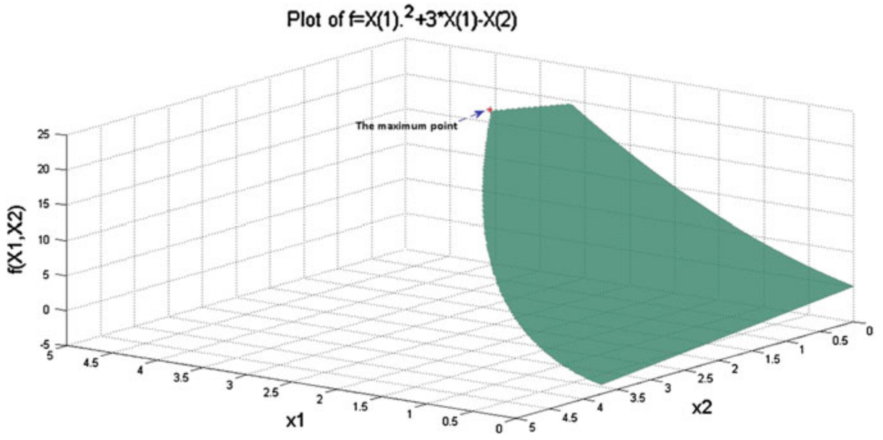


Fig. 5.47 The function f(x) and its optimal solution after applying constraints

It should be noted that the @Objfunc, and the @Confunc call defined objective function and desired constraints respectively, are used to find the optimal solutions. The optimal solution for this nonlinear problem occurs at $x_1 = 3.3710$, $x_2 = 0.9091$, and $f(x_1, x_2) = -20.5675$. Hence, the maximum of function f(x) is 20.5657. Figure 5.47 shows the function f(x) and its optimal solution after applying the constraints.

4. As already noted, the Optimtool of MATLAB also can be applied to find the optimum solution of desired nonlinear optimization problem. Figure 5.48 shows the process of solving this problem using this toolbox.

5.2.4 Solving Multiobjective Optimization Problems

The aforementioned weighting and ε-Constraint methods in Chap. 4 are two of the most traditional optimization approaches to solve the multiobjective optimization problems by transferring a multi-objective problem to a single objective one and solving the problem until finding the set of solutions. Although many researchers have used these techniques in optimization analyses, there are some practical limitations and difficulties about them in particular for solving complex optimization problems. For example, in the case of the weighted method, although it is simple to implement, a selection of weights requires prior information in which lack of this information can result in unacceptable solution and resolving the problem by considering the new weights. In addition, this method is not capable to determine points on the concave portion of the frontier and sometimes it is sensitive to the shape of Pareto front. In contrast to the traditional techniques, evolutionary methods like Genetic algorithm (GA) are known as robust and powerful methods that have been used so far by many researchers to solve complex

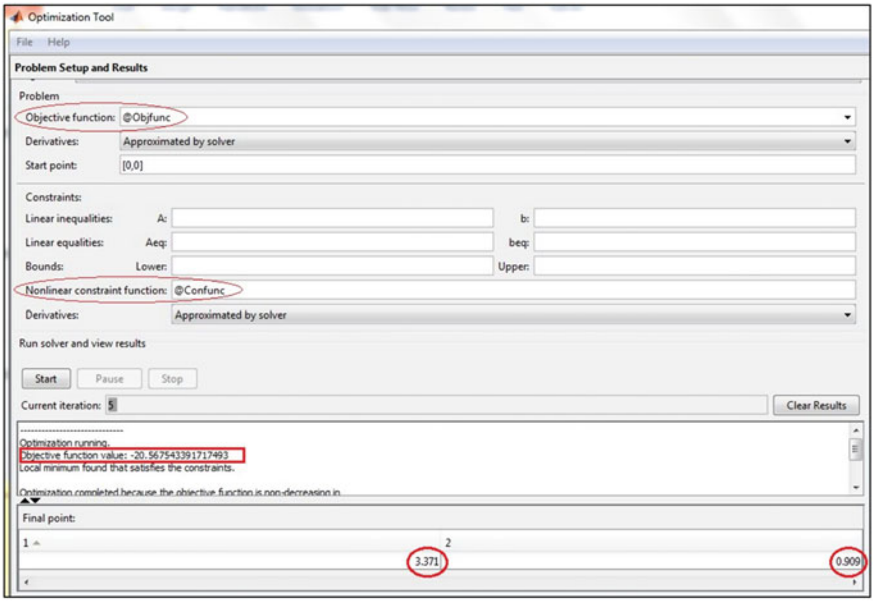


Fig. 5.48 The optimal solutions using Optimtool of MATLAB

optimization problems with multiple conflicting objectives. The Genetic algorithms methods are population based and they use a population of solution in each iteration and outcome also will be a population of solutions, while the traditional approaches are able to produce only one solution in each iteration and so, the outcomes is a single value. The main advantages of evolution methods for multiobjective optimization problems can be written as (Simonovic 2009; Deb 2008, 2009);

1. Less sensitivity to the shape of Pareto front,
2. Ability to produce multiple-objective solutions in a single iteration,
3. The outcomes are not affected by the initial solution (unlike non-linear optimization methods which are gradient based and need initial starting points).

MATLAB applies genetic algorithm to find the Pareto optimal solution for a multiobjective optimization problem. The appropriate function of MATLAB in this case is "gamultiobj" that uses genetic algorithm to perform a multiobjective minimization (optimization) and obtains local Pareto set. This function can be used in the following forms;

1. $\mathrm{X = gamultiobj(FINESSFCN, NVARS)}$; determine Pareto set X for the objective function which is described in FITNESSFCN. The second factor NVARS shows the number of decision variables for the problem.
2. $\mathrm{X = gamultiobj(FINESSFCN, NVARS, A, b)}$; find the local Pareto front subject to linear inequality $Ax \leq \mathrm{b}$.

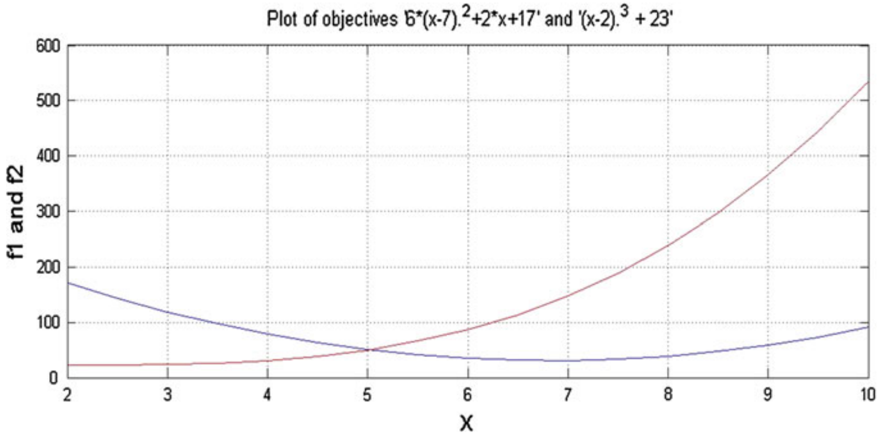


Fig. 5.49 Objective functions $f_1(x)$ and $f_2(x)$ versus decision variable x

3. $X = \text{gamultiobj}(\text{FINESSFCN}, \text{NVARS}, \text{A}, \text{b}, \text{Aeq}, \text{beq})$; obtain local Pareto set X subject to linear inequality $Ax \leq \text{b}$, and linear equality $Aeq\ x = beq$.
4. $X = \text{gamultiobj}(\text{FINESSFCN}, \text{NVARS}, \text{A}, \text{b}, \text{Aeq}, \text{beq}, lb, ub)$; find the optimal solution in the form of Pareto curve subject to linear inequality $Ax \leq \text{b}$ and linear equality $Aeq\ x = beq$ with the lower and upper bounds lb and ub, repectively.
5. $[\text{X}, \text{FVAL}] = \text{gamultiobj}(\text{FITNESSFCN, NVARS}, \ldots)$; returns the value of all of the objective function which are described in FITNESSFCN.

The following examples shows the application of "gamultiobj" solver in solving multiobjective optimization problems.

Example 5.16 Solve Example 4.1 using MATLAB and find the Pareto front as well. The objective functions for the example problem are;

$$f_1(x) = 6(x-7)^2 + 2x + 17$$
$$f_2(x) = (x-2)^3 + 23$$

Subject to the following constraint;

$$2 \leq x \leq 10$$

Solution: Figure 5.49 illustrates how two objective functions are changing against the only decision variable in this problem. As it can be seen in the figure, by increasing variable x, one objective is decreasing while the other one is increasing.

At first, we need to generate an m-file for these two objective functions as shown in Fig. 5.50 and save it with the same name of the function f which is "Mobjfunc".

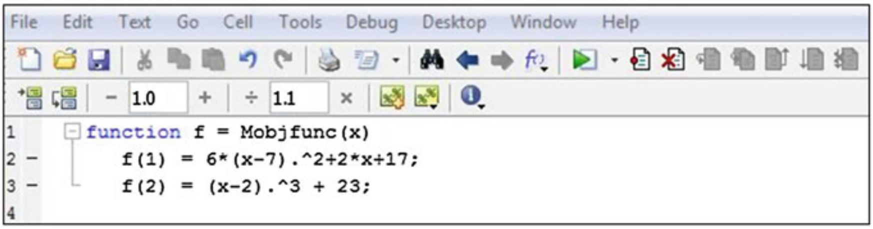


Fig. 5.50 The objective function m-file

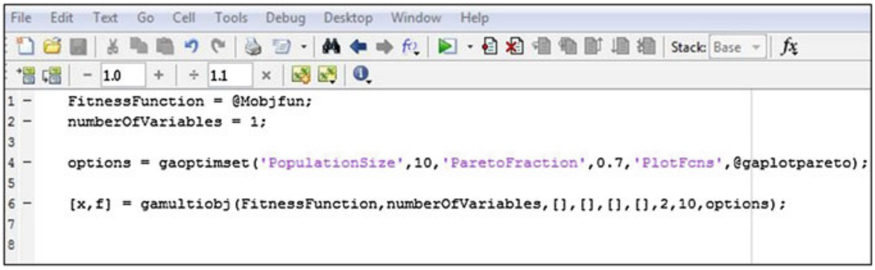


Fig. 5.51 Apply "gamultiobj" solver

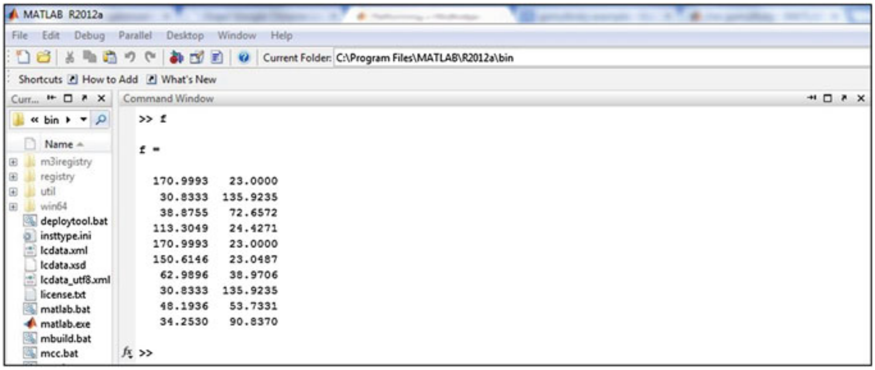


Fig. 5.52 Multiobjective optimization results

In the next step, the function "gamultiobj" should be applied to find the optimal solutions and Pareto front. Figure 5.51 shows the appropriate command lines for solving this multiobjective optimization problem.

For the sake of convenience, the results of this optimization analysis are only presented for a limited number of population size (Fig. 5.52). However, the Pareto front for a bigger population size (population of size 50) are presented in Fig. 5.53.

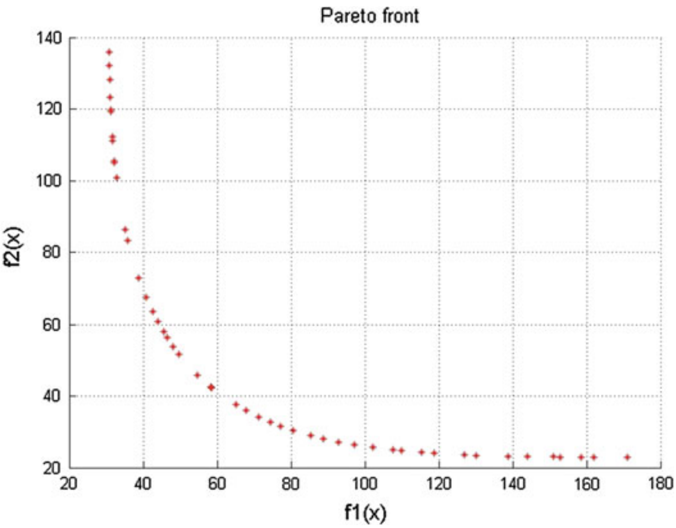


Fig. 5.53 The pareto front

Example 5.17 Apply MATLAB to solve Example 4.2 and determine the Pareto optimal solutions. The objective functions for the example problem are;

$$\min\ f_1(x_1, x_2) = 1.5(x_1 - 1)^2 + (x_2 + 1)^2$$
$$\min\ f_2(x_1, x_2) = 0.35(x_1 + x_2 - 1)^2 + (2x_2 - x_1)^2 + 4$$

Subject to the following constraints:

$$0 \le x_1 \le 5$$
$$0 \le x_2 \le 6$$
$$2x_1 - x_2 \le 6$$
$$x_1 - 4x_2 \le 0$$

Solution: Figure 5.54 illustrates how the objective functions varies against two decision variables x_1 and x_2 in the specific range of x_1, $x_2 \in [-10,10]$.

To arrive at a solution, the same process as was applied in the previous example is followed. First, we need to write an m-file for all objective functions and constraints, and then, apply “gamultiobj” solver to find the optimal solutions. Figure 5.55 shows the appropriate m-file for both objective functions $f_1(x_1,x_2)$ and $f_2(x_1,x_2)$.

As constraints are linear in this problem, it is not necessary to write a separate m-file and they can be written as shown in Fig. 5.56.

The problem is solved using the MATLAB and a part of estimated results are presented in Fig. 5.57. As it can be seen in this figure, by reducing the first objective, the values of second one are increased simultaneously.

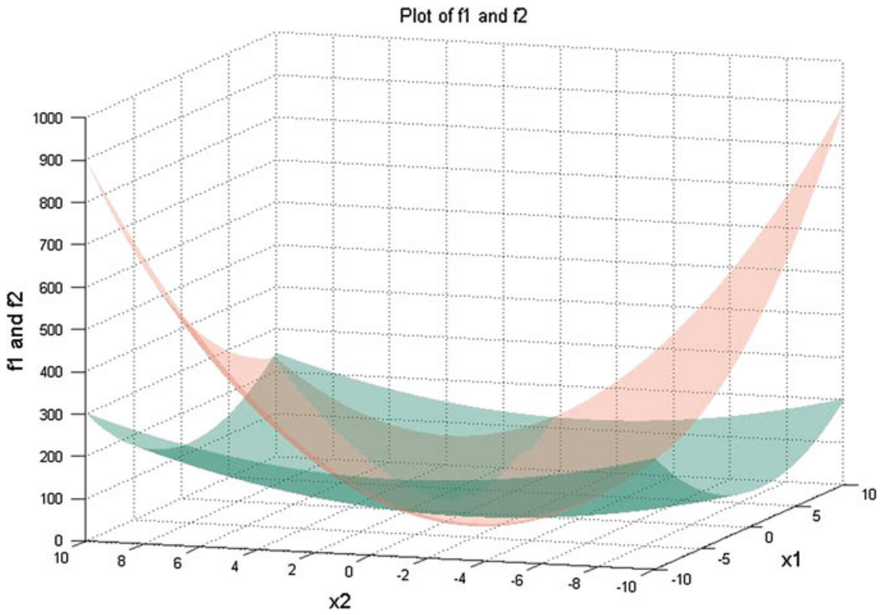


Fig. 5.54 Objective functions $f_1(x)$ and $f_2(x)$ versus decision variables x_1 and x_2

```
function f = Mobjfunc(x)
f(1) = 1.5*((x(1)-1).^2)+(x(2)+1).^2;
f(2) = 0.35*((x(1)+x(2)-1).^2)+((2*x(2)-x(1)).^2)+4;
```

Fig. 5.55 The objective functions m-file

```
FitnessFunction = @Mobjfunc;
numberOfVariables = 2;

options = gaoptimset('PopulationSize',100,'ParetoFraction',0.7,'PlotFcns',@gaplotpareto);
A=[2 -1;1 -4];
b=[6;0];
lb=[0;0];
ub=[5;6];

[x,f,exitflag] = gamultiobj(FitnessFunction,2,A,b,[],[],...
lb,ub,options);
```

Fig. 5.56 The constraints and "gamultiobj" solver

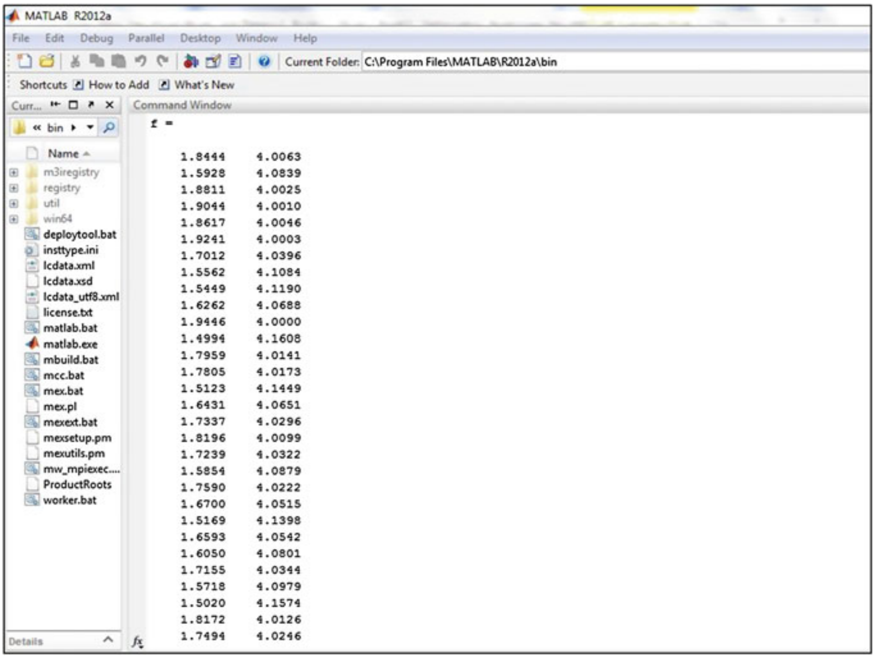


Fig. 5.57 The optimization results

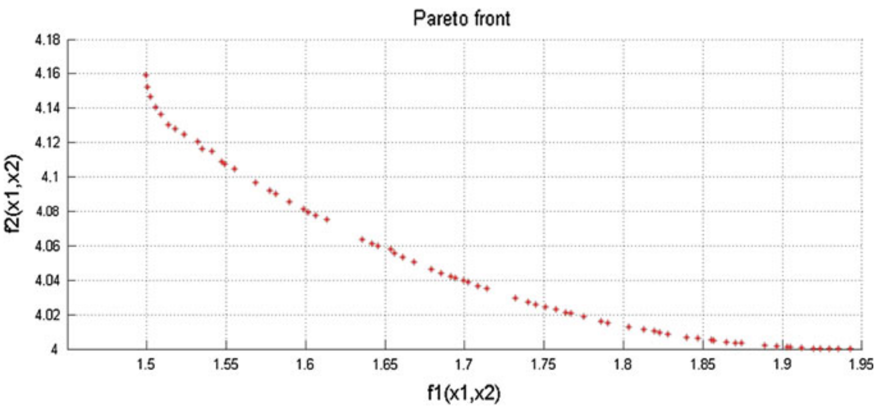


Fig. 5.58 The pareto front

Figure 5.58 shows the Pareto optimal solutions for this example. Based on the Pareto curve, the decision-maker can make a trade-off decision for the problems that the optimum value cannot be simply determined at a single point in the design space.

5.3 Problems

Problem 5.1 Solve Problem 2.1 using LINGO and MATLAB.
Problem 5.2 Find the maximum value of function f(x) in Problem 2.3 using LINGO and MATLAB.
Problem 5.3 Minimize function f(x) using LINGO and MATLAB an compare your results with the outcomes of Simplex method.
Problem 5.4 Apply LINGO and MATLAB to solve Problem 2.5.
Problem 5.5 Determine the optimal pumpage for a confined aquifer with one-dimensional steady-state flow and fixed hydraulic heads along the boundaries in Problem 2.6 by applying LINGO and MATLAB.
Problem 5.6 Solve Problem 2.7 using LINGO and MATLAB.
Problem 5.7 Minimize f(x) on the interval [−4,2] using LINGO and MATLAB in Problem 3.5 and compare your results with the Fibonacci method.
Problem 5.8 Find the minimum of function f(x) by applying LINGO and MATLAB in Problem 3.7 and compare your results with the outcomes of Newton method.
Problem 5.9 Solve Problem 3.12 using LINGO and MATLAB and see how your results are different from the results of Lagrange multiplier method.
Problem 5.10 Apply MATLAB to find the Pareto front for Problem 4.1.
Problem 5.11 Find the optimal solutions in Problem 4.2 using MATLAB and compare your results with the results of weighted method.
Problem 5.12 Solve Problem 4.3 by applying MATLAB.
Problem 5.13 Apply MATLAB to find the optimal solution in Problem 4.8.

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Chapter 6
Reservoir Optimization and Simulation Modeling: A Case Study

Abstract This chapter presents a combination of optimization (LINGO) and simulation (HEC-ReSim) models to determine monthly operating rules for the Zayandehrud reservoir system in Iran. Based on the optimized flow determined in the single-objective framework, system behavior was simulated over 47 years. The results show that optimizing the operation of Zayandehrud reservoir could increase its storage by 88.9 % as well as increase the reliability index of regulated water for all downstream demands by more than 10 %.

6.1 Introduction

A reservoir is a natural or artificial lake to storage water; it keeps the water level at a controlled level, and releases it regularly to supply downstream requirements. The most important applications of reservoirs are: flood control, agricultural and environmental water supply, domestic and industrial water supply, hydroelectric power generation, and recreational activities. However, due to increasing water demands across the world and difficulties in building new dams, it is important to enhance the efficiency of reservoir operation based on optimization analyses. In other words, we need to determine the appropriate operating policies to find the amount of water that should be released in different periods according to downstream needs. The major points that need to be considered in any reservoir optimization analysis are:

A part of this chapter is published in the Water International journal as follows:
Ziaei. M, TeangShui. L, Goodarzi. E, (2012). Optimization and simulation modeling for operation of the Zayandehrud reservoir. Water International Journal, Taylor & Francis. Vol. 37, No. 3. pp. 305–318.

E. Goodarzi et al., *Introduction to Optimization Analysis in Hydrosystem Engineering*, Topics in Safety, Risk, Reliability and Quality 25, DOI: 10.1007/978-3-319-04400-2_6,

1. Determine the main objective function in a reservoir optimization analysis (e.g., minimizing spills, maximizing releases, or minimize the cost to maximize benefits),
2. Obtain decision variables that should be optimized (e.g., water levels or releases),
3. Determine the constraints of the problem (e.g., inflows, outflows, dead storage) properly.

In general, various mathematical programming methods such as linear or nonlinear techniques are applied to optimize operation of reservoirs. However, regarding the uncertainties of hydrological and hydraulic variables, it is difficult to draw a solid operational program that controls all decision variables. It should be noted that each reservoir system includes a number of unique characteristics due to its particular geographic location, local climate conditions, and downstream requirements; and hence it needs to be studied individually.

Optimizing the operation of existing water resources and structures is of particular importance in arid and semi-arid countries such as Iran where water demand is on the rise. Because of the high geographical variability of rainfall in Iran, reservoir operation occupies an important place in the usage of water resources. An efficient approach to defining reservoir operation is to use optimization models in combination with simulation models (Ngo et al. 2007). The main advantage of simulation models is that they provide insight into how the real system might perform over time under varying conditions.

In this chapter the combination of optimization and simulation studies for long periods on ZayandehRud reservoir located in central part of Iran is presented. Many areas of Iran, in particular central regions, have recently been suffering draught, with large proportions of the country's crops and livestock perishing while it is difficult to supply the industrial and agricultural water demand. Thus, obtaining appropriate operation policies and scenarios can help managers with decision making to attain optimum allocation of water resources based on priorities and downstream demands. The main objectives of the ZayandehRud reservoir study regarding a monthly simulation–optimization model are:

1. To derive an optimal operational policy for assessing the amount of allocated water to all downstream demands (agricultural, domestic, industrial, and environmental) with regards to minimizing shortages, and
2. To simulate reservoir conditions using optimized data record of 47 years (1957–2003) for the Zayandehrud reservoir.

6.2 Optimization Analysis

Optimization or optimality is the expression that is referring to the study of minimizing or maximizing a real function by selecting the values of real or integer variables systematically from within an acceptable interval. This concept essentially is used for improving the efficiency of system and gains the best available values of some objective function in the problem area. As described in previous chapters optimization problems can be divided into two fundamental parts: the objective function, and the set of constraints. The objective function describes the performance criteria of the system. Constraints describe the boundaries and restrictions under which the system or process is being analyzed. In general, constraints include physical characteristics of the reservoir system such as storage capacities, diversion or stream flow requirements for various purposes, and mass balance. An optimal solution is a set of values of the decision variables that satisfy the constraints and provides an optimal value of the objective function.

LINGO, one of the simplest tools commonly used to formulate, solve, and analyze different linear and non-linear optimization problems, has been applied as an optimization model throughout this study. LINGO is capable of modeling all systems (large or small) for linear or non-linear problems. It creates related groups for solving the problem in which these groups are determined based on the inherent defined problems such as discharge, precipitation, demand, time period, etc. Then, LINGO allows the placing of similar objects into a *set* and uses a single statement for all elements of a set. This model allows a user to quickly input model formulation, assess the correctness or appropriateness of the formulation based on the solution, quickly make minor modifications to the formulation, and repeat the process. Many researchers such as Bozorg Haddad et al. (2008) and Montazar et al. (2010) have applied LINGO to arrive at an optimal allocation plan of surface and ground water for various types of hydrosystems.

Application of optimization techniques to reservoir operation problems has been a major focus of water resource management for some time (for comprehensive surveys, see Wurbs 1993 and Labadie 2004). Bower et al. (1962) recommend two rules for determining releases over a specific period: a Standard Operation Policy (SOP) and a hedging rule. The SOP calls for a target release in each period, if possible. If insufficient water is available to meet the objective, the reservoir releases all the available water and becomes empty; if too much water is available, the reservoir can fill up and spill the excess water. Different optimization models include linear, nonlinear and dynamic programming, which have been used to recognize the hedging rules with respect to the economic return or other system products such as water supply reliability (Hashimoto et al. 1982; Shih and ReVelle 1995; Neelakantan and Pundarikanthan 1999; Shiau and Lee 2005). The linear-based models are still popular and effective tools in dealing with optimization problems (Rani and Moreira 2009). Linear Programming (LP) is concerned with solving problems where all relations among the variables include the constraints and the objective function, and that all underlying models of real-world processes are linear. Latif and James (1991) presented a linear programming-based conjunctive model and applied it to the Indus basin in Pakistan to maximize the net income of irrigators. Peralta et al. (1995) developed a linear programming-based simulation optimization model to obtain sustainable groundwater extractions over a period of five decades, under a conjunctive water use scenario. Shih and Revelle (1995) investigated a discrete hedging rule for water supply operation during droughts and impending droughts by applying a mixed integer linear programming model. Devi et al. (2005) presented a linear programming model for optimal water allocation in a large river basin system. They applied the model to the trans-boundary Subernarekha River in India. Loucks and Beek (2005) introduced and compared various methods of water resource system optimization based on linear programming in the LINGO model. Sudha et al. (2007) studied the effects of optimization on the efficiency of water use in agriculture and highlighted what is needed for optimizing reservoir operation.

6.3 Simulation Analysis

One of the most efficient ways of analyzing water resource systems is applying simulation models. These models work based on physical relatives with a series of operational rules to simulate new conditions and system behavior under a specified policy. HEC's ResSim reservoir simulation program is a computer program applicable for hydrologic and hydraulics of reservoir system simulation. This model is also used for research in water resources management to survey the connection between decisions support system and GIS. HEC-ResSim developed by the United States Army Corps of Engineers (USACE) is the new extension of the HEC-5. This model is commonly used for simulation of flood control and conservation systems alternative analysis. HEC-ResSim, reservoir simulation program applies reservoir operation for critical state variables with operational investigation or variance purposes as constraints. The reservoir simulation models for flood control are generally defined based on single guide curves that cause for the creation of optimum realization of benefits (Timothy and Curran 2003). This program simulates reservoir operation, including all characteristics of a reservoir and channel routing downstream. The model also allows users to define alternatives and run simulations simultaneously to compare results. On this basis, HEC-ResSim as a simulation tool is able to manage drought situations where the objective is to access the impacts of different drought rules, their timing, and impacts to activate emergency measures. Computations can be performed, and the results are viewed within the simulation module. In this case, many problems require determination of the properties of the output of a system given the input and transfer function. When the transfer function is simple, the properties of the output can be obtained analytically. But when transfer function is complex, the derivation of the properties of output maybe difficult (Olani 2006). Hec-ResSim involves three main functions that are called modules and are briefly described as:

1. Watershed setup, contains system elements and basic geographic information,
2. Reservoir network, used to create a water resource network, and
3. Simulation module that performs, identifies, and manages outputs of simulation or optimization runs.

Each module has a unique purpose and an associated set of functions accessible through menus, toolbars, and schematic elements. HEC-ResSim allows the change of the background layer to show the physical layout of the system. i.e., it is possible to import an Arc GIS shape file of watershed into the HEC-ResSim and put in the background of program. Input data of HEC-ResSim include stream flow, demands (i.e., domestic, Industrial, agricultural withdrawals), power generation (if is available) and reservoir operations. Necessary data for reservoir operation includes providing adequate data of reservoir capacity, evaporation and diversions, capacity of spillway and elevation-volume curves. The main parts of ResSim program can be written as bellow:

1. Schematic which is a part of watershed module and include a schematic network of streams and rivers of watershed.
2. Module Elements to show different element in a network such as reservoirs, reaches, junctions, and diversions.
3. Operation Scheme that includes necessary criteria for reservoir release decisions.
4. Alternatives; this option allows to user to compare estimated results.
5. Analysis Tools to analyze the results of simulation and also preparing summary reports and HEC-DSSVue.

This model has been applied for simulating the history of events, especially for flood and drought periods (Hanbali 2004). Babazadeh et al. (2007) considered the performance of a storage dam by HEC-ResSim simulation model under various scenarios in present conditions and different periods considering sedimentation. Other researchers who have used this model for simulation are Olani (2006) and Klipsch (2003). The main computer programs for reservoir system modeling using simulation and optimization were reviewed by Ejeta and Mays (2002) and Wurbs (1993). Karamouz and Vasiliadis (1992) investigated a non-linear optimization model along with simulation model to analyze the long-term performance of a reservoir system. Reservoir simulation models for flood control are generally defined based on single rule curves that bring about optimized benefits (Timothy and Curran 2003).

6.4 Case Study

Water demands change from year to year and month to month. There are many physical, social, economic, and political reasons for these alternations. In recent years, significant climatic changes have been observed in many parts of the world, including more severe floods, greater precipitation, and even unusual droughts in many areas of the world. These changes have considerably influenced the water demands in many parts of the world including Iran that many regions of which are classified as arid ore semi arid. Iran has a variable climate and it has an arid

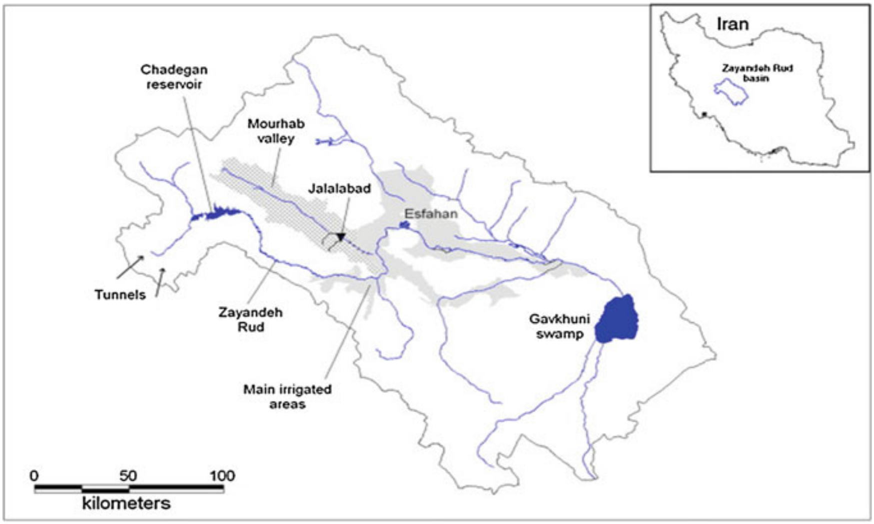

Fig. 6.1 The schematic view of ZayandehRud basin (Molle and Mamanpoush 2012)

climate in the central regions and most of the relatively scant annual precipitation falls from October through April and in some parts of the country, annual precipitation average is 25 cm (∼8 in.) or less. Due to limited water resources, optimal operation of these resources is unavoidable. Latest estimates show, the demand for water in Iran will be 116.2 billion cubic meters in 2020 with a population of 100 millions where agriculture and fishery fields are the greatest water consumers (Babazadeh et al. 2007). Obviously, it is necessary to do more research toward storage and management of water resources to meet different demands in the country. In Iran, reservoirs are usually constructed to provide multiple purposes, such as irrigation, municipal and industrial water supply, and hydropower generation. Because of the high geographical variability of rainfall in the country, reservoir operation occupies an important position in the usage of water resources. So, understanding reservoir behavior and optimal release are crucial for envisaging drought period and maximizing the annual net benefit. The final output expected from water management studies is an optimization and simulation model that will be used to assess given situations and constraints.

The Basin of ZayandehRud River is located in the west central part of Iran and is the major water source for Isfahan Province. This river is one of the most important rivers of Iran and the largest river in Isfahan Province. It starts in the Zagros Mountains and flows 400 km eastward before ending in the Gavkhouni Marsh, a seasonal salt lake, in the south-east of Isfahan City (Fig. 6.1). It is important to note that the ZayandehRud reservoir also is known as Chadegan reservoir since it is located in Chadegan area.

The River basin has an area of 41,500 km^2, altitudes change from 3,974 to 1,466 m above mean sea level (msl), annual average rainfall (precipitation) is 130,

Fig. 6.2 The schematic view of ZayandehRud basin (Molle and Mamanpoush 2012)

Table 6.1 Physical characteristics of ZayandehRud dam

Characteristics	Description
Type of dam	Concrete arch dam
Elevation from foundation (m)	100
Crest length (m)	450
Type of spillways	Gated spillway
Reservoir gross capacity (MCM)	1,470
Min elevation of operation (MCM)	210
Effective capacity (MCM)	1,250
Regulated annual water (MCM)	1,200
Irrigated area (ha)	95,000
Area of reservoir (km^2)	4,130

and monthly average temperature range of 3–29 °C. Managing optimum operation of Zayandehrud Reservoir is unavoidable because of high limitations of available water resources and recently severe droughts in the province of Isfahan. The ZayandehRud Dam and its physical characteristics are shown in Fig. 6.2 and Table 6.1, respectively.

Isfahan is a generally arid region, with agriculture, industry, and municipalities all dependent on the river as an economical source of water that seems insufficient to meet the need for water. So, there are many transbasin diversions constructed from other basins which are delivering water to the reservoir. Since many years ago there has been severe shortage of water, an optimal exploitation from available water sources has become the most important and intricate problem in the Zayandehrud basin.

To deal with varying flow of the river, reservoirs have been built and inevitably the government has done several transbasin diversion projects such as; Koohrang1, Koohrang 2, and Cheshme Langan with a total of 900 million cubic meters annual input has been implemented. Another transbasin diversion plan, the Koohrang 3 which is ongoing will have annual input of 250 MCM. The most important project of transferring water which is under study is called the Behesht Abad, consisting of, reservoir dam, tunnel with 5.5 m diameter and 65 km length with 1,100 MCM average annual input.

6.5 Optimization Model

The main objective of this study is to maximize the total reservoir release by considering domestic, industrial, and environmental aspects as major priorities over the planning horizon. Hence, the objective function of the problem can be written as follows:

$$\max Z = \sum_{i=1}^{N} R_i \tag{6.1}$$

where, R_i is the regulatory water release of the ith month and N is the planning horizon (total months of optimization, in this case $N = 564$).

It is important to note that Eq. 6.1 is used for single-objective optimization and solving it results in maximizing the dam's total regulatory water within 564 months. The general form of the intended linear optimization model can be written as:

$$F(x) = c_1 x_1 + c_2 x_2 + \cdots + c_N x_{N=} \underline{c}^T \underline{x} \tag{6.2}$$

where c_1, c_2, …, c_N, are real numbers, c^T is the transposed vector of vector c, and vectors c and x are defined as:

$$\underline{c} = \begin{bmatrix} c_1 \\ \vdots \\ c_N \end{bmatrix} \quad \text{and} \quad \underline{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix}$$

where x_1, x_2, …, x_N are the problem decision variables. So, Eq. 6.1 can be written as:

$$\max Z = \sum_{i=1}^{N} R_i = (R_1 + R_2 + \cdots + R_N) = \underline{c}^T \underline{x} = K \tag{6.3}$$

The requirement values for each month of the period are constant and known, and hence, the second term of Eq. 6.3 is constant and its value is considered as K. Therefore, the amounts of coefficient matrixes and decision variable are as follows:

$$\underline{c} = \begin{bmatrix} +1 \\ \vdots \\ +1 \end{bmatrix} \quad \text{and} \quad \underline{x} = \underline{R} \begin{bmatrix} R_1 \\ \vdots \\ R_N \end{bmatrix}$$

As reservoir water balance must be preserved in all stages of optimization, the reservoir continuity equation is considered the main constraint in this case. The reservoir continuity equation can be written as:

$$S_{i+1} = S_i - R_i + I_i \quad i = 1, 2, \ldots, N \tag{6.4}$$

where S_i is the reservoir volume in the month, S_{i+1} is the reservoir volume in the $(i + 1)$th month, R_i is the released volume of water from the reservoir in the ith month, and I_i is the inflow to the reservoir in the ith month.

6.5.1 Boundary Conditions

1. Based on the policies of the Ministry of Energy of Iran, the priority demands in ZayandehRud basin are domestic, industrial and environmental, and should be fully supplied in the planning horizon. In other words, the minimum allowed release of the dam must supply the total needs of the mentioned priorities in each month. Equation 6.5 shows the boundary conditions in this case:

$$(D_I + D_E + D_D)_i \leq R_i \leq (D_I + D_E + D_D + D_A)_i \tag{6.5}$$

where D_I is the industry requirement, D_E is the environmental requirement, D_D is the domestic requirement, and D_A is the agricultural requirement.

Equation 6.5 demonstrates that the agricultural water demand will be sacrificed during shortage and all the water in this sector will be dedicated to other sectors to minimize priority deficiencies.

2. The maximum reservoir capacity equals 1,250 MCM and $0 \leq S_i \leq S_{max}$.
3. The starting month for optimization is January and the Zayandehrud reservoir is almost half full in January of different years. So, it is assumed that the reservoir is half full at the beginning of optimization, or $S_1 = 580$ (MCM).

6.6 Finding Outlier Data

Outlier data means the data that are significantly higher or lower than normal range of time-series data. For extreme data that appear to be high or low outliers they can be tested with the Bulletin 17B detection procedure as follows. Based on long term optimization in this study (47 years), finding outlier data is necessary.

Table 6.2 Initial information for outlier test

Statistical parameters	Quantity
Mean	2.099
Standard deviation	0.337
Y_{oh}	3.158
Y_{ol}	1.041
n	564
k	3.138

1. Use the sample size (n) to obtain the value of the detection deviate K_0 (in this study, regarding to the 564 inflow data, K_0 is 3.148,
2. Compute the mean ($\bar{Y}$) and standard deviation (S_y) of the logarithms of the series data,
3. Compute the value of the detection criterion for high outliers (Y_{oh}):

$$Y_{oh} = \bar{Y} + K_0 S_y \tag{6.6}$$

4. Compare the logarithm of the extreme data being considered (Y_h) with the criterion (Y_{oh}). If $Y_h > Y_{oh}$, then the data can be considered a high outlier,
5. For low outlier data, compute the value of the detection criterion as follows.

$$Y_{ol} = \bar{Y} - K_0 S_y \tag{6.7}$$

6. Compare the algorithm of the extreme data being considered (Y_1) with the criterion (Y_{ol}). If $Y_1 < Y_{ol}$, then the data can be considered a low outlier. In this case, the coefficients and data are as follows (McCuen 2005).

For this example, Table 6.2 illustrates the initial information that is required for Bulletin 17B procedure.

Based on the initial information in Table 6.2, the outlier test was performed and the results are shown in Table 6.3. The results of outlier test show that there is no outlier data for ZayandehRud basin.

Table 6.3 Outlier test for inflow data to ZayandehRud reservoir

Rank	Inflow to ZayandehRud	Log of data	High outlier test	Low outlier test
1	82	1.913	No high outlier data	No low outlier data
2	93	1.968	No high outlier data	No low outlier data
3	140	2.146	No high outlier data	No low outlier data
4	301	2.478	No high outlier data	No low outlier data
5	286	2.456	No high outlier data	No low outlier data
6	228	2.357	No high outlier data	No low outlier data
7	157	2.195	No high outlier data	No low outlier data
8	92	1.963	No high outlier data	No low outlier data
9	52	1.716	No high outlier data	No low outlier data
10	48	1.681	No high outlier data	No low outlier data
11	57	1.755	No high outlier data	No low outlier data
12	104	2.017	No high outlier data	No low outlier data
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
557	315.5242	2.499	No high outlier data	No low outlier data
558	206.4787	2.3148	No high outlier data	No low outlier data
559	138.0758	2.140	No high outlier data	No low outlier data
560	75.99744	1.880	No high outlier data	No low outlier data
561	44.21952	1.645	No high outlier data	No low outlier data
562	37.11744	1.569	No high outlier data	No low outlier data
563	43.23456	1.635	No high outlier data	No low outlier data
564	58.4496	1.766	No high outlier data	No low outlier data

6.7 Simulation by HEC-ResSim

Simulation of reservoir system by ResSim is based on utilizing physical information of the reservoir, importing inflow data and downstream demands. The steps of simulating process in this study can be summarized according to following steps:

1. Collecting necessary data and reservoir modeling,
   The required data for the ZayandehRud reservoir has been collected from the administration for 47 water years (1957–2003).
2. Develop a schematic view of the watershed and create major parts of basin such as location of reservoir(s), junctions, and etc. shown in Fig. 6.3. Geo-referenced map files of the ZayandehRud basin (identified in step one) is used as schematic background of the model.
3. HEC-ResSim contains seven methods for routing streamflow (Coefficient, Muskingum, Muskingum-Cunge 8-pt Channel, Muskingum-Cunge Prismatic Channel, Modified Puls, SSARR, and Working R&D Routing), for flow routing in the main channel and major tributaries. As the study area is located in an arid climate, the effect of flooding is ignored.

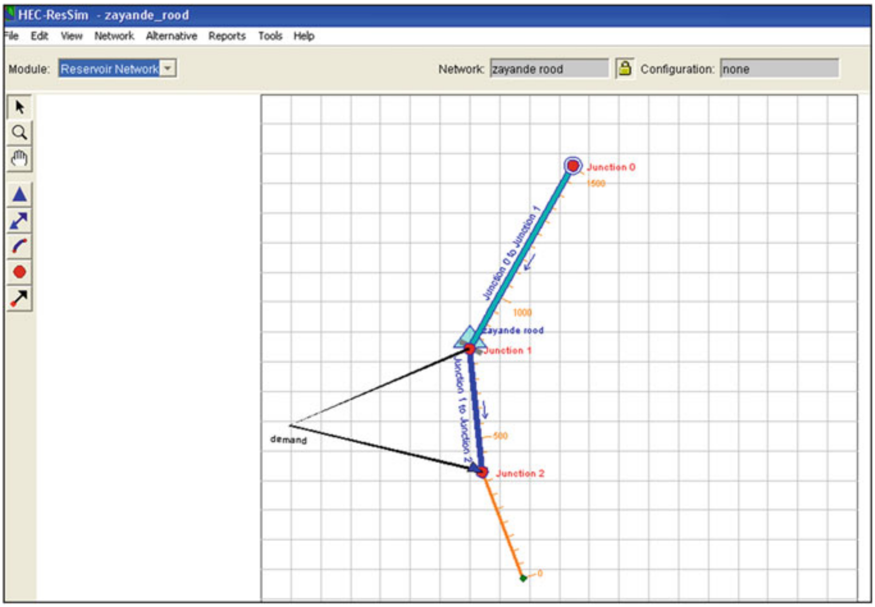

Fig. 6.3 Reservoir network modules of HEC-ResSim for ZayandehRud reservoir

4. Providing operational and physical data for any reservoirs in the watershed. The physical data of reservoir involves; the length and elevation of crest of dam, the capacities of outlet structures, pool storage definition of reservoir, etc.

During simulation by ResSim, various types of data should be applied in which they are classified as follows in this study:

1. Time-series data; monthly inflow to ZayandehRud reservoir is used for 47 years (1957–2003), and
2. Physical data include; elevation-storage-area of the ZayandehRud reservoir, reach between reservoir and sources of demands in downstream, outlet capacity curves for spillway, and junctions and diversions between the reach and demands (agricultural, environment, and industrial). The top and bottom elevations of the Zayandehrud reservoir are 2,060 and 2,005 m, respectively.

In addition, the following constraints are considered for desired case of study:

1. Evaporation of surface water,
2. The sediment profile of 50 years has been used at the beginning of the simulation period (administration of dam),
3. Simulation period is 47 years (1957–2003),
4. Operation policy is based on demands in downstream of reservoir, and
5. Based on the previous studies, the amount of seepage in the ZayandehRud reservoir is negligible, so, in this study it has been ignored.

Table 6.4 The values of current and new rule curve based on the optimized data

Month	New rule curve	Current rule curve
January	73	73
February	74	74
March	132	185
April	197	276
May	218	314
June	296	395
July	237	311
August	243	315
September	192	241
October	190	250
November	131	179
December	75	75

6.8 Study Results

6.8.1 Reservoir Operation Policy

Reservoir-river operation is based on specific policies that present practical guidelines for the amount of stored or released water from the reservoir to meet project requirements. A rule curve comprises static policies and practical guidelines to determine specific operation policies for downstream flow requirements and reservoir operation. In this study, LINGO 11.0 was applied for the single-objective optimization, and total releases were optimized by the model from 1957 to 2003.

After that, the optimized monthly averages of regulatory release were used to attain the new rule curve for ZayandehRud reservoir which will be used as a guideline for dam administration to find the best water allocation to downstream demand, and optimal water distribution to different sectors to minimize deficiency. Table 6.4 and Fig. 6.4 show the proposed and the new rule curves for different months of the year.

6.8.2 Operation Policy Performance

Evaluating reservoir operation policy performance is an important step in an optimization model. A major indicator is the reliability index (a), which was defined as the probability that the system output is satisfactory or the probability that the system will not fail in a given period. Reliability can also be defined as a probability of providing a specific percentage of water for demand in the given time period. Hashimoto et al. (1982) investigated reservoir operation system performance with a reliability index and presented Eq. 6.8:

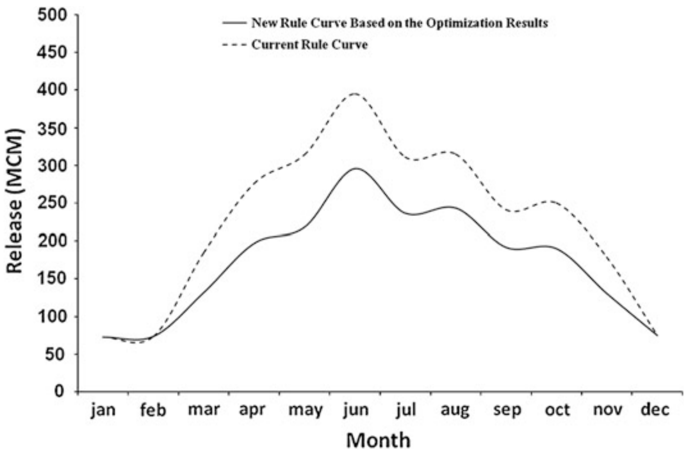

Fig. 6.4 The proposed and the new rule curve

Table 6.5 Reliability index for ZayandehRud reservoir

Reliable months		Percentage of reliable months (%)	
Opt.	Non-opt.	Opt.	Non-opt.
463	402	82.1	71.3

$$a = \frac{\text{The number of months with standard supply}}{\text{Total months}} \times 100 \quad (6.8)$$

Reliability analysis has shown that the reliability index of regulatory water increased 10.8 % for priority demands (Table 6.5).

6.8.3 Optimization Outcomes

The regulatory dam releases were evaluated by executing a simulation model for optimized and non-optimized conditions. The values of water elevation and storage volume before and after optimization analysis (from 1999 to 2001), as sample results, are presented in Tables 6.6 and 6.7. The trend of varying water elevation and water storage volume under optimization and non-optimization conditions also are shown in Figs. 6.5 and 6.6, respectively.

As the volumes of reservoir storage are 636.1 and 336.8 MCM for optimized- and non-optimized operation, respectively, the reservoir storage volume increased about 88.9 % under the optimized operation condition.

Increasing the average storage in the reservoir allows dam administrators to distribute water based on priorities and decrease deficiencies in draught seasons. Furthermore, optimization results signified that the reservoir is full for 33 months

Table 6.6 The variations of water elevation under optimized and non-optimized conditions in 1999–2001

	1999		2000		2001	
Months	Opt.	Non-opt.	Opt.	Non-opt.	Opt.	Non-opt.
January	2,009.7	2,009.7	2,009.5	2,009.5	2,014.7	2,014.7
February	2,009.8	2,009.8	2,010.1	2,010.1	2,016.4	2,016.4
March	2,013.3	2,011.4	2,012.9	2,010.2	2,019.5	2,017.4
April	2,024.2	2,018.1	2,016.7	2,009.6	2,027.3	2,021.3
May	2,027.3	2,014.5	2,016.3	2,009.5	2,032.9	2,022.3
June	2,020.6	2,009.5	2,010.0	2,009.5	2,031.6	2,013.1
July	2,009.8	2,009.5	2,009.5	2,009.5	2,024.7	2,009.5
August	2,009.5	2,009.5	2,009.5	2,009.5	2,012.9	2,009.5
September	2,009.5	2,009.5	2,009.5	2,009.5	2,009.5	2,009.5
October	2,009.5	2,009.5	2,009.5	2,009.5	2,009.5	2,009.5
November	2,009.5	2,009.5	2,009.5	2,009.5	2,009.5	2,009.5
December	2,009.5	2,009.5	2,010.8	2,010.8	2,009.5	2,009.5

Table 6.7 The variations of storage volume (MCM) under optimized and non-optimized conditions in 1999–2001

	1999		2000		2001	
Months	Opt.	Non-opt.	Opt.	Non-opt.	Opt.	Non-opt.
January	99.1	99.1	97.5	97.5	147.7	147.7
February	99.8	99.8	102.7	102.7	166.4	166.4
March	134.3	115.1	129.5	103.9	204.3	177.8
April	268.7	186.2	169.6	98.8	319.4	226.6
May	317.6	147.2	164.9	97.5	421.3	239.8
June	220.7	97.5	102.3	97.5	396.1	133.5
July	99.8	97.5	97.5	97.5	276.6	97.5
August	97.5	97.5	97.5	97.5	131.7	97.5
September	97.5	97.5	97.5	97.5	97.5	97.5
October	97.5	97.5	97.5	97.5	97.5	97.5
November	97.5	97.5	97.5	97.5	97.5	97.5
December	97.5	97.5	109.7	109.7	97.5	97.5

(5.9 %) and four months (0.7 %), and empty for 76 months (13.5 %) and 181 months (32.1 %) under optimized and non-optimized conditions, respectively (Table 6.8).

6.9 Discussions

As the Zayandehrud reservoir is located in a semi-desert area and the annual average precipitation in Esfahan Province is only 130 mm per year, there is a constant shortage in the Zayandehrud basin; and even with optimized operation,

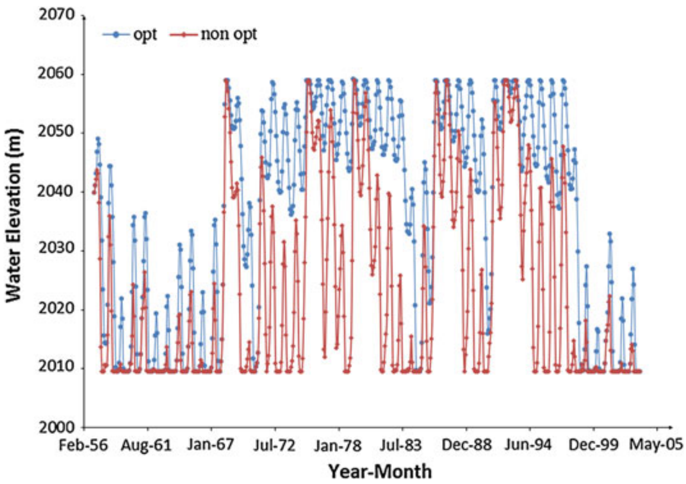

Fig. 6.5 Water elevations in optimized and non-optimized conditions

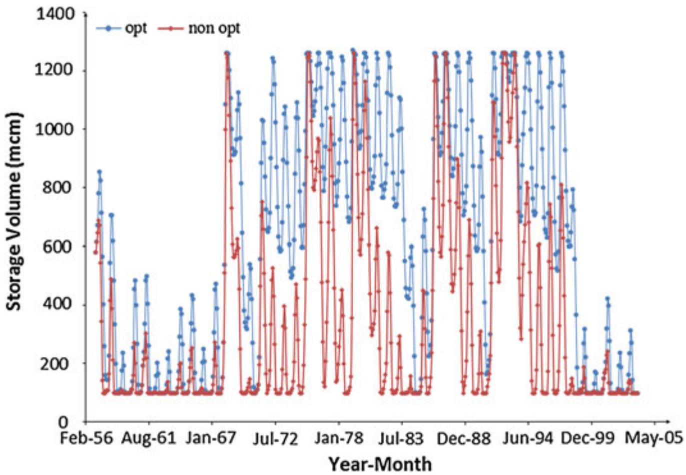

Fig. 6.6 Water storage volume in optimized and non-optimized conditions

Table 6.8 The number of months that reservoir is full or empty

	Opt.	Non-opt.
Number of months reservoir is full	33	4
Percentage that the reservoir is full (%)	5.9	0.7
Number of months reservoir is empty	76	181
Percentage that the reservoir is empty (%)	13.5	32.1

Table 6.9 The mean of regulated water in 47-year for total demands (MCM)

Months	Demands	Optimized	Non-optimized
January	73	69	67
February	74	72	72
March	185	136	178
April	276	219	271
May	314	255	303
June	395	302	335
July	311	225	232
August	315	209	186
September	241	149	116
October	250	138	110
November	179	106	95
December	75	70	68
Annual (sum)	2,688	1,952	2,033
Supply (percent)		72.6	75.7
Number of month with deficit		101	162
Reliability (%)		82.1	71.3

downstream needs cannot be completely accommodated. However, the important point is finding the best policy to allocate water between downstream demands using optimization analysis considering the priorities (drinking water, industrial and environmental).

Table 6.9 shows the total regulatory volume to supply downstream operation demand under both non-optimized and optimized operation conditions. The results demonstrate that the annual regulating volumes of the Zayandehrud dam are 2,033 MCM for non-optimized conditions, but decrease to 1,952 MCM under optimized operation. Under the optimized and non-optimized operating conditions on average, 72.6 and 75.7 % of downstream demand would be met, respectively. The results reveal that water allocated for the agricultural sector is sacrificed by getting distributed among other sectors in the optimization process, so the total release is reduced here. According to the results, about 70 % of downstream requirements were supplied under optimal and non-optimal conditions in different months of the 47 year period.

Figure 6.7 shows the average of the total regulatory volume under the non-optimized and optimized operating conditions in conjunction with total downstream demands. By considering 70 % supply of downstream demand in all months, there would be 101 months of shortage in optimized condition, while, there are 162 months of shortage under non-optimized conditions. It can be concluded that the reliability of system would be 82.1 and 71.3 % in the optimized and non-optimized conditions, respectively. Figure 6.8 shows the average of optimized and non-optimized regulatory volume for drinking, industrial and environmental purposes of ZayandehRud reservoir. The results demonstrate that the annual regulatory volume to meet the drinking, industrial and environmental needs under

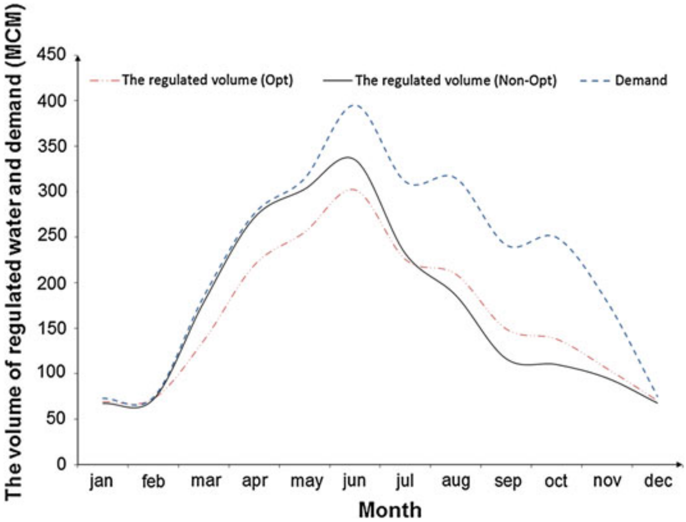

Fig. 6.7 Total downstream demands and the average of total regulatory volume

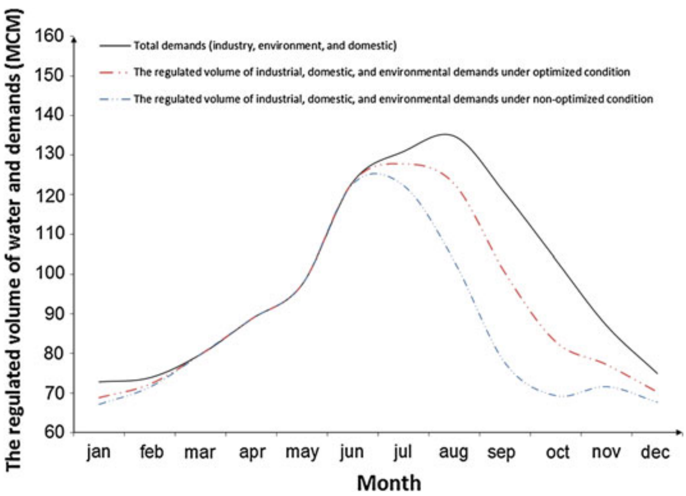

Fig. 6.8 Average of regulatory volume for drinking, industrial and environmental needs

non-optimized and optimized operational conditions are 1,039 and 111 MCM per year, respectively. On average, 93.6 and 87.5 % of the required water for domestic, industrial and environmental purposes was met under the optimized and non-optimized operating conditions, respectively. Although deficiency still exists under the optimized condition for all priorities and it is only 6.4 %; however,

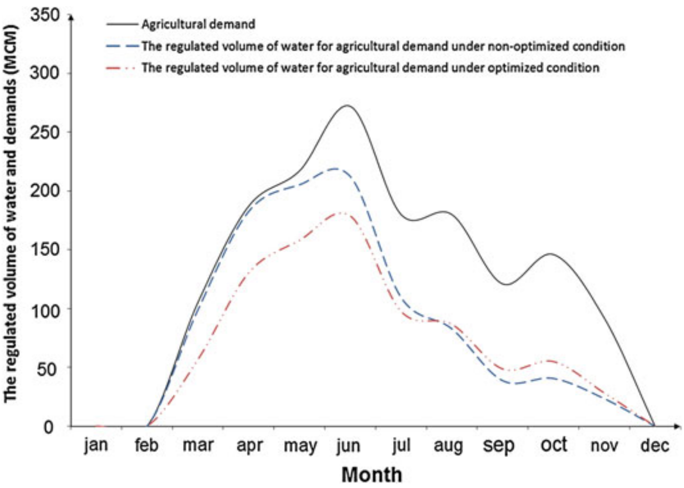

Fig. 6.9 The regulated volume of water for agricultural sector

under the non-optimized condition it is 12.5 %. This was overcome by the use of supplementary wells in the domestic and industrial water supply network and mostly applied during peak water requirement.

Furthermore, it can be observed that the Zayandehrud reservoir cannot supply the required needs of priorities for 99 and 206 months under optimized and non-optimized operating conditions, respectively. So, the reliability index is 82.4 and 63.5 % under optimized and standard operating conditions, correspondingly. In other words, under standard condition, 63.5 % of the water supply would be on the safe side, while in the optimal operating condition 82.4 % of the required water can be provided.

Finally, the achieved results revealed that the annual regulatory volume of the ZayandehRud dam to meet agricultural needs under non-optimized and optimized conditions were 994 and 840 MCM per year, respectively. The agricultural demand was supplied by 56 % under the optimized and 66.3 % under the non-optimized conditions. In other words, during the planting season, the agricultural sector would have faced 44 and 33.7 % deficit in irrigation supply in optimized and non-optimized operating conditions, respectively. Increased deficiency under the optimized condition is due to considering the lowest priority in the agricultural sector and allocating its water to other parts with higher priorities during shortage. The reliability indexes of agricultural water supply were 71.3 and 52.1 % under non-optimal and optimal operation conditions, correspondingly. These values demonstrate that the water allocated to the agricultural sector is sacrificed and distributed among other priorities (Fig. 6.9).

6.10 Conclusions

This chapter focused on the operation of the ZayandehRud reservoir using a combination of LINGO and HEC-ResSim optimization and simulation models. Study results prove that the applied methods can efficiently optimize the rule curves for operating the existing reservoir in a single-objective framework. In addition, optimizing resulted in increasing reservoir storage by about 88.9 %, increasing the time that the reservoir is full by about 5.2 %, and decreasing the time that the reservoir is empty by about 18.6 %. Although optimizing ZayandehRud reservoir results in a 3.1 % reduction of total supply, it also causes 10.8 % increased reliability index of regulatory water for all requirements. Furthermore, optimization resulted in an increase of 6.1 % of water supply and 19 % reliability index to supply priorities (drinking water, industrial and environmental).

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Chapter 7
Reservoir Operation Management by Optimization and Stochastic Simulation: A Case Study

Abstract Population increase and socio-economic mobility has escalated the water demand for various purposes and has put stress on existing water resources across the world, in particular in arid and semi-arid regions. Hence, managing the optimum use of water resources is a crucial issue and it is imperative to adopt realistic policies to ensure water is used more efficiently in various sectors. This chapter presents an optimization analysis to determine monthly operating rules for the Doroudzan Reservoir located in southern Iran. Different strategies under limited water availability conditions have been analyzed by running an optimization model based on observed and synthetic inflow data, and the performance indicators of each strategy are presented. Each strategy includes a minimum requirement release in the optimization process and results in a specific operation policy. In this study, LINGO is applied to determine optimum operational parameters and the synthetic inflows are generated using the Monte Carlo simulation method. The results demonstrated that the applied methods could efficiently optimize the current operational policy of an existing reservoir in a single-objective framework.

7.1 Introduction

Increasing water demands, higher standards of living, growing population, climate variability, and water resource limitations have caused conflicting issues among water consumers and put stress on existing water resources across the world

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A part of this chapter is published in the Water International journal as follow; Goodarzi. E, Ziaei. M, Shokri. N, (2013). Reservoir operation management by optimization and stochastic simulation. Journal of Water Supply: Research and Technology (AQUA) IWA Publishing, 62.3: 138–154.

E. Goodarzi et al., *Introduction to Optimization Analysis in Hydrosystem Engineering*, Topics in Safety, Risk, Reliability and Quality 25, DOI: 10.1007/978-3-319-04400-2_7,

(Ganji et al. 2007). Arid and semi-arid areas of the developing world are suffering from insufficient water supply and lack of adequate quantities and quality of water resources. Therefore, proper management of water resources and providing comprehensive programs to optimize available water supplies plays an important role in satisfying existing demands. Constructing dams to create reservoir and storing water allow distribution at the right time in downstream districts. Reservoirs have significant roles in water resource engineering in which their proper design, construction and maintenance contribute considerably toward fulfilling water supply requirements and minimizing the risk of water shortages. In recent years, applying optimization techniques to reservoir operation by mathematical tools has become a major focus of water resource engineers. Reservoir operation consists of several control variables that define strategies for guiding a sequence of releases to meet downstream demands. Wurbs (1993) and Labadie (2004) presented comprehensive reviews about reservoir operation models and their applications in water resources engineering. Bower et al. (1962) recommended Standard Operation Policy (SOP) and the hedging rule to determine necessary releases over desire planning horizon. The SOP releases only require demand in each time period. In other words, if sufficient water is not available to meet the objective, the reservoir releases all the available water and empties, and if there is excess water the reservoir can fill and spill the surplus water. Hence, applying SOP method will not result in preserving water for future requirements, while the hedging rule attempts to store available water and use it in the upcoming periods.

Different optimization models including linear, nonlinear and dynamic programming are using the hedging rule with respect to the economic return or the other system products such as water supply reliability (Hashimoto et al. 1982; Shih and ReVelle 1995; Neelakantan and Pundarikanthan 1999; Shiau and Lee 2005). However, the linear based models are still popular and effective tools in dealing with optimization problems (Rani and Moreira 2009). A Linear Programming (LP) solves problems that have linear relations among their variables including the constraints, objective functions, and all of the underlying models. Latif and James (1991) maximized the net income of irrigators using a linear programming-based conjunctive model for the Indus basin in Pakistan. The main objective of their study was finding the optimal ground-water extraction for stabilizing the water table below land surface, while supplementing the surface irrigation supply at the same time. Peralta et al. (1995) developed a linear programming-based simulation optimization model to obtain sustainable groundwater extractions over a period of five decades, under a conjunctive water use scenario for the Mississippi River Valley alluvial aquifer in northeastern Arkansas. Based on the results of this study, a number of optimal water-use strategies are computed for alternative management scenarios from 1990 to 2039. Shih and Revelle (1995) investigated a discrete hedging rule for water supply operations during droughts and impending droughts by applying a mixed integer linear programming model. Devi et al. (2005) presented a linear programming model for optimal water allocations in a large river basin system and applied the model to the Transboundary Subernarekha River in India. The main purpose of their study was finding the maximum annual benefits from irrigation and hydropower and also determining optimal water allocations during the dry and wet years. Loucks and Beek (2005) compared various optimization methods in water resources engineering based on linear programming. In this study, they have tried to address specific water resources planning and management problems, and also introduce optimization methods for infrastructure design and operating policies. Sudha et al. (2008) developed a mixed integer linear programming (MILP) model and used five different management strategies to test their new developed model. The results of their study showed that an appropriate management strategy with deficit irrigation, which supplies less water in non-critical growth periods and maximum water during stress sensitive periods, is the best viable solution to increase the performance of system. Ngo et al. (2007) discussed a combination of optimization and simulation models as an efficient approach for defining reservoir operation. In this study, a simulation model with real-coded GA and shuffled complex evolution (SCE) is applied for optimizing a reservoir operation in Vietnam. The results of this study demonstrated that the applied method can be used efficiently to optimize the rule curves for operating the reservoir in a multi-objective framework. Ejeta and Mays (2002) and Wurbs (1993) reviewed the main simulation and optimization computer programs for reservoir system modeling. Karamouz and Vasiliadis (1992) investigated a non-linear optimization model, along with a simulation model, to analyze the long-term performance of existing reservoirs. In another study, Sattari et al. (2009) investigated the efficiency of the Eleviyan irrigation dam in Iran by setting up the optimization model that maximized the water release for irrigation purposes after municipal water needs were met. In their study, three phases were considered to investigate the efficiency of desired irrigation dam as; (1) setting up the optimization model using recorded inflows prior to the construction of the reservoir, (2) applied inflows generated by the Monte Carlo simulation method, and (3) using inflows after the construction of the reservoir. The results of their study demonstrated that the operation policy was effectively attained during the operation period.

This study presents an optimization analysis to determine monthly operating rules for the Doroudzan Reservoir in the semi-arid area of Iran. The efficiency of this reservoir was investigated in seven different strategic alternatives by maximizing amounts of water released downstream. Each strategy includes a minimum required release in the optimization process and resulted in a specific operational policy. In other words, an optimal operational policy for assessing the amount of allocated water to all downstream demands (domestic-industrial, agricultural, and power generation) are derived based on available data in the period 1986–2006 (21 years). It is important to note that demands are considered to be constant over the desired planning horizon and they have not been changed from year to year. In addition, due to the complete development of downstream areas of the Doroudzan Reservoir over previous years and also the recent droughts in Iran, building any new industries or expanding agricultural area has been halted. Therefore, the applied data for downstream demands are approximately the same as the demands in 2012.

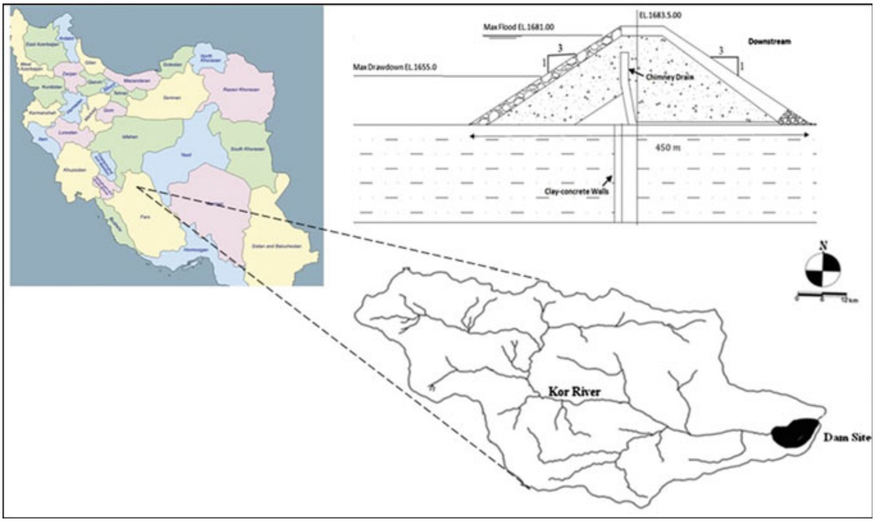

Fig. 7.1 Schematic view of Doroudzan Reservoir basin that extends between 51° 43′ and 52° 43′ E longitude and 30° 08′ and 31° 00′ N latitude

Furthermore, the optimization model was re-run using synthetic inflow data to determine the effect of alternative scenarios on the reservoir operation in a period longer than the observed data. Then, the achieved results through both observed and synthetic inflow data were compared to evaluate the optimized results in both conditions.

7.2 Materials and Methods

7.2.1 Study Area

Doroudzan Dam is one of the most important dams in southern Iran. The preliminary studies and investigations of the dam site were carried out between 1963 and 1966 and the dam construction was started in 1970 and completed in 1974. The basin of this multipurpose earth-fill dam is situated near the city of Shiraz on the Kor River and in the Bakhtegan lake catchment area (Fig. 7.1). The elevation of the highest watershed point is 3,749 m above mean sea level (MSL) and is located in the northwestern region of the watershed. The total volume and dead storage of the reservoir are 993 and 133 million cubic meters (MCM), respectively.

Basic technical information concerning Doroudzan Dam is shown in Table 7.1. The dam is a major source of water, supplying 112,000 hectares of agricultural land and the domestic-industrial and power plants requirements of Shiraz, the capital of Fars province, and Marvdasht and Zargham, two other main cities in the province.

Table 7.1 Physical characteristics of Doroudzan Reservoir

Type	Earth-fill
Height	57 m
Crest length	710 m
Crest width	10 m
Fill volume	4.8×10^6 m^3
Volume	993×10^6 m^3
Dead storage	133×10^6 m^3
Spillway type	Ogee spillway

Table 7.2 The constant monthly downstream demands of Doroudzan Reservoir (MCM)

Months	Domestic-industrial	Agriculture	Power plant	Total demands
Jan	2.82	0	48.41	51.24
Feb	3.21	0	55.82	59.03
Mar	3.74	103.42	56.15	163.31
Apr	3.91	104.04	61.41	169.36
May	3.74	167.91	59.33	230.99
Jun	4.03	155.32	125.14	284.49
Jul	4.1	136.3	114.86	255.26
Aug	4.22	144.34	117.25	265.81
Sep	4.18	141.4	56.91	202.49
Oct	3.47	25.32	53.76	82.55
Nov	3.33	20.1	37.54	60.97
Dec	3.41	0	39.42	42.82

All inflows, reservoir storage, evaporation, and releases from 1986 to 2006 have been collected by the Fars Ministry of Energy Data Center land based/surface data collection. Team members collected all available hydro-meteorological data including inflows, water elevation, rainfall, temperature, etc. for each station along the Kor River, and the recorded data were ported in Microsoft Excel for data quality assurance and quality control. Table 7.2 and Fig. 7.2 present the constant monthly downstream demands and monthly inflow data over the observation period of 252 months. As monthly inflows are less than the average of inflows (97.49 MCM) in 172 of 252 months, it can be concluded that dry periods are more dominant than wet periods in the area of study.

7.2.2 Linear Programming and LINGO

Linear programming (LP) is a popular method and the most widely used technique for optimization models. The popularity of linear programming is because of its efficient solution algorithms, its availability of generalized computer software packages, and its applicability to wide ranges of water resources problems (Wurbs 2005). Problems such as determining the size of a reservoir, finding the best

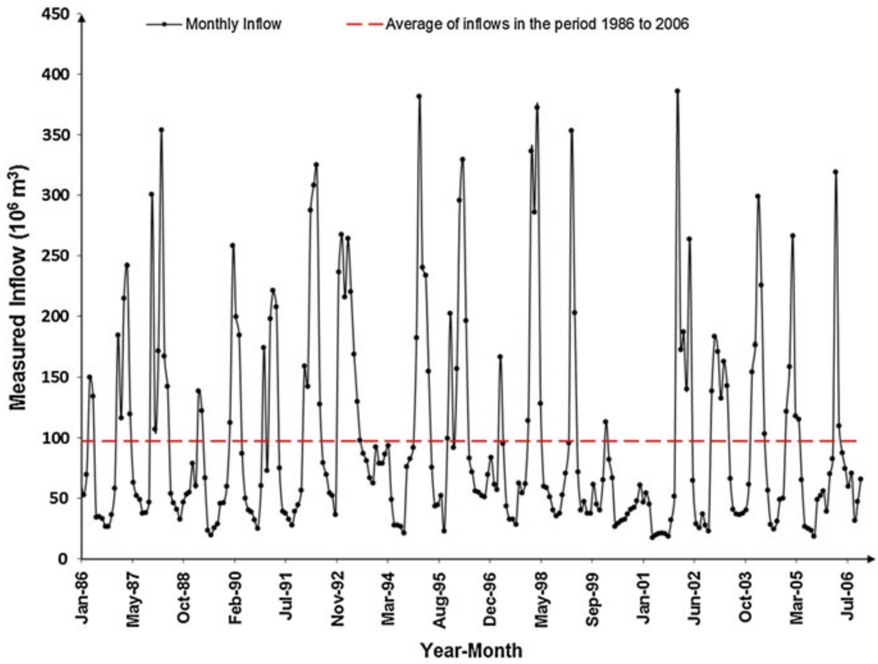

Fig. 7.2 Inflow time series and its average in the period 1986–2006

system yield, and obtaining optimum releases are handled commonly through LP application (Loucks et al. 1981). LINGO is a comprehensive tool for modeling all systems (large or small) for linear or non-linear problems as decribed previously in Chap. 5. It provides a fully integrated package that includes a powerful language for expressing optimization models and a full featured environment for building and editing problems. Furthermore, it creates related groups for solving the problem based on the inherent defined problems such as discharge, precipitation, demands, and time period. LINGO allows the placing of similar objects into a *set* and uses a single statement for all elements of a desire set. This model allows a user to quickly input the model formulation, assess the correctness or appropriateness of the formulation based on the solution sought, quickly make minor modifications to the formulation, and repeating the process until a solution is reached. Many researchers such as Bozorg Haddad et al. (2008) and Montazar et al. (2010) applied LINGO to evolve an optimal allocation plan of surface and ground water for various hydrosystem types. In another study, Ziaei et al. (2012) combined LINGO and HEC-ResSim models to determine monthly operating rules for the Zayandeh-Rud Reservoir system in central part of Iran as described in detail in Chap. 6. In their study, system behavior was simulated over 47 years and the results showed that optimizing the operation of Zayandeh Rud Reservoir could increase its storage by 88.9 %, and increase the reliability index of regulated water for all downstream demands by over 10 %. In this study, LINGO is used to determine optimum operational parameters of the Doroudzan Reservoir for different strategies and the results are presented in the following sections.

7.2.3 The Optimization Model and Constraints

Optimization methods are designed to provide the best values of system design and obtain high performance solutions. Hence, the results can increase efficiency of system outcomes and reduce conflict in operating policies (Loucks and Beek 2005). The main objective of this study is to maximize the total reservoir release after fully meeting the domestic-industrial demands and considering different priority coefficients for agricultural and power plant segments over the desired planning horizon. The mathematical form of objective function in this study is considered as follow:

$$\max Z = \sum_{i=1}^{21} \sum_{j=1}^{12} R_{i,j} \tag{7.1}$$

subjected to

$$S_{i,j+1} = S_{i,j} + I_{i,j} - E_{i,j} - R_{i,j} - SP_{i,j} \tag{7.2}$$

and

$$\left.\begin{matrix} S_{i,j} \le S_{max} \\ S_{i,j} \ge S_{min} \end{matrix}\right\} \forall_i \text{ and } \forall_j \tag{7.3}$$

where Z is a target function, $R_{i,j}$ is release supplies to downstream, $S_{i,j}$ is the initial storage in the reservoir, $I_{i,j}$ is inflow into the reservoir, $E_{i,j}$ is evaporation from reservoir surface, $R_{i,j}$ is release supplies to downstream, and $SP_{i,j}$ is spill from the reservoir in year i and month j. S_{max} and S_{min} are the maximum and minimum storage volumes of the reservoir. In addition, it is assumed that reservoir volume is not sensitive to the precipitation variable.

Equation 7.1 has been used for single-objective optimization and its solution will result in maximizing total regulatory releases for 252 months period of record. It should be noted that reservoir water balance must be preserved in all stages of optimization, and thus the reservoir continuity equation is considered as the main constraint in this case study. The assumed constraints for the applied LP model in this study are as follows:

1. The water budget equation that includes reservoir input (inflow and precipitation), outflow (domestic-industrial, agricultural, power plant releases, evaporation and spill from the reservoir), and stored water at the end of previous storage period (Eq. 7.2).
2. As the portion of reservoir capacity below dead storage is not used for operational purposes, the water volume in the reservoir should always be above the

dead storage. In the case of Doroudzan Reservoir, the dead storage is 133 MCM and then $S_{i,j} \geq 133$ MCM.

3. To minimize unnecessary spills from the reservoir at the time that the stored water in the reservoir exceeds the total capacity, the maximum water volume in the reservoir is assumed to be equal to the total reservoir volume. Thus, $S_{i,j} \leq 993$ MCM.
4. As the maximum inflows into the reservoir typically occur between January and April (Fig. 7.3a), January is assumed as the initial optimization month and the average water volume in January over 21 years was considered as the initial condition in the optimization model. Therefore, $S_{1,1} = 654.77$ MCM.
5. Different management strategies have been applied to select the appropriate release policy for Doroudzan reservoir. According to downstream demands (domestic-industrial, agricultural and power plant sectors), the minimum allowable releases are considered as follows:

$$D_{i,j} + \alpha A_{i,j} + \beta V_{i,j} \leq R_{i,j} \leq D_{i,j} + A_{i,j} + V_{i,j} \tag{7.4}$$

where $D_{i,j}$ is the sum of domestic and industrial demands, α is priority coefficient of agricultural segment $A_{i,j}$, and β is priority coefficient of power plant sector $V_{i,j}$ in year i and month j. It is important to note that the hydroelectric generation is not only an in-stream water user, it is also a large consumptive user of water at the plant.

The Doroudzan reservoir cannot supply the necessary water for all demands simultaneously and there is always a deficiency in providing downstream needs. Therefore, different management policies have been considered to find the appropriate operational policy with the maximum reliability for monthly releases. The model was run for seven management strategies which imply different minimum requirements, as follows:

Strategy 1: Only supplying domestic-industrial requirements ($\alpha = 0$, and $\beta = 0$).

Strategy 2: Supplying domestic-industrial needs plus 25 % of agricultural requirements ($\alpha = 0.25$, and $\beta = 0$).

Strategy 3: Supplying all domestic-industrial needs plus 50 % of agricultural demand ($\alpha = 0.5$, and $\beta = 0$).

Strategy 4: Supplying all domestic-industrial requirements plus 75 % of agricultural needs ($\alpha = 0.75$, and $\beta = 0$).

Strategy 5: Supplying domestic-industrial needs plus 25 % of agricultural and 25 % of power plant requirements ($\alpha = \beta = 0.25$).

Strategy 6: Supplying domestic-industrial needs plus 50 % of agricultural and 25 % of power plant requirements ($\alpha = 0.5$, and $\beta = 0.25$).

Strategy 7: Supplying domestic-industrial needs plus 25 % of agricultural and 50 % of power plant demand ($\alpha = 0.25$ and $\beta = 0.5$).

Table 7.3 The agricultural and power plant coefficients in different strategies

Strategy	Agricultural coefficient (α)	Power plant coefficient (β)
1	0.0	0.0
2	0.25	0.0
3	0.5	0.0
4	0.75	0.0
5	0.25	0.25
6	0.5	0.25
7	0.25	0.5

The values of α and β are summarized for all adopted strategies in Table 7.3. In this study, the values α and β have been determined according to the dam's administrative recommendations and demands pattern history in the study area. As the highest priority in this study is domestic-industrial, the coefficient of $D_{i,j}$ is considered one in all strategies. The second and third priorities are agricultural and power plant, respectively, and so the allocated priority coefficients for these two demands are considered less than domestic-industrial needs. The values of α and β demonstrate that the agricultural and power plant segments will be sacrificed during shortage and a portion of the available water in these sectors will be dedicated to other sectors to minimize deficiencies in main priority areas.

7.2.4 Reliability Index

In order to assess the operational performance of reservoir water delivery systems, several performance criteria can be applied to characterize demand scenarios, system alternatives and operation policies. In the example of reservoir system considered herein, there is a single variable deciding whether the system performance is reliable or not over the desired planning horizon. If water supply is not lower than water demand, the system is reliable and downstream demands will be met (Kundzewicz and Kindler 1995). In this case, the reliability index (η) is a major indicator which is defined as the probability of the system not failing in a given period. Hashimoto et al. (1982) investigated reservoir operation system performance with a reliability index as follow:

$$\eta = \frac{N_M}{N_T} \times 100 \tag{7.5}$$

where, N_M is the number of months with standard supply, and N_T is the total months over desired planning horizon.

In this study, the total and monthly reliability of the system have been considered to find the most reliable strategies and also the most supply deficit months over the desired planning horizon.

7.3 Results and Discussion

The main purpose of this chapter is to obtain monthly operational rules for Doroudzan Reservoir in southern Iran. The optimization model was run using observed and synthetic inflows data, and optimal operation policies were derived for assessing the amount of allocated water to all downstream demands including domestic-industrial, agricultural, and power plants.

7.3.1 Optimization Analysis Based on Observed Inflows

The operation of reservoirs is based on some specific policies that present practical guidelines for the amount of stored or released water to meet project requirements. A rule curve is a kind of static policy and practical guideline for determining specific operational policies based on downstream needs. In this study, LINGO was applied for a single-objective optimization, and the total releases were optimized by the model for 252 months. The optimized monthly averages of regulatory release in seven water supply strategies are presented in Table 7.4. These results can be applied as guidelines to find the appropriate way to distribute water among different sectors with minimum deficiency.

Figures 7.3 and 7.4 compare the total monthly demands, the monthly demands in each strategy, the average monthly inflows, and the monthly averages of optimized and non-optimized releases in all adopted strategies and demonstrate how optimization changes the monthly distribution of regulatory releases in different strategies compared to the total demands and non-optimized releases. Based on the results, the Doroudzan Reservoir cannot supply the necessary water for all demands simultaneously and dam administrators have to choose the appropriate operation policy based on downstream needs. Therefore, different strategies are considered to supply downstream demands based on historical inflows, existing water in the reservoir, and minimum downstream requirements.

According to Fig. 7.3a, the inflows into the reservoir decrease from April to June, while the total demands increase during this period. These months are the most critical months and the optimized releases have downward trends in strategies 1, 2, 3, and 5 (see Figs. 7.3b–d and 7.4b). In these cases, low inflows force the optimization model to only provide the minimum requirements in desired strategy and store more water to be released in the following months, such as July, August, and September that include high demands. On the other hand, the optimized releases are increased by increasing minimum requirements in strategies 4 and 7, and so, dam administrators have to release much more water in these strategies to supply downstream needs. As much more water should be released, the volume of stored water will be decreased significantly (see Fig. 7.5 and Table 7.6).

In addition to release, the yearly averages of stored water in the non-optimized and optimized conditions are estimated for all adopted strategies and the results are

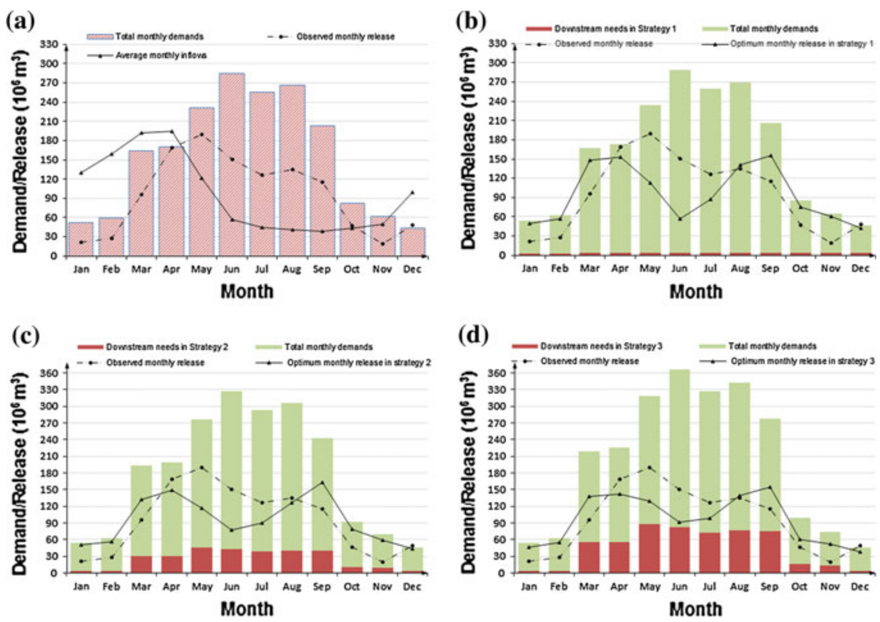

Fig. 7.3 Comparison of total monthly demands and monthly average of observed release with total demands in each strategy and **a** monthly average of inflows, **b** monthly average of optimum releases in strategy 1, **c** strategy 2, and **d** strategy 3

Table 7.4 Monthly average of optimum reservoir releases in different strategies (MCM)

Month	Strategies						
	1	2	3	4	5	6	7
Jan	49.90	50.84	46.63	30.47	49.51	45.48	33.95
Feb	56.37	56.37	54.03	34.11	57.04	52.38	41.09
Mar	148.11	131.47	137.63	119.13	136.05	128.10	119.06
Apr	153.60	149.44	142.35	124.71	139.81	143.47	123.99
May	113.35	116.30	128.64	164.94	117.37	97.62	157.29
Jun	57.45	77.38	91.35	161.59	84.17	122.48	158.62
Jul	87.82	89.87	98.39	147.62	111.74	126.02	145.65
Aug	141.24	126.21	139.53	145.37	135.02	146.61	145.32
Sep	155.27	163.69	153.88	122.41	145.83	125.63	113.28
Oct	75.02	79.09	60.41	31.32	71.25	67.26	37.67
Nov	60.97	58.46	51.91	34.35	52.74	49.69	37.28
Dec	42.82	42.82	37.19	25.93	41.41	37.19	28.74

presented in Table 7.5. According to this table, optimization increased the total stored water in the reservoir by 2.9, 4.54, 7.04, 6.69, and 1.75 % in strategies 1, 2, 3, 5, and 6, respectively, while stored water decreased about 34.9 and 33.92 % in strategies 4, and 7, respectively.

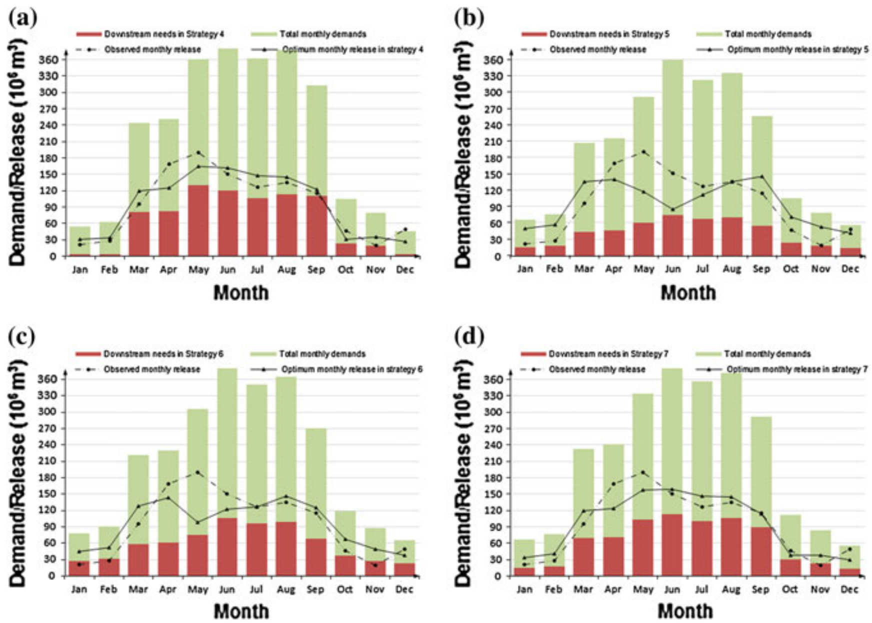

Fig. 7.4 Comparison of monthly demands and monthly average of observed release with total demands in each strategy and monthly average of optimum releases in **a** strategy 4, **b** strategy 5, **c** strategy 6, and **d** strategy 7

These results showed that much more water must be released to provide downstream needs in strategies 4 and 7. Figure 7.5a–d shows the yearly variations of optimized and non-optimized stored water in strategies 1, 4, 5 and 7, respectively.

Besides the monthly averages of releases and stored water, the monthly values of optimized and non-optimized releases and stored water during two certain dry and wet years are also presented based on a particular criterion. In this case, if more than 75 % of monthly inflows through a specific year are less than the average of inflows in the period of 1986–2006 (97.49 MCM), that year is considered as a dry year; and if less than 50 % of monthly inflows in a certain year are less than the average of inflows during planning horizon, the desired year is considered as a wet year.

As the study area is located in a semi-arid region where dry periods are more dominant than wet periods, two different thresholds are considered to determine the wet and dry years. In the case of Doroudzan Reservoir, the years 1993 and 2001 are selected as wet and dry years, respectively, and the associated inflows of each year in conjunction with the average of inflows in the period of 1986–2006 are shown in Fig. 7.6.

Figures 7.7 and 7.8 show the monthly values of optimized and non-optimized releases and stored water for strategies 3 and 6 for desired dry and wet years. As

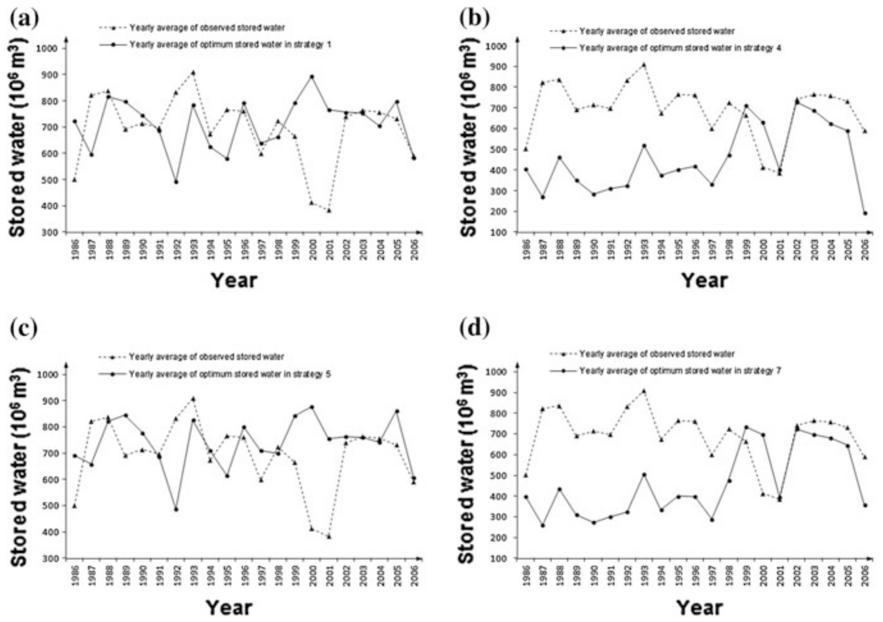

Fig. 7.5 Comparison of yearly average of observed stored water and optimum stored water in **a** strategy 1, **b** strategy 4, **c** strategy 5, and **d** strategy 7

can be seen from these figures, the optimized releases during a dry year can only provide the minimum requirement of downstream, while there are more releases and additional storages in the reservoir during a wet year.

7.3.2 Reliability Analysis

If no water is discharged from the reservoir or if the allocated water to the downstream area is below demand requirements, the released flow does not meet demands and a deficit exists. In this study, the performance of Doroudzan Reservoir was assessed before and after optimization by applying Eq. 7.5. Results showed that there are 211 deficit months and the reliability of system was only 16.27 % before optimization, while optimization analysis increased the reliability index in all adopted strategies (Table 7.6). Although there is still deficiency under optimized conditions, optimization resulted in fewer deficit months, and the reliability of the system has increased in all strategies. However, the reliability is considerably lower in strategies 4 and 7 rather than the other adopted strategies. This decreased reliability indicates that the system will face serious problems when supplying domestic-industrial and 75 % of agricultural needs or domestic-industrial, 25 % agricultural needs, and 50 % power plant requirements in

Table 7.5 Yearly average of observed and optimum stored water in different strategies (MCM)

Date	Observed	Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6	Strategy 7
1986	501.34	723.35	703.05	718.84	405.93	692.73	697.55	398.22
1987	821.36	596.85	618.11	682.41	268.84	658.29	684.70	258.59
1988	836.53	815.49	840.26	850.78	462.36	822.50	809.45	433.48
1989	691.67	797.49	806.13	813.76	351.32	845.32	792.95	309.59
1990	714.21	744.56	726.39	696.04	283.99	777.96	747.66	273.49
1991	696.23	685.19	656.81	637.85	311.41	685.84	729.64	300.18
1992	832.32	491.65	501.17	526.83	324.87	486.91	514.98	323.11
1993	908.84	785.58	761.43	855.59	518.20	827.70	803.99	505.05
1994	672.02	624.63	682.97	695.02	375.02	709.24	651.64	333.28
1995	764.99	580.63	615.00	651.36	400.57	614.25	589.70	399.15
1996	761.82	793.06	757.22	837.51	418.05	799.31	784.83	398.77
1997	599.58	637.30	714.50	696.98	330.11	709.22	688.32	288.37
1998	723.18	663.40	691.48	700.43	472.61	699.54	730.99	476.66
1999	663.91	793.75	812.07	849.16	709.10	843.63	824.71	733.76
2000	412.78	894.41	898.00	848.95	628.35	876.44	760.72	696.05
2001	383.37	765.08	751.35	729.31	402.05	756.45	471.67	394.05
2002	740.06	756.67	769.13	816.35	725.82	763.77	685.18	724.74
2003	762.72	752.14	742.80	764.62	685.58	759.67	734.52	696.48
2004	756.66	704.76	715.47	771.58	624.46	742.43	717.78	680.55
2005	731.39	798.39	861.70	832.90	590.17	861.40	816.67	644.74
2006	589.70	582.91	600.40	613.85	193.33	606.58	581.70	356.33
Ave.	693.56	713.68	725.02	742.39	451.53	739.96	705.68	458.32

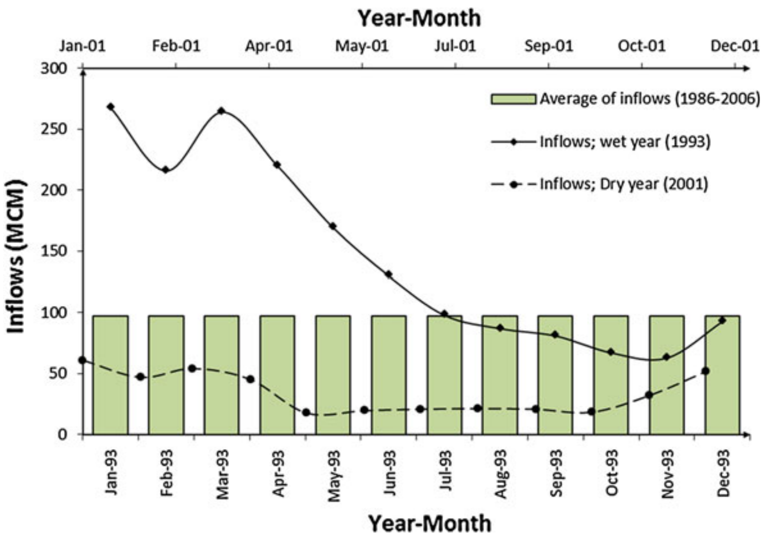

Fig. 7.6 Monthly inflows in two certain wet and dry years in conjunction with monthly average of inflows in the period 1986–2006

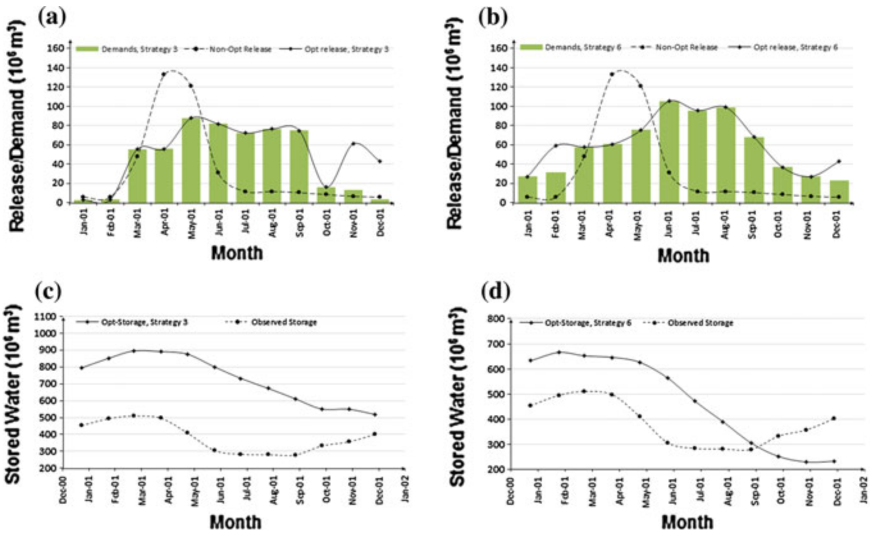

Fig. 7.7 Monthly values of optimized and non-optimized releases and stored water in strategies 3 and 6 during dry year (2001)

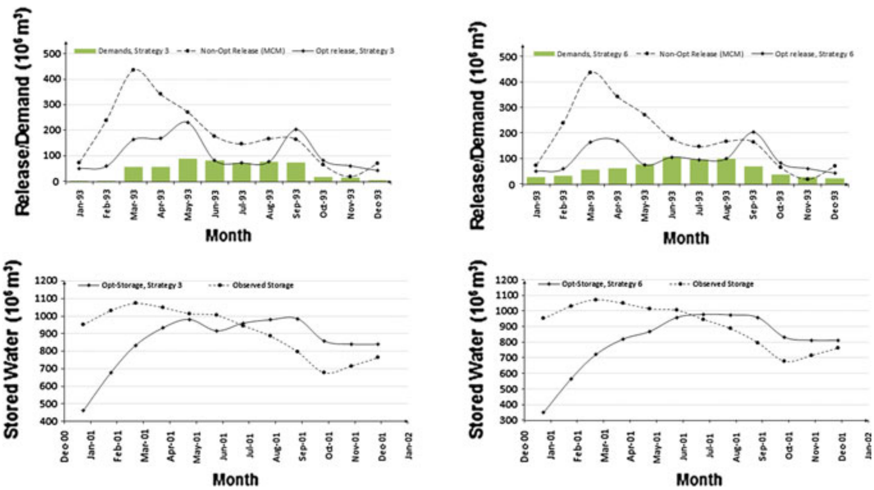

Fig. 7.8 Monthly values of optimized and non-optimized releases and stored water in strategies 3 and 6 during wet year (1993)

strategies 4 and 7, respectively. In this condition, the dam's administrator has to provide the necessary water from other available water sources to supply downstream needs with higher reliability. For example, supplementary wells can be used to overcome deficiency during peak water demand periods.

Table 7.6 The number of deficit months, reliability of system, and changing the mean of total stored water in comparison to non-optimized condition

Strategy	Number of deficit months	Reliability (%)	Variation of stored water (%)
Observed	211	16.27	–
1	65	74.00	2.90
2	77	69.00	4.54
3	103	59.00	7.04
4	177	29.76	−34.90
5	93	63.10	6.69
6	123	51.19	1.75
7	169	32.94	−33.92

Table 7.7 Monthly reliability in optimized and non-optimized conditions

Monthly reliability (%)								
Month	Observed release	Strategies						
		1	2	3	4	5	6	7
Jan	19.05	95.24	95.24	90.48	52.38	95.24	76.19	52.38
Feb	14.29	95.24	95.24	90.48	57.14	95.24	76.19	57.14
Mar	23.81	90.48	76.19	76.19	47.62	76.19	66.67	47.62
Apr	52.38	90.48	85.71	76.19	38.10	76.19	76.19	47.62
May	23.81	47.62	38.10	28.57	28.57	33.33	14.29	28.57
Jun	0.00	19.05	14.29	4.76	19.05	4.76	9.52	23.81
Jul	0.00	33.33	23.81	14.29	19.05	23.81	19.05	28.57
Aug	0.00	52.38	38.10	33.33	14.29	33.33	28.57	14.29
Sep	0.00	76.19	76.19	61.90	14.29	61.90	42.86	19.05
Oct	0.00	90.48	95.24	66.67	0.00	80.95	66.67	4.76
Nov	4.76	100.00	95.24	80.95	9.52	80.95	66.67	19.05
Dec	57.14	100.00	100.00	85.71	57.14	95.24	71.43	52.38

On the other hand, the importance of deficit magnitude has also been considered to find the most harmful months over the desired planning horizon. Hence, the optimized releases are compared with the total water demands in all 252 months and monthly reliabilities are presented in Table 7.7. Although optimization increased the reliability index in most months, its values are almost under 40 % in May–August. Since there are high water requirements and also low inflows between May and August (Fig. 7.3a), these months are recognized as the most harmful months, in particular for agricultural and power plant segments. However, it can be concluded that the achieved results prove the success of optimization analysis to supply minimum requirements associated with each strategy and also provide appropriate operational policies for Doroudzan Reservoir.

Figures 7.9a–d compare the monthly reliability in optimized and nom-optimized conditions for all adopted strategies. These figures show how the reliability of the system in each strategy varies in comparison to the other strategies. For

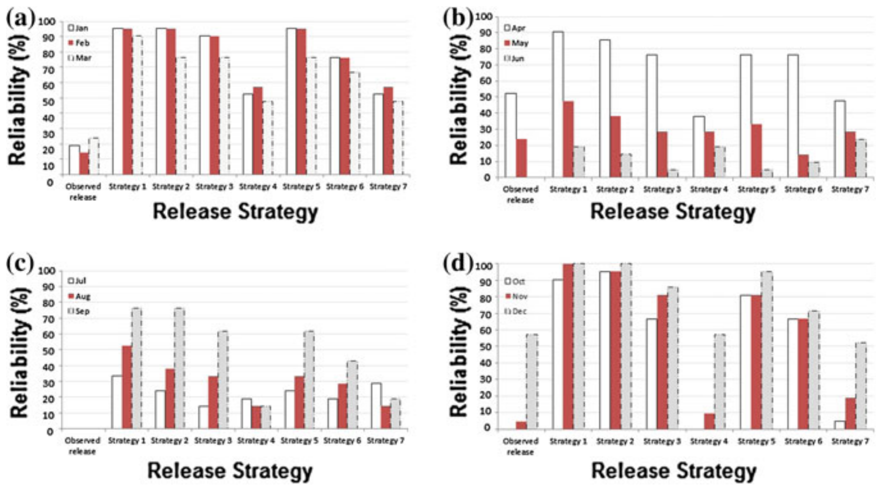

Fig. 7.9 Monthly values of optimized and non-optimized releases and stored water in strategies 3 and 6 during wet year (1993)

example, Fig. 7.9c shows the system is not very reliable in Strategy 7 in comparison to strategy 1, while it is more reliable than Strategy 4.

7.3.3 Optimization Analysis Based on Synthetic Inflow Data

In this study, in addition to the observed inflow data which were applied to evaluate optimum releases for different strategies by an LP model, a period longer than the recorded 252 months is examined by using synthetic data to determine the effect of alternative scenarios on the reservoir operation. It is important to note that using synthetic data in optimization analysis is necessary to see the changes in the reservoir operation policies in a longer period. Synthetic inflows were generated for a longer period of 432 months by the Monte Carlo method based on the statistical characteristics of the 252 months of observed inflow data. Then, the LP model was re-run in strategies 1 and 5. For this alternative, only strategies 1 and 5 were considered for optimization analysis based on synthetic inflow data. In the next step, the optimized values of releases were obtained from historical and synthetic inflow data for both strategies 1 and 5.

In the first phase, the most common statistical distributions including Normal, Gamma, Gumbel, Log-normal, and Pareto were fitted to 252 monthly observed inflow data and the best distribution was selected based on the goodness-of-fit test. To find the best fitting statistical distribution, the Kolmogorov–Smirnov (KS) and Chi Square tests were applied and the results are presented in Table 7.8.

Table 7.8 Goodness-of-fit test for observed data

Distribution	Kolmogorov–Smirnov	Chi square
Normal	0.207	240.35
Gamma	0.135	62.49
Gumbel	0.155	115.37
Lognormal	0.086	31.73
Pareto	0.223	84.95

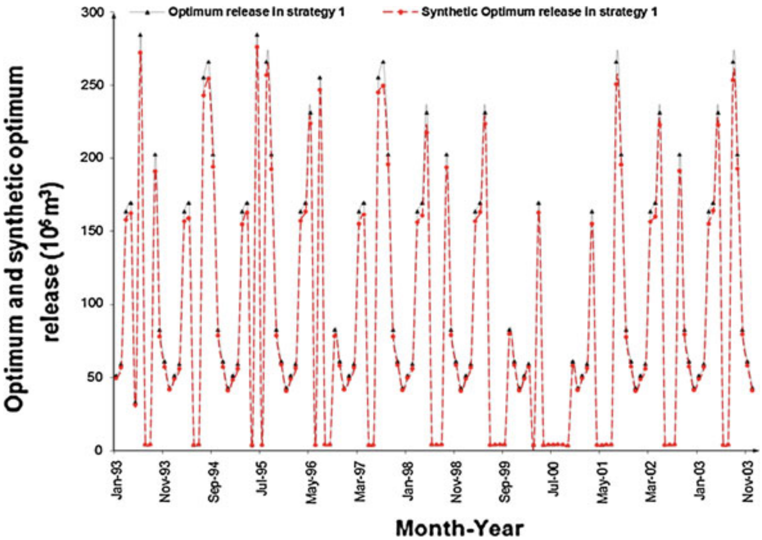

Fig. 7.10 Optimum releases and synthetic optimum releases in strategy 1 (1993–2003)

According to this table, the distribution yielding the smaller KS value, herein the Log-normal distribution, was selected and synthetic inflow data were generated for 432 months. Figures 7.10 and 7.11 compare the optimum releases and synthetic optimum releases in strategies 1 and 5, respectively, from 1993 to 2003. Furthermore, variations of stored water in strategy 1 were calculated by the applied optimization model using both data sets, and the results are plotted in Fig. 7.12 for the period of 1993–2003.

7.3.4 Performance of the Model

There are a number of ways in statistics to quantify the difference between values implied by an estimator and the true values of the quantity being estimated. In this case, three common techniques are the mean error (ME), mean absolute error (MAE) and root-mean-square error (RMSE), which can be applied to measure differences between values predicted by a model or an estimator and the observed

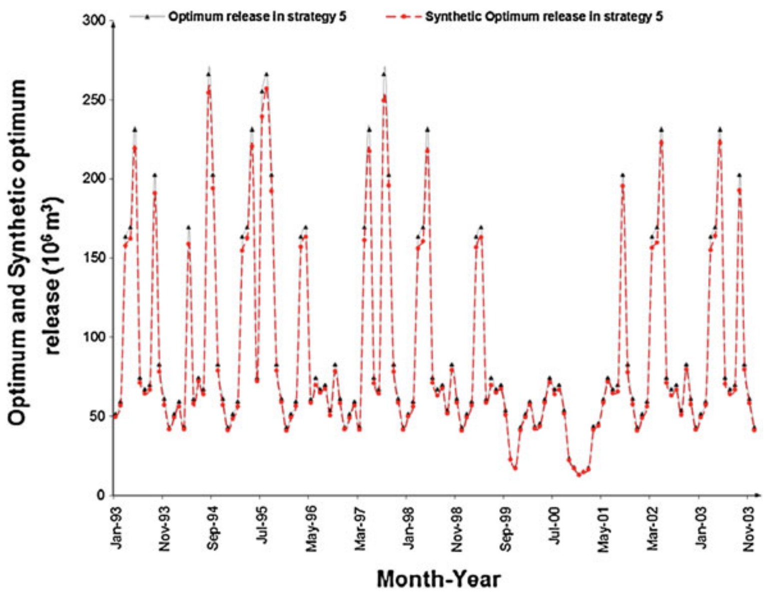

Fig. 7.11 Optimum releases and synthetic optimum releases in strategy 5 (1993–2003)

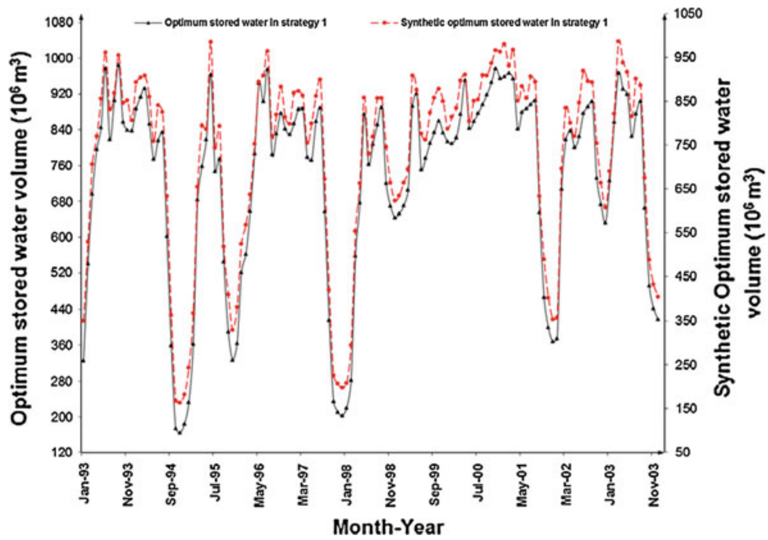

Fig. 7.12 Variations of stored water in strategy 1(1993–2003)

values. Mean Error indicates whether the forecasts are biased, MAE measures the average magnitude of the errors in a set of forecasts, and RMSE is a quadratic scoring rule which measures the average magnitude of the error. Lower values of

Table 7.9 Performance summary of the synthetic and observed values

	RMSE	ME	MAE
$R_{Opt.}$ versus R_{opt-s} in strategy 1	1.26	−0.36	0.79
$R_{Opt.}$ versus R_{opt-s} in strategy 5	1.18	−0.32	0.82
$S_{Opt.}$ versus S_{opt-s} in strategy 1	22.34	4.11	19.31

Table 7.10 Monthly average of optimum and synthetic optimum releases in strategies 1 and 5

Month	Strategy 1 (MCM)		Strategy 5 (MCM)	
	Optimum release	Synthetic optimum release	Optimum release	Synthetic optimum release
Jan	49.90	50.12	49.51	49.68
Feb	56.37	56.66	57.04	57.36
Mar	148.11	148.46	136.05	136.41
Apr	153.60	154.18	139.81	140.41
May	113.35	114.13	117.37	117.71
Jun	57.45	57.81	84.17	84.67
Jul	87.82	88.52	111.74	112.25
Aug	141.24	141.55	135.02	135.51
Sep	155.27	155.29	145.83	145.76
Oct	75.02	75.23	71.25	71.42
Nov	60.97	61.09	52.74	52.83
Dec	42.82	43.15	41.41	41.72

RMSE, ME, and MAE indicate a better fit between the model's predictions and the observed data. In this study, ME, MAE, and RMSE are used to quantitatively evaluate the performance of the applied model and estimate deviations of optimum releases (R_{opt}) and stored water (S_{opt}) from synthetic optimums release (R_{opt-s}) and stored water (S_{opt-s}). A performance summary of the predicted and observed values is presented in Table 7.9.

RMSE, MAE, and ME values between optimum release (R_{opt}) and synthetic optimum releases (R_{opt-s}) are calculated as 1.26, 0.79, and −0.36 in strategy 1, and 1.18, 0.82, and −0.32 in strategy 5, respectively. As it can be seen from Figs. 7.10, 7.11 and 7.12, and also Table 7.10, the outcomes of the LP optimization model using observed and synthetic data are well resembled and confirm each other. Table 7.10 presents the monthly averages of synthetic and normal optimized releases in strategies 1 and 5. Although the results of simulation do not always get used because of inflow regime variations due to climate changes and also recent droughts in Iran, they can be applied by dam administrators for future implementation operation policies to create new operational plans as part of novel management strategies within an acceptable range.

7.4 Conclusions

This chapter focused on the Doroudzan Reservoir operation using observed and synthetic inflow data in different strategies. The objective function of the applied LP model was maximizing allocated water for various downstream needs by considering the domestic-industry as main priority and assuming various priorities coefficients for agriculture and power plant segments. Different strategies were analyzed by running the optimization model for observed inflows during 252 months, and then seven different operation policies within each strategy's performance figured out. The achieved results can be briefly summarized as;

1. There were 211 deficit months before optimization and the reliability of system was only 16.27 %, while optimization analysis increased the reliability index from a minimum of 29.76 % in strategy 4 to a maximum of 74 % in strategy 1.
2. The optimization increased the stored water in the reservoir by 2.9, 4.54, 7.04, 6.69, and 1.75 % in strategies 1, 2, 3, 5, and 6, respectively.
3. The Doroudzan Reservoir cannot supply the necessary water for all demands simultaneously and dam administrators have to choose the appropriate strategy based on available water and downstream priorities. However, the optimization analysis increased efficiency and decreased the conflict in the management of tradeoffs between available water and downstream demands.
4. Furthermore, the optimization model was re-run based on the synthetic inflow data to consider the performance of the model based on synthetic inflows and obtaining appropriate operation policies. The results demonstrated that the outcomes of the LP optimization model using observed and synthetic data are well resembled and confirm each other. Therefore, it can be concluded that applying synthetic data in optimization analysis is useful to see the changes in the reservoir operation policies in a longer period.

It can be concluded that optimization with mathematical modeling techniques can enhance reservoir operation efficiency throughout scientific allocation of the available water, and determine the appropriate operational releases regarding downstream demands.

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Chapter 8
Using Optimization in Wellfield Operations: An Implementation Case Study at Tampa Bay Water

Nisai Wanakule and Alison Adams

Abstract This chapter includes simulation–optimization analysis of water supply in central Florida using integrated surface and groundwater modeling to allocate groundwater pumping that is protective of the natural ecosystem while meeting water supply demands of over 2 million people using a mix of surface water, groundwater and desalinated water.

8.1 Introduction

Tampa Bay Water is Florida's largest wholesale water supplier, serving more than 2.3 million people with annual average demand between 220 and 262 million gallons per day (mgd). It was established in 1974 as the West Coast Regional Water Supply Authority by State Legislation through a five-party agreement among Hillsborough, Pinellas and Pasco counties and the cities of St. Petersburg and Tampa. The city of New Port Richey joined the Agency in 1984 as a non-voting member. The Agency's mission has been to develop, store, and supply water for municipal purposes in such a manner that gives priority to reducing adverse environmental effects of excessive or improper withdrawals of water from concentrated areas. Conflict between meeting water demands and preventing harm to wetland and lake systems was intensified in the early 1990s, making it difficult under the existing Authority's organization to manage wellfields effectively. In 1996, the Authority was mandated by the Legislature to develop regional water solutions and a comprehensive answer to the water needs of the Tampa Bay area. As a result, two actions occurred, (1) the Authority was reorganized into Tampa Bay Water and obtained ownership and control over the regional groundwater supply facilities, and (2) Southwest Florida Water Management District

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E. Goodarzi et al., *Introduction to Optimization Analysis in Hydrosystem Engineering*, Topics in Safety, Risk, Reliability and Quality 25, DOI: 10.1007/978-3-319-04400-2_8,

(SWFWMD) and the Authority and its Member Governments entered into a new agreement, the Partnership Agreement, which required new water sources be developed. It also required a reduction in pumpage from eleven existing wellfields in three phases: an immediate reduction from 192 mgd to 158 mgd; then to 121 mgd by 2003; and finally to 90 mgd by 2008. In return, SWFWMD committed up to $183 million to assist with developing new, alternative water supply sources.

In response to groundwater cutbacks to relieve wetland stress and develop an environmentally sustainable water supply system, Tampa Bay Water developed the following initiatives:

(a) The revised Master Water Plan, which included the surface water supply system and surface water treatment plant to treat water withdrew from area rivers and from the then to-be-constructed off-stream reservoir.
(b) The Optimized Regional Operations Plan (OROP), an integrated ground water-surface water model to schedule well pumpage among eleven wellfields with an objective of maximizing surficial aquifer water levels.
(c) The Phase I Mitigation Plan, which provided a rehydration plan for hydrologically-stressed wetlands and lakes.

As the new water supply sources of the Master Water Plan came on-line, the OROP model and the Phase I Mitigation Plan were designed to ensure that water production would not result in unacceptable adverse environmental impacts and that historical impacts from groundwater production were addressed.

A Master Water Plan developed prior to the reorganization in 1998 was modified to include many new projects with a very challenging time schedule. The Plan included new water treatment facilities and conveyance for that supply to be developed in the Tampa Bay area to enable mandated wellfield cutbacks and meet growing demand. Components of the plan also included additional infrastructure to interconnect the 11 consolidated wellfields, the hemisphere's largest seawater desalination plant with a permitted capacity of 28.75 mgd; an "enhanced surface water system" comprised of three supply sources, a 15.5 billion gallon (BG) off-stream reservoir and a new 72 mgd regional surface water treatment facility (which has been expanded to a 120 mgd permitted surface water treatment facility); a redesigned and refurbished wellfield; and more than 80 miles of interconnecting pipelines. The program was projected to meet the regions needs with environmentally sustainable, diversified, drought proof and/or drought resistant and cost effective new sources as well as aggressive water conservation efforts to reduce demands by up to 17 mgd. Figure 8.1 shows the current facilities after Phase I of the Master Plan had been completed.

The OROP was designed to minimize production impacts to wetlands and lakes by rotating among sources in response to target levels set in surficial aquifer monitoring wells. These target levels were determined by statistical correlations between minimum levels established for wetlands and lakes and surficial aquifer water levels. The establishment of minimum wetland and lake levels was based on

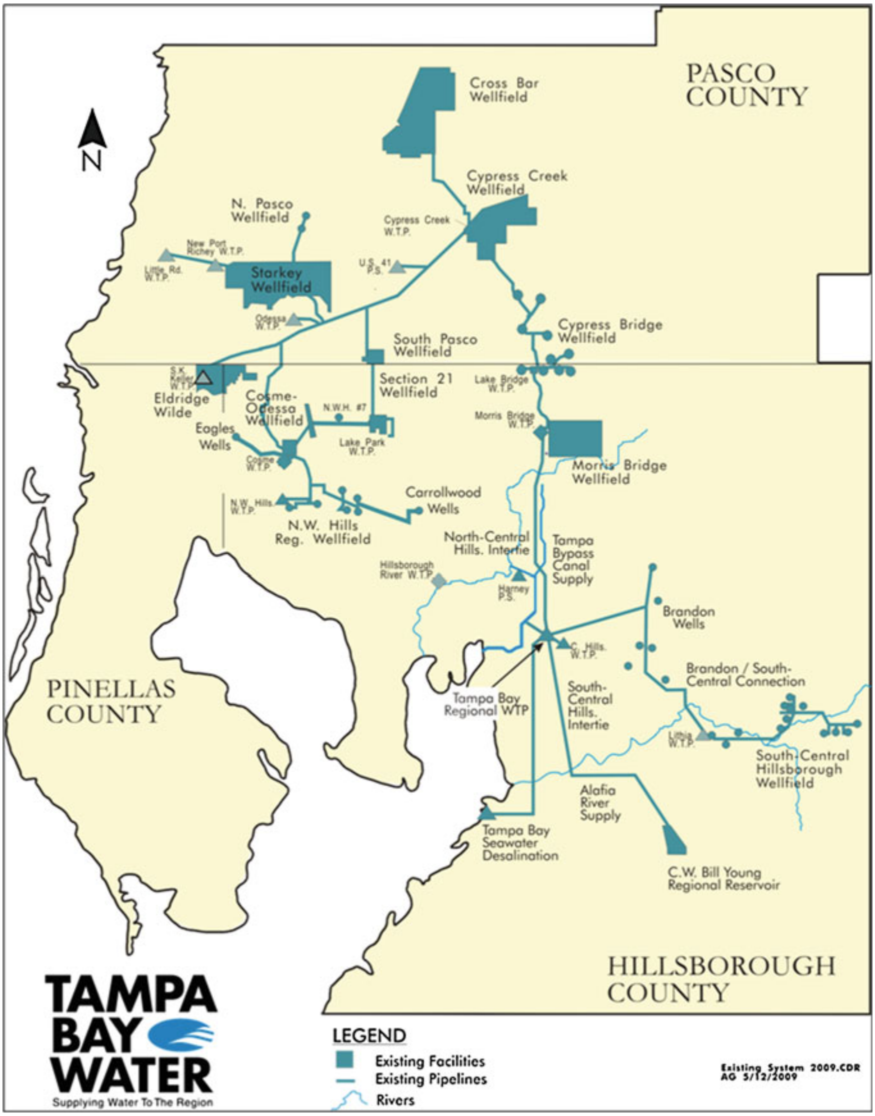

Fig. 8.1 Locations of Tampa Bay Water facilities in 2013

regulatory criteria that relate environmental health to indicators of historical wetland and lake levels (known as historical normal pool).

The Phase I Mitigation Plan began with a study that assessed wetland hydrologic impacts, and predicted wetland water level recovery, based on aerial photointerpretation, GIS analysis and integrated surface water—groundwater modeling. Based on the results of that study, rehydration plans were being developed for those wetlands that were predicted not to recover to their minimum levels (on a long-term average basis) after the Master Water Plan projects were operational. These mitigation plans included surface drainage alterations (e.g. ditch blocks, water control structures), rehydration with reclaimed water or excess surface water, and possibly groundwater augmentation.

In 1998, about 71 % of the demand was met by 12 regional wellfields with the remaining 29 % met by the use of a surface water reservoir owned and operated by City of Tampa. Eight of the 12 Tampa Bay Water's regional wellfields were interconnected which, if the supplies were adequate, would facilitate the rotation of pumpage based on environmental, physical and regulatory constraints. It was obvious that Tampa Bay Water had to start implementing an innovative water supply management program to effectively manage the existing wellfields and the up-coming new water sources in an environmentally sustainable manner.

8.2 Optimized Regional Operation Plan

8.2.1 Background

The Optimized Regional Operations Plan (OROP) is a key component of the Operations Plan. The OROP is a custom-built application which incorporates an optimization model and utilizes output from various models, current hydrologic and pumpage data, and a set of operating constraints to manage the 11 wellfields under the Consolidated Permit (also known as the Central System Facilities), the Brandon Urban Dispersed Wells (BUDW), and the Carrollwood wells (Fig. 8.2) through the development of weekly production schedules. The models used to provide input to the optimization model include the Integrated Hydrologic Model (through the development of a unit response matrix or URM), a group of artificial neural network models, surface water forecasting tools, and short-term demand forecasting models. Input to the optimization model includes demands, surface water availability and scheduled withdrawals from the Hillsborough River/Tampa Bypass Canal system, Alafia River and Regional Reservoir, and scheduled production from the seawater desalination facility. The optimization model schedules production from the Central System wellfields based on current hydrologic conditions, operational constraints, permit limits, forecasted treated surface water reliably available from the regional surface water treatment plant, and reliably available desalinated seawater, to meet forecasted Member Government demands, and seeks to optimize groundwater levels based on targets at a selected set of surficial aquifer and Upper Floridan Aquifer monitoring wells called control points. It also adheres to operating policies and infrastructure physical limits as well as complies with conditions of the Consolidated and other water use permits. Policy issues are addressed by using weights to assign preferences to maximize groundwater levels at the control point locations.

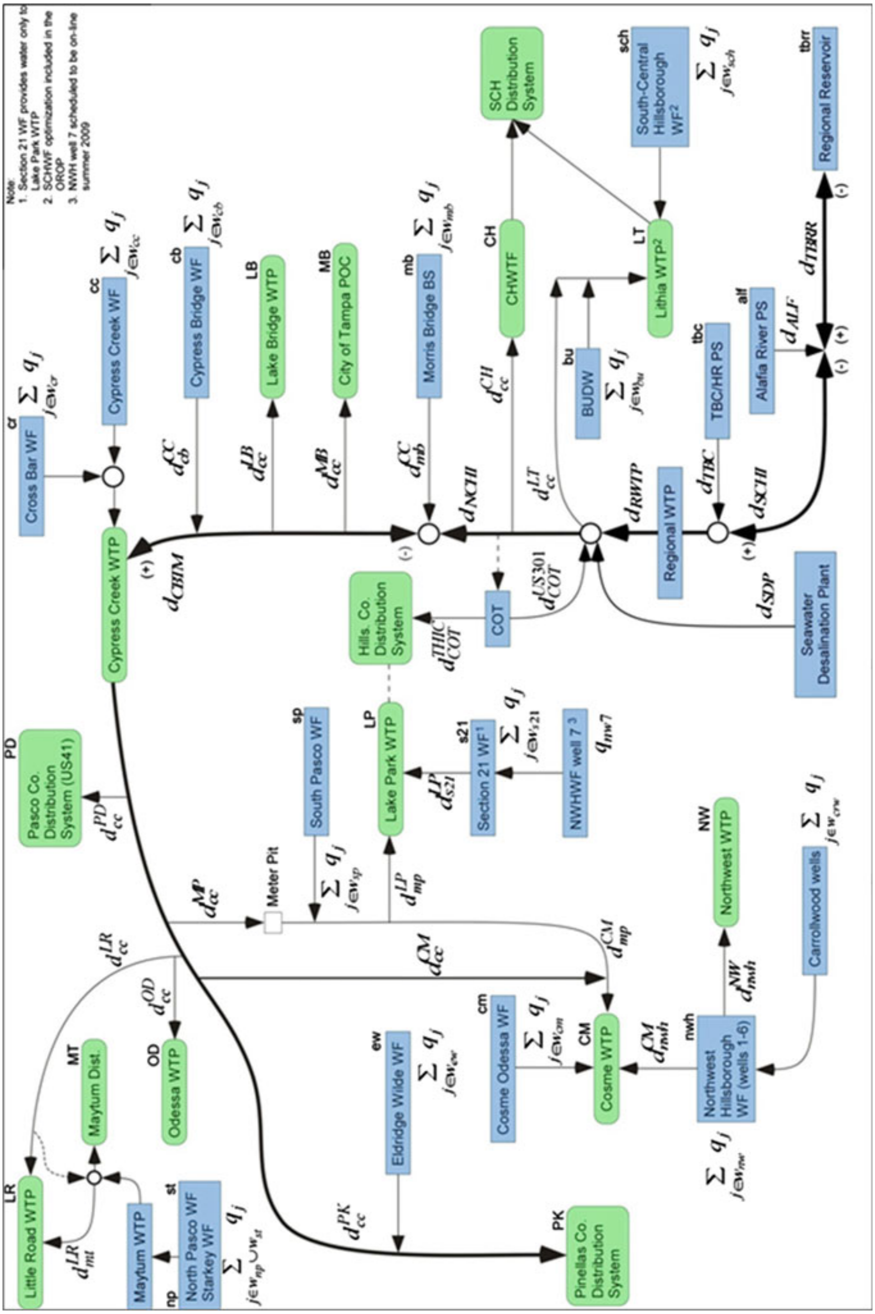

Fig. 8.2 Schematic diagram of OROP infrastructure

The output of the optimization routine is a weekly schedule prioritizing pumpage from all active production wells of the Central System Facilities.

The OROP is formulated as a linear programming (LP) model. Constraints that govern the optimization model generally fall into one of four categories—physical constraints (e.g., pump capacities, conveyance facilities), regulatory constraints (e.g., wellfield pumping limits, specified water levels), operational constraints (e.g., water quality, minimum production limits), and demand constraints. An additional set of constraints that represents the integrated surface/groundwater hydrologic system is required to complete the optimization formulation.

The hydrologic model, which is based on the physical characteristics of the surface and groundwater systems, simulates changes in water levels due to changes in pumpage and rainfall. The pumpage/water-level relationships are based on the Integrated Hydrologic Model (IHM) Northern Tampa Bay application providing a unit response for each production/monitor well combination which relates pumpage changes to water-level changes.

Water quality constraints are included in both the physical constraint and regulatory constraint categories. Tampa Bay Water produces a "finished" product that meets not only Safe Drinking Water act requirements but also additional standards and requirements defined in the Agency's governance documents.

8.2.2 Optimization Formulation

The objective of this optimization problem is to maximize ground-water levels at specified locations in the surficial aquifer system (SAS) while satisfying the projected water demands and complying with regulatory requirements, given the system constraints. The primary decision variables for each time period are the pumping rates at each production well withdrawing from the Upper Floridan Aquifer system (UFAS). The secondary decision variables (also called state variables) are the ground-water levels in monitoring wells for both the SAS and UFAS. The problem is subjected to two general constraint sets and three specific constraint sets. The general constraint sets consist of a system of equations describing the surface and ground-water hydrology and the variable bounds. Tampa Bay Water's Integrated Hydrologic Model of the Northern Tampa Bay area is currently used to simulate the physical system hydrology. The specific constraint sets consist of the demand constraints, the regulatory constraints on water levels and pumpage specified in the water use permits (WUPs), and operation/maintenance and water quality constraints of the infrastructure system. The optimization routine determines the wellfield and well production schedule based on the water demands projected for each of the points of connection and reliably available treated surface water and desalinated seawater. Before the problem is formulated mathematically, a set of notations must be defined as;

i	An index of an element in the set R^u or R
t	An index for time period corresponding to the week number in the simulation model, ($t \leq 0$ refers to time index prior to the start of simulation)
$h_{i,t}^u$	The SAS water level at location i at the end of time period t
$h_{i,t}$	The UFAS water level at location i at the end of time period t
ω_i	The assigned weight to enable priority factors applied to reduce environmental stress preferentially at location i
R^u	A set of monitoring wells in SAS where water levels are being maximized
R	A set of regulatory monitoring wells specified in WUPs
H_i	Regulatory water level in the UFAS at the monitoring well i
$q_{j,t}$	Average weekly pumping rate from the jth well for time period t
$d_{n,t}^r$	Pipe flow from the nth source to the rth point of connection during time period t
D_t^r	Water demand for the rth point of connection for time period t
w_n	A set of production wells for the nth wellfield
j	An index of an element in the set w_n
P_n^y	Regulatory 12-month average withdrawal for wellfield n
P_n^m	Regulatory peak month withdrawal for wellfield n
P_n^m	Regulatory peak month withdrawal for well j
W_t	The week number in water year (commence on Oct 1 each year) for time period t
C_j^{Fe}	Level of measuring iron concentration (mg/l) in production well j
C_j^{H2S}	Level of measuring hydrogen sulfide concentration (mg/l) in production well j
$\underline{Q}_j, \bar{Q}_j$	The lower and upper production limits (by the maintenance requirement, or well capacities, or the peak shaving program) for the jth well,
Q^{cc}	The Cypress Creek Pumping Station capacity
$\underline{Q}_n^{wf}, \bar{Q}_n^{wf}$	Minimum and maximum limits (required to maintain line pressure or to stay in the venturi calibration ranges) for the nth wellfield

All pumping rates, production limits, demand requirements, and flow quantities are in mgd, and also all of water levels are in feet NGVD. In addition to the above notation, the following abbreviations are used to identify source and demand points (Table 8.1).

Moreover, WF^c is a set of Consolidated Permit Wellfields as *cr*, *cc*, *cb*, *sp*, *mb*, *s*21, *nwh*, *cm*, *ew*, *st* and *np*, and also *SCHI* is South-Central Hillsborough Intertie.

The problem in this case can be expressed mathematically as follows:

$$\max Z = \sum_{i \in R^u} \sum_{t=1}^{T} \omega_i h_{i,t}^u \tag{8.1}$$

Table 8.1 Abbreviations to identify source and demand points

cr	Cross Bar Ranch Wellfield	*cc*	Cypress Creek Wellfield	*cb*	Cypress Bridge Wellfield
sp	South Pasco Wellfield	*mb*	Morris Bridge Wellfield	*s*21	Section 21 Wellfield
nwh	Northwest Hillsborough Wellfield	*cm*	Cosme Odessa Wellfield	*ew*	Eldridge Wilde Wellfield
st	Starkey Wellfield	*np*	North Pasco Wellfield	*bu*	Brandon Urban Dispersed Wells
sch	South-Central Hillsborough Wellfield	*crw*	Carrollwood wells	*cot*	Purchased water from City of Tampa
CC	Cypress Creek Water Treatment Plant (WTP)	*MB*	Morris Bridge WTP	*LB*	Lake Bridge WTP
LP	Lake Park WTP	*NW*	Northwest Hillsborough WTP	*CM*	Cosme WTP
LR	Little Road WTP	*MT*	Maytum WTP	*PD*	West Pasco Point of Connection–Pasco Distribution System
OD	Odessa Water Treatment Plant–Pasco Distribution System	*CH*	Central Hillsborough Regional Water Treatment Facility (replaced Highview)	*MP*	South Pasco Meter Pit
PK	Pinellas County Distribution System (Keller WTP and Regional System)	*LT*	Lithia Water Treatment Plant	*RWTP*	Regional Surface Water Treatment Plant
SDP	Seawater Desalination Plant	*TBC*	Hillsborough River/Tampa Bypass Canal pump station	*ALF*	Alafia River pump station
TBRR	Tampa Bay Regional Reservoir	*CBTM*	Cypress Bridge Transmission Main	*NCHI*	North-Central Hillsborough Intertie

Subject to the following constraints;

1. Demand constraints:

Some wellfields and Points of Connection (POC) are interconnected to the "Regional System" as shown in Fig. 8.3. This constraint set must satisfy not only demands at Points of Connection but also the physical representation of the Regional System, namely, the quantity and direction of pipe flow. All demands used in this formulation are projected demands obtained from the Short-Term Water Demand Forecasting System Model.

The pipe flow and demand constraints are written at each POC as follows:

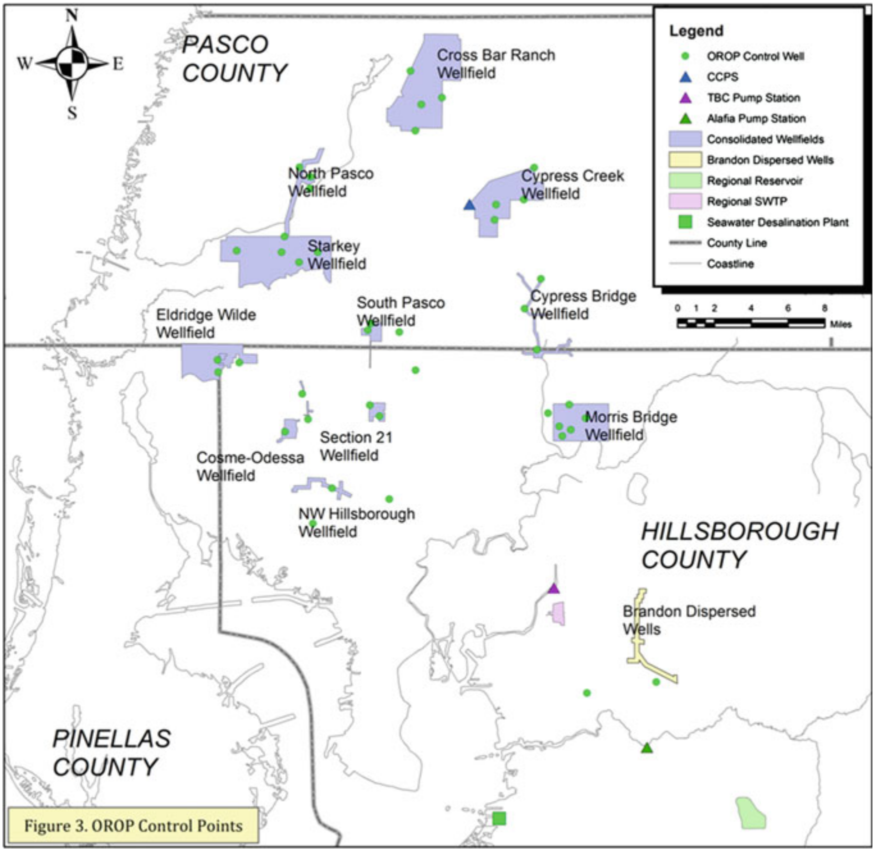

Fig. 8.3 OROP control points

(a) Morris Bridge WTP

$$d_{cc,t}^{MB} \geq D_t^{MB} \quad \forall t = 1, \ldots, T \tag{8.2}$$

$$\sum_{j \in w_{mb}} q_{j,t} - d_{mb,t}^{CC} = 0 \quad \forall t = 1, \ldots, T \tag{8.3}$$

(b) Lake Bridge WTP

$$d_{cc,t}^{LB} \geq D_t^{LB} \quad \forall t = 1, \ldots, T \tag{8.4}$$

$$\sum_{j \in w_{cb}} q_{j,t} - d_{cb,t}^{CC} = 0 \quad \forall t = 1, \ldots, T \tag{8.5}$$

(c) Lake Park WTP

$$d^{LP}_{mp,t} + d^{LP}_{s21,t} + d^{THIC}_{cot,t} \geq D^{LP}_{t} \quad \forall t = 1, \ldots, T \tag{8.6}$$

$$\sum_{j \in w_{s21}} q_{j,t} - d^{LP}_{s21,t} + q_{nw7,t} = 0 \quad \forall t = 1, \ldots, T \tag{8.7}$$

(d) Cosme WTP

$$d^{CM}_{nwh,t} + d^{CM}_{mp,t} + d^{CM}_{cc,t} + \sum_{j \in w_{cm}} q_{j,t} \geq D^{CM}_{t} \quad \forall t = 1, \ldots, T \tag{8.8}$$

(e) Northwest Hillsborough WTP

$$d^{NW}_{nwh,t} \geq D^{NW}_{t}, \quad \forall t = 1, \ldots, T \tag{8.9}$$

$$\sum_{j \in w_{nwh}} q_{j,t} - q_{nw7,t} + \sum_{j \in w_{crw}} q_{j,t} - d^{NW}_{nwh,t} - d^{CM}_{nwh,t} = 0, \quad \forall t = 1, \ldots, T \tag{8.10}$$

(f) Keller WTP (has been combined with Pinellas County Distribution System)
(g) Pinellas County Distribution System

$$d^{PK}_{cc,t} + \sum_{j \in w_{ew}} q_{j,t} \geq D^{PK}_{t}, \quad \forall t = 1, \ldots, T \tag{8.11}$$

(h) Little & Maytum WTPs—Equations (8.12) and (8.13) reflect the new West Pasco Infrastructure

$$d^{LR}_{cc} + d^{LR}_{mt} \geq D^{LR}_{t}, \quad \forall t = 1, \ldots, T \tag{8.12}$$

$$\sum_{j \in \{w_{st} \cap w_{np}\}} q_{j,t} - d^{LR}_{mt} \geq D^{MT}_{t}, \quad \forall t = 1, \ldots, T \tag{8.13}$$

(i) Pasco Interties

$$d^{PD}_{cc} \geq D^{PD}_{t}, \quad \forall t = 1, \ldots, T \tag{8.14}$$

$$d^{OD}_{cc} \geq D^{OD}_{t}, \quad \forall t = 1, \ldots, T \tag{8.15}$$

(j) Regional (Cypress Creek WTP) and sub-regional pipe flow water balance

$$d^{PK}_{cc,t} + d^{MP}_{cc,t} + d^{CM}_{cc,t} + d^{PD}_{cc,t} + d^{OD}_{cc,t} + d^{LR}_{cc,t} = \sum_{j \in w_{cr}} q_{j,t} + \sum_{j \in w_{cc}} q_{j,t} + d_{CBTM,t}, \quad \forall t = 1, \ldots, T \tag{8.16}$$

$$d^{LB}_{cc,t} + d^{MB}_{cc,t} + d_{CBTM,t} - d^{CC}_{mb,t} - d^{CC}_{cb,t} - d_{NCHI,t} = 0, \quad \forall t = 1, \ldots, T \tag{8.17}$$

$$\sum_{j \in w_{sp}} q_{j,t} + d_{cc,t}^{MP} - d_{mp,t}^{CM} - d_{mp,t}^{LP} = 0, \quad \forall t = 1, \ldots, T \tag{8.18}$$

(k) Finished water clear wells

$$d_{NCHI,t} = d_{SDP,t} + d_{RWTP,t} + d_{COT,t}^{US301} - d_{cc,t}^{CH}, \quad \forall t = 1, \ldots, T \tag{8.19}$$

(l) Raw water tanks and high service pump station

$$d_{SCHI,t} = d_{RWTP,t} - d_{TBC,t}, \quad \forall t = 1, \ldots, T \tag{8.20}$$

(m) Flow from Tampa Bay Regional Reservoir

$$d_{TBRR,t} = d_{SCHI,t} - d_{ALF,t}, \quad \forall t = 1, \ldots, T \tag{8.21}$$

(n) Central Hillsborough Water Treatment Facility and Lithia WTP

$$d_{cc,t}^{CH} \geq D_t^{CH}, \quad \forall t = 1, \ldots, T \tag{8.22}$$

$$d_{cc,t}^{LT} + \sum_{j \in w_{bu}} q_{j,t} + \sum_{j \in w_{sch}} q_{j,t} \geq D_t^{LT}, \quad \forall t = 1, \ldots, T \tag{8.23}$$

In constraints (8.20) and (8.21), $d_{TBC,t}, d_{ALF,t}, d_{SDP,t}$, and $d_{TBRR,t}$ are the forecasted availability of surface water supply at Tampa Bypass Canal pump station, Alafia River pump station, Seawater Desalination Plant, and the C.W. Bill Young Regional Reservoir, respectively.

2. Regulatory and facility constraints:

(a) WUP's regulatory levels for non-cumulative weekly average (swing level)

$$h_{i,t} \geq H_i - 3, \quad \forall i \in R; t = 1, \ldots, T \tag{8.24}$$

(b) WUP's regulatory levels for cumulative weekly average (minimum level)

$$\left(\sum_{\tau = -W_{-1}}^{t} h_{i,\tau} \right) / W_t \geq H_i, \quad \forall i \in R; t | W_t > 8 \tag{8.25}$$

(c) Weekly minimum and maximum production by wellfield (facility constraints, rule-curves, venturi limits)

$$\underline{Q}_n^{wf} \leq \sum_{j \in w_n} q_{j,t} \leq \overline{Q}_n^{wf}, \quad \forall n \in WF^c \cup \{bu\}; t = 1, \ldots, T \tag{8.26}$$

(d) 12-month running average total pumpage from the Consolidated Permit Wellfields. This permit constraint can be expressed as,

$$\sum_{n \in WF^c} \sum_{j \in w_n} \sum_{\tau = t-51}^{t} q_{j,\tau} \leq 52 P^y, \quad \forall t = 1, \ldots, T \tag{8.27}$$

(e) Peak month for each wellfield (4-week basis)

$$\sum_{j \in w_n} \sum_{\tau=\max(1,t-3)}^{t} q_{j,\tau} \leq \min(4,t) P_n^m, \quad \forall n \in WF^c \cup \{bu\}; t = 1, \ldots, T \tag{8.28}$$

(f) Peak month for each well (4-week basis)

$$\sum_{\tau=\max(1,t-3)}^{t} q_{j,\tau} \leq \min(4,t) P_j^m, \quad \forall j \in w_n; n \in WF^c \cup \{bu\}; t = 1, \ldots, T \tag{8.29}$$

(g) Cypress Creek Pumping Station capacity

$$\sum_{j \in w_{cr}} q_{j,t} \sum_{j \in w_{cc}} q_{j,t} \leq Q^{cc}, \quad \forall t = 1, \ldots, T \tag{8.30}$$

(h) Saltwater intrusion

Constraint equations to address saltwater intrusion are expressed in the same manner as regulatory wells. Equations (8.24) and (8.25) are applied at saltwater intrusion monitoring wells using the long-term mean values of water levels as the minimum levels.

(i) South Central Hillsborough Wellfield permit condition

$$\sum_{j \in w_{sch}} \sum_{\tau=t-51}^{t} q_{j,\tau} \leq 52 P_{sch}^y, \quad \forall t = 1, \ldots, T \tag{8.31}$$

$$\sum_{j \in w_{sch}} \sum_{\tau=\max(1,t-3)}^{t} q_{j,\tau} \leq \min(4,t) P_{sch}^m, \quad \forall t = 1, \ldots, T \tag{8.32}$$

(j) Carrollwood Wells peak month limitation based on Lake Carroll stage

$$\sum_{j \in w_{crw}} \sum_{\tau=\max(1,t-3)}^{t} q_{j,\tau} \leq \min(4,t) P_{crw}^m, \quad \forall t = 1, \ldots, T \tag{8.33}$$

$$P_{crw}^m = \begin{cases} 0.820 \text{ mgd, if Lake Carroll monthly level } \geq 34.5 \text{ ft NGVD} \\ 0.707 \text{ mgd, otherwise.} \end{cases} \tag{8.34}$$

(k) Water quality

One of Tampa Bay Water's obligations is to deliver Quality Water to its Member Governments. In order to meet this requirement, the Operations Department staff identified four wellfields in which certain wells exhibit a history of poor raw water quality with respect to iron and hydrogen sulfide concentrations. In order to address this raw water quality issue which is not addressed in the treatment of groundwater, Operations Department staff developed maximum concentrations of iron and hydrogen sulfide for the effluent from these wellfields. Constraint set 35 was formulated based on long-term observations of iron and sulfide concentrations (C_j^{Fe} and $C_j^{H_2S}$) from wells Cross Bar Ranch Wellfield, Morris Bridge Wellfield and South Pasco Wellfield.

$$\begin{aligned} \sum_{j \in w_n} q_{j,t} C_j^{Fe} \leq 0.2 \sum_{j \in w_n} q_{j,t}, \quad \forall n \in \{cr, mb\};\ t = 1, \ldots, T \\ \sum_{j \in w_n} q_{j,t} C_j^{H_2S} \leq 1.0 \sum_{j \in w_n} q_{j,t}, \quad \forall n \in \{mb, sp\};\ t = 1, \ldots, T \end{aligned} \tag{8.35}$$

3. Other operating constraints: (requested by Operations Department)
   (a) Minimum flow from Eldridge-Wilde wellfield

$$\sum_{j \in w_{ew}} q_{j,t} \geq 10, \quad \forall t = 1, \ldots, T \tag{8.36}$$

   (b) Balance flows in pipelines for COSME cutoff

$$d_{cc,t}^{MP} = d_{cc}^{CM}, \quad \forall t = 1, \ldots, T \tag{8.37}$$

   (c) Lake Park venturi minimum

$$d_{mp,t}^{LP} \geq 4, \quad \forall t = 1, \ldots, T \tag{8.38}$$

   (d) Flow range for Maytum water treatment plant

$$2.5 \leq \sum_{j \in \{w_{st} \cap w_{np}\}} q_{j,t} \leq 10, \quad \forall t = 1, \ldots, T \tag{8.39}$$

4. Physical System: (derived from IHM model)

$$g(h^u, h, q) = 0 \tag{8.40}$$

5. Upper and lower bounds:

$$\underline{Q}_j \leq q_{j,t} \leq \overline{Q}_j, \quad \forall j \in w_n; n \in WF^c \cup \{bu, sch\};\ t = 1, \ldots, T \tag{8.41}$$

6. Non-negativity: (unidirectional flow pipes)

$$\begin{aligned} & d_{cc,t}^r \geq 0, \quad \forall r \in \{PK, OD, PD, CM, MP, LB, MB, CH, LR, LT\};\ t = 1, \ldots, T \\ & d_{nwh,t}^{CM} \geq 0,\ d_{mp,t}^{CM} > 0,\ {}_{nwh,t}^{NW} \geq 0,\ d_{mp,t}^{LP} \geq 0,\ d_{s21,t}^{LP} > 0,\ {}_{mb,t}^{CC} \geq 0 \quad \forall t = 1, \ldots, T \\ & d_{mt,t}^{LR} > 0,\ {}_{cb,t}^{CC} = 0,\ d_{COT,t}^{THIC} \geq 0,\ d_{COT,t}^{US301} \geq 0, \quad = 1, \ldots, T \\ & d_{ALF,t} \geq 0,\ d_{TBC,t} \geq 0,\ d_{RWTP,t} \geq 0,\ d_{SDP,t} \geq 0,\ d_{NCHI,t} \geq 0 \quad \forall t = 1, \ldots, T \end{aligned} \tag{8.42}$$

8.3 Implementation Details

8.3.1 Unit Response

Equation (8.40) represents the physical system constraint that consists of the set of equations describing the surface and ground-water hydrology. Theoretically, the Integrated Hydrologic Model (IHM) could be embedded as a constraint function within the optimization routine. Due to the run times of the IHM, this is not practical since each optimization iteration requires as many functional evaluations as the number of decision variables. In addition, the embedded approach would require a nonlinear optimizer to solve the optimization problem. An alternative, the *unit response* method, is used to represent the functional constraint with an equivalent linear response system of equations predetermined using IHM. The development of the unit response matrix for OROP is described in Wanakule (2009).

Let,

$\varphi_{i,j}$ a matrix element of the (SAS or UFAS) water level from the base scenario at location i at the end of time period t

$u_{i,j}$ a (SAS or UFAS) unit response matrix as defined in Eq. (8.3) for the monitoring location i and production well j

$\Delta q_{j,t}$ a matrix element represents the increase in pumpage from the base scenario at the jth well during the time period τ

The constraint (8.40) is replaced by two sets of system equations relating pumpage increments to water levels in each aquifer layer and is expressed as follows:

$$h^u_{i,t} = \varphi^u_{i,t} - \sum_j \sum_{k=1}^{t} u^u_{i,j,k}\, \Delta q_{j,t-k+1}, \quad \forall i \in R^u; t = 1, \ldots, T \tag{8.43}$$

$$h_{i,t} = \varphi_{i,t} - \sum_j \sum_{k=1}^{t} u_{i,j,k}\, \Delta q_{j,t-k+1}, \quad \forall i \in R; t = 1, \ldots, T \tag{8.44}$$

The incremental pumpage, $\Delta q_{j,t}$, is related to the decision variables $q_{j,t}$ as $\Delta q_{j,t} = q_{j,t} - v_{j,t}$ where $q_{j,t}$ is the pumpage from the base scenario or the initial projection of the pumpage schedule.

Note that the objective function can now be explicitly expressed in terms of pumpage decision variables by substituting the expression (8.43) into equation (8.1) to yield;

$$Minimize\, Z = \sum_{i \in R^u} \omega_i \sum_{t=1}^{T} \left(\sum_j \sum_{k=1}^{t} u^u_{i,j,k}\, \Delta q_{j,t-k+1} \right) \tag{8.45}$$

Investigation of the unit response time profiles found that SAS drawdowns at various monitoring wells recover from a pumpage pulse differently depending on the nearby hydrogeology and the distance between the pulsed and observation well pair. For a longer lag-time response, rainfall will play a major role in influencing water-level responses. Hence, the resting (e.g., non-pumping) drawdown profile or the calculated responses from the pumpage no longer apply. To take advantage of differences in drawdown volumes at various monitoring sites, the time summation in (8.45) should be shortened to include the time profile up to the next expected effective rainfall (EERF) week. The expected number of weeks for effective dry-spell (T′) will vary from week to week and can be predetermined from the twelve long-term rainfall gages in the region. This can be achieved by changing the terminal week number on the time summation, T, in the above equation to T''. In addition, the equation can be simplified if this time summation is predetermined such that:

$$\hat{u}_{i,j,\tau} = \sum_{k=\tau}^{T'} u^{u}_{i,j,k}, \quad \text{for } \tau = 1\ldots4 \tag{8.46}$$

and the objective function can be rewritten as:

$$Minimize\, Z = \sum_{i\in R^{u}} \omega_i \sum_{\tau=1}^{4} \left(\sum_{j} \hat{u}^{u}_{i,j,\tau}\, \Delta\, q_{j,\tau} \right) \tag{8.47}$$

8.3.2 Solving the Optimization Problem

Each week OROP is run. The solution is optimized over the upcoming four-week period using the incremental analysis approach. With the incremental analysis, the prevailing hydrologic conditions are not used directly to derive the optimum solution. Instead, a set of preferential weights for control points is used to establish priority pumping sites. The current formulation provides for this preferential selection through a set of weighting factors, ω_i, which are assigned based on the surficial aquifer status at each control point at the start of the four-week period. A modified approach to the weighting formulation was approved as part of the July 2003 OROP annual report. The original function was modified to consider the natural range of wetland water-level fluctuations at all associated OROP control points.

Basically, the weight at each monitoring site is calculated by applying the current field measured water level to the functional relationship for that site. Since weights are predetermined and constant over the duration of the optimization routine, the optimum solution is limited to only a short-term (4-week) projection. Since the solution is optimized over a four-week period, a sequence of these short-term solutions may not yield the optimal operation in the long run. This is because the short-term solution lacks some knowledge of seasonal demand patterns. To overcome this constraint, the optimization model is run in two steps, a long-term (52-week) projection and the short-term (4-week) projection. The long-term projection is made without the weighting factors to first establish the upper and lower bounds of production at each wellfield, taking into account seasonal variations in demand. These bounds become the operational rule-curves for the short-term projection. All constraints for the short-term and long-term cases are the same with the following exceptions:

1. The time index and the summation for constraints with a time-averaged function must be adjusted (corresponding to time span of the stress period); that is, since the time span for a stress period of the short-term (one week) and long-term (4 week) model are different, the constraint function involving time-averages in the two models will have different running and terminal indices (e.g., the annual average for the one-week stress period will be averaged over 52 values compared with 13 values for the four-week stress period) and,
2. The upper and lower bounds for production values by well and wellfield are different (the short-term case being constrained by the rule-curve results determined in the long-term case).

8.3.3 Current OROP Implementation Procedure

8.3.3.1 Inputs to OROP

1. Demand at 12 Points of Connection (POC)

Each Friday morning, weekly demands for each of the 12 delivery points used in OROP are forecasted using the Short-term Demand Forecast application. Results are reviewed by Source Rotation or Operations Department staff and either accepted or changed. Factors for consideration to make a change to the demand forecast include recent weather trend or an infrastructure change at a POC that has not been captured by the model (e.g., increased hydraulic capacity, new connection, temporary connection, and temporary maintenance activity by member government such as free chlorine burn). The OROP data base automatically picks up the results and stores them for use in the weekly OROP production run. Staff can further revise the demand forecast prior to actually running OROP.

2. Alternative sources availability and use

(a) On Wednesdays Operations Manager and Source Rotation Manager discuss SWTP production options for the upcoming week, decide the appropriate production quantity and use of reservoir (storage or withdrawal). On Fridays, Source Rotation staff enters this quantity into the OROP database for use in the weekly OROP production run. Factors for consideration include annual budget and current (year to date) production, near term (next week) and next month surface water availability, reservoir level, season, total system demand, infrastructure constraints (e.g., scheduled maintenance, source water quality, chemical deliveries).

(b) Each Friday morning, weekly rates in mgd of surface water availability for the Alafia River, Lower Pool TBC, and Middle Pool TBC are forecasted for four weeks into the future by Source Rotation staff; the OROP data base automatically picks up the results and stores them for use in the weekly OROP production run.

(c) Each Friday morning, weekly rates in mgd for the desalination facility are determined and entered into the OROP data base. Factors for consideration include water quality, intake water temperature, blending ratios with treated surface water, seasonal demands, scheduled maintenance and TECO activities which affect production.

(d) Operations staff informs Source Rotation staff if Tampa Bay Water plans to purchase water from the City of Tampa and the quantity. Source Rotation staff enters this data into the OROP data base. Factors for consideration include season, surface water availability, the City's ability to deliver, and budget.

3. Wellfield production constraints

When scheduling the weekly OROP production run, the Operations staff has the opportunity to consider additional constraints at the wellfield level, either turning a wellfield off, setting a production minimum or production maximum. These are not permanent constraints and are available to handle short term operational problems. If there are not additional specific constraints for the week, then this information is not used by OROP.

4. Wells on-line status

Within the OROP database are the well status tables. Data is stored regarding the status of all production wells, regarding on-line or off-line, permanent or temporary, and the reason for being off-line (e.g., bacteriological testing, water quality, mechanical problems, electrical problems). The Operations staff maintains the wells on-line/off-line status, which can be updated prior to the weekly OROP production run.

5. Water level data and predicted water levels at control points and 18 UFAS wells

   (a) Continuous water-level data are collected at all OROP control point monitor wells and sent to the Enterprise database through wireless transmission. The data are subjected to automated quality control/quality assurance procedures and stored. The OROP database retrieves the most current water-level data for all control point wells automatically through stored-procedures in the database.

(b) Predictive water levels for the 18 UFAS wells and 42 control points are currently generated by the groundwater artificial neural network (ANN) models developed by staff. The OROP database retrieves these predicted water levels and stores them for use in the weekly OROP production run.

6. Current pumpage

(a) Daily production for all active wells is collected and stored in Tampa Bay Water's SCADA database. This production data is processed through our automated quality control/quality assurance procedures and stored in the Enterprise database for use. The OROP database retrieves current pumpage data for all production wells in order to determine well peak month current quantities and the 12-month running average to compare against program constraints.

(b) Initial value for well production is taken from the last seven days of actual production prior to the OROP weekly run.

8.3.3.2 OROP Output

1. OROP Detailed Report includes demand summary, surface water availability, wellfield pumpage rates, well priorities, control point weights, etc.
2. OROP Operators Report provides well priorities and pumpage rates, and wellfield rates for first two weeks of four week schedule; summarizes alternate sources availability for the upcoming two weeks.
3. OROP Schematic of Weekly Flows.

8.4 Summary of Models

The current OROP uses Gurobi Optimization program as, a commercial solver for linear programming (LP), quadratic and quadratically constrained programming (QP and QCP), and mixed-integer programming (MILP, MIQP, and MIQCP). The OROP programming code is written in AMPL (Fourer et al. 2002), a high level, comprehensive, and powerful algebraic modeling language for mathematical programming. AMPL is the software originally developed by AT&T Bell Laboratories that uses common notation and familiar concepts to formulate optimization models and examines solutions while the software manages communication with an appropriate solver and databases. The language acts as a shell script that allows efficient prototyping, change and/or experimentation with the model. AMPL supports most commercial solvers including Gurobi. The OROP model application has been re-developed and deployed under a Microsoft Windows™ application using Visual Basic Dotnet programming language. This allows the application to conform to Tampa Bay Water's Information System technical requirements, facilitates use of the application by the Operations staff, and improves software maintenance and documentation. The OROP solution or the weekly pumping schedule is obtained via Tampa Bay Water's Decision Support System (DSS). The optimization model was approved by the District as part of the original OROP report (1998).

OROP develops an optimized well production schedule for the upcoming four-week period. In addition to constraint parameter data and current well production rates, the optimization model requires weekly information for the forecasted inputs. These inputs include weekly demands forecasts at each of the points of connection, projected UFAS and SAS water levels, and weekly forecasted surface water availability. Since the original OROP was implemented, Tampa Bay Water has developed additional modeling tools which provide weekly input to OROP.

8.4.1 Weekly Demand Forecast Models

Demand delivered to the points of connection is one piece of input data that is required to be forecasted. In 2002, Tampa Bay Water developed a set of short-term forecasting models for eleven points of connection. These models were subsequently implemented as part of the OROP process beginning in 2003. In 2005, the performance of these models was evaluated. This evaluation concluded that reasonable weekly forecast could be generated from the models using the average of the six daily models. The study included a recommendation to evaluate alternate forecasting methods. Performance of these models was highly dependent upon obtaining reliable real-time rainfall data for three NOAA rainfall stations and rainfall forecasts. Not all of the NOAA stations used to develop the models provide real time rainfall data accessible to Tampa Bay Water. In some cases delays of up to three months were experienced. In addition, Tampa Bay Water explored several approaches for obtaining improved rainfall forecast for one-week, two-weeks and four-week periods, but to date have not found suitable rainfall products readily available. Documentation and evaluations of these models can be found in previous OROP annual reports.

In 2006, Tampa Bay Water implemented a new set of short-term demand forecasting models. Seven autoregressive with exogenous variable (ARX) models for 11 points of connections were developed (Asefa and Adams 2007). The models estimate aggregated demands by demand planning areas and use a disaggregation algorithm to determine demands at point of connections. Variables include recent demands, several rainfall measures (including rain amount, number of rainy days in a week, and number of consecutive dry days), and a temperature threshold. Model inputs are based on observed data; no forecasts of model inputs are currently conducted. For two points of connection (Central Hillsborough and Morris Bridge) there are insufficient data available to develop ARX models; the naïve forecast (previous week's demand) is used to forecast Central Hillsborough demands. The Morris Bridge POC demands are currently based on the City of Tampa's request for water. The agency continues to investigate short-term rainfall forecasting methods which could be incorporated into the new ARX models to improve the near-term demand forecast. During Water Year 2010, these models will be evaluated and revised, if necessary, based on additional period of record data.

8.4.2 Groundwater Level Forecast Models

In 2004, Tampa Bay Water developed a set of artificial neural network models to predict water levels at the set of surficial aquifer control points and set of UFAS monitor wells. In 2005, Tampa Bay Water began implementing these models to replace the use of the ISGW model for predicted UFAS and SAS water levels used in the optimization model.

8.4.3 Surface Water Availability Models

Tampa Bay Water has two water use permits authorizing withdrawals from surface water sources. The Hillsborough River/Tampa Bypass Canal (HRTBC) water use permit was originally issued in 1999 and authorized diversions from the Hillsborough River during high flow times (Hillsborough Reservoir discharge > 100 cfs) and withdrawals from the Tampa Bypass Canal Lower and Middle Pools while requiring a minimum flow of 11 cfs over TBC structure S-160. In 2007, this water use permit was modified to remove the minimum flow over S-160 requirement, base withdrawals from the Tampa Bypass Lower Pool on pool stage, and increase the Hillsborough River diversion percentage while maintaining a minimum flow of 100 cfs over the dam. Tampa Bay Water placed the Tampa Bypass Canal withdrawal facilities and pump station into service in September 2002.

The Alafia River water use permit was issued in 1999 and authorized withdrawals of up to 10 % of available flow to a maximum of 51.8 mgd when the river flow is above the permit threshold of 80 mgd.

The following Alafia withdrawal rules (SWFWMD 2012) establish the baseline flow:

1. Average daily river flow at Lithia gage is multiplied by a factor of 1.117 to account for additional watershed contribution between the gage and Bell shoals withdrawal point,
2. Recent weekly spring flows of Buckhorn. and Lithia Spring major are then added,
3. Annual average daily withdrawal of 5.06 mgd of existing Mosaic water use permit is added.

If baseline flow is less than 128 cfs (82.7 mgd) for the previous day, there will be no withdrawal. If baseline flow for previous day is between 128 cfs and 142 cfs (92.4 mgd), the difference between baseline flow and 128 cfs is the allowable withdrawal. If baseline flow for previous day is more than 142 cfs, 10 % of baseline flow up to a maximum of 60 mgd will be allowed.

Tampa Bay Water placed the Alafia River withdrawal facilities and pump station into service in February 2003.

In 2002, Tampa Bay Water began development of models to forecast surface water availability from the Hillsborough River/Tampa Bypass Canal system. The Hillsborough River/TBC watershed is a very complex hydrologic system including groundwater and surface water interactions, several major tributaries, spring discharges, and man-made routing and flow control structures. The purpose of the HRTBC models was to generate streamflow predictions for the major tributaries to the lower Hillsborough River basin and to route these flows through the lower Hillsborough River and Tampa Bypass Canal. The resulting predicted TBC flow rates and associated water surface elevations were used to predict the quantity of surface water supply available for withdrawal, treatment and distribution. Flow generation models for the Hillsborough River gauges were developed using artificial neural network (ANN) modeling techniques. An assessment of these neural network models was performed for the July 2005 OROP annual report. Results of this assessment showed that the models did not perform as well during Water Year 2004 as during the initial testing and validation of the models. A second evaluation of these ANN models was conducted in 2006. The results of this evaluation indicated that the surface water flow models demonstrated good performance based on known upcoming rainfall and the hydraulic models showed good performance based on known stream flow. However, once upcoming rainfall was considered unknown, stream flow model performance degraded considerably.

A weekly Markov flow model was first developed for the Alafia River at the Lithia gauge as described in the OROP Annual Report for Water Year 2001. The focus of the Alafia River water availability model was on prediction of flow for the Alafia River at the Lithia gauge. Since the flow component from Lithia Springs is both small and relatively invariant, when compared to Alafia River flow at the Lithia gauge, short term predictions for Lithia Springs flows are treated as a constant equal to the last weekly observation.

Each week the Markov model was used to forecast Alafia River flow at the Lithia gauge for the next four upcoming weeks. These results along with the last measurement made for Lithia Springs were entered into the equation to determine the forecasted flow at the Alafia River Pump Station. The last step of the weekly forecast was to apply the WUP withdrawal rules to the forecasted flow to obtain the projected surface water availability for the next 4 weeks. This procedure was followed every week, i.e. updating the last 3 weeks of the previous weekly forecast and projecting one more week into the future. Staff discontinued use of this model after new surface water forecasting models were developed.

In 2007, Tampa Bay Water developed new surface water artificial neural network models to forecast river flows for the Hillsborough River (Morris Bridge gauge), Trout Creek, Cypress Creek, and Alafia River. The current models used to provide input into OROP were developed using a GLUE-based (generalized likelihood uncertainty estimate) neural network approach and generate weekly forecast for up to 4 weeks. Inputs to the models include past stream flow, rainfall and water levels of shallow and deep aquifers. Documentation of this approach is provided in Appendix D. The models are developed using MATLAB®. Water use permit withdrawal rules are applied to the results of the forecasted flows to determine the amount of surface water expected to be available for the upcoming four-week OROP period. These models are currently used to provide surface water availability input data for OROP.

8.5 Control Points

Thirty-one surficial aquifer monitor wells were established as control points for the initial optimization model as described in the first revision of OROP report (Tampa Bay Water 1998c). Historical data were used to perform correlation analyses and to develop regression relationships that formed the basis for the weighting function at each site. Since implementation of the OROP in January 1999, changes have been made to the original set of 31 control points. These changes are documented in subsequent OROP annual reports. Currently, there are forty SAS and two UFAS monitor wells which are used as control points in the optimization routine (Fig. 8.3). Target groundwater levels have been established for all of the control points in the vicinity of the 11 wellfields, the Brandon Urban Dispersed Wells and the Carrollwood Wells.

One of the tasks for the OROP annual update was a re-evaluation of the correlation and regression analyses that were performed at the control point locations. An evaluation was conducted later to determine if the wetland/control point regression analyses needed to be updated annually. The results indicated that conducting regression updates every other year is sufficient for control points that have been active for several years. As a result, the bi-annual update to the control point target levels has been adopted as the current practice since.

8.5.1 Preferential Weights for Objective Function

The primary purpose of the optimization problem is to seek a pumpage scenario that given demands, operational and system constraints and availability of alternative supplies will minimize water-level drawdown at specific locations (i.e., control points). An optimization routine has been setup with an objective function that will maximize the weighted sum of the water levels at all 42 OROP control points.

The preferential weights ω_i enable priority factors to be applied to enhance water levels preferentially at the wetland associated with the ith SAS monitoring site or control point. This weighting factor is predetermined for each control point based on the most recent water-level reading, and remains constant throughout the optimization simulation period (4 weeks). Actual water levels at the monitor wells (based on observed data) are compared to the target levels every week. Individual weighting factors for each site are updated every week based on observed water levels, and are used in revising the four-week short-term analysis for pumping distributions. The weights are based on relative measures of water levels compared to the target levels set at each monitor well and are applied to reflect the deviation between actual and target levels. The weights function as a ranking system for the optimization algorithm that causes the search for an optimal solution to preferentially reduce drawdown (in support of increased water levels) at locations with greater weight, thereby driving those water levels toward their target levels. Equal weights apply to all cases in which current water levels are equal to or above their respective targets. The weighting system is strongly non-linear where sites with large water-level deficits receive considerably higher weight than those where current water levels are near their targets. In certain cases, actual water levels may be above their target levels, which would result in a preference for production in that vicinity as compared to other locations in the region where water levels are below target levels. The current weighting function is expressed in the functional form of a piecewise linear on semi-logarithmic scale as follows:

$$\log(\omega) = \begin{cases} \left[\frac{(H_{max}-b)}{(H_{max}-H_T)}\right] & if\, b > H_T \\ 1 + \left[\frac{(H_T-b)}{(H_T-H_{NL})}\right] & if\, H_{NL} \le b \le H_T \\ 2 + 2\left[\frac{(H_{NL}-b)}{6}\right] & if\, b < H_{NL} \end{cases} \tag{8.48}$$

where H_{max} is the period of record (POR) maximum water level and H_{NL} is the lowest elevation of the natural fluctuation range, which has been determined to be 8 feet below the H_{max}.

This weighting function provides three different semi-logarithmic linear equations for three regimes of water-level fluctuations. The piecewise weighting function will bind the weighting factor at H_{max}, H_T, H_{NL}, and $(H_{NL} - 6)$ to 1, 10, 100, and 10,000, respectively. The rate of change in weighting factor after the water level drops below H_{NL} will be the same for all wells. When H_T is lower than H_{NL}, the function reduces to two equations since the second piece of the linear equation is no longer applicable. The remaining third function is modified to maintain a constant slope and becomes:

$$\log(\omega) = 1 + 2\left[\frac{(H_T - b)}{6}\right] \quad if\ b < H_T \tag{8.49}$$

Figure 8.4 depicts the current functional relationship of the weighting factor and water level for the same OROP well and wetland pair. Under this functional

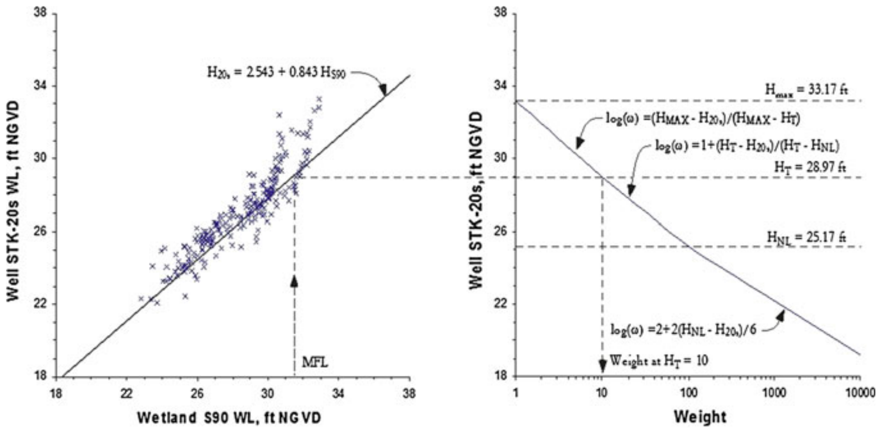

Fig. 8.4 The piecewise linear weighting function on semi-logarithmic scale

relationship, the weighting factor is unbounded or undefined when measured water level in the control well is above H_{max} or below H_{NL}. If the water level is above H_{max}, the weighting factor becomes insignificant which will rotate production to nearby wells. If the water level drops below H_{NL}, the weighting factor becomes very large and will force production away from nearby wells, even if the drawdown response is relatively small.

8.6 Environmental Management Plan Wetland Referrals

As part of the Consolidated Permit for the 11 Central System wellfields, Tampa Bay Water implements an Environmental Management Plan (EMP). The EMP requires monitoring of wetland hydrology and ecology and periodic review of environmental conditions at wetlands that could potentially be affected by water production. Hydrologic parameters at monitored wetlands are statistically compared to reference and control sites semi-annually at the end of both the spring (dry) and fall (wet) seasons. Sites that fail this statistical test are called "outliers" and are tabulated and tracked during future semi-annual tests. In compliance with Special Condition 3 of the 2011 Consolidated Water Use Permit, Tampa Bay Water staff modified the protocol for the interaction between the EMP and OROP. Based on this protocol, no action is required for the first two consecutive failures of the outlier test. If a wetland site fails a third consecutive outlier test a site-specific analysis is performed to determine if there is an adverse environmental impact and if it is attributable to wellfield pumpage. If adverse impacts due to wellfield pumpage are confirmed, then the wetland site is referred to OROP to attempt to relieve the impact. Actions undertaken within OROP could include the adjustment of an OROP control point target level or the addition of a new control point. (If it is determined that a change in OROP will not have a "meaningful effect", a referred wetland may also go directly to the Phase 2 Mitigation program, with no recommended change in OROP.)

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Appendix

The following tables briefly show the most of important optimization studies in water resources engineering.

Author(s)	Date	Title
Buras	1963	Conjunctive operation of dams and aquifers
Young	1967	Finding reservoir operation rules
Hall and Butcher	1968	Optimal timing of irrigation
Hall et al.	1968	Optimization of the operation of a multipurpose reservoir by dynamic programming
Aron	1969	Optimization of conjunctively managed surface and groundwater resources by dynamic programming
ReVelle et al.	1969	The linear decision rule in reservoir management and design 1. Development of the stochastic model
Burt	1970	Groundwater storage control under institutional restrictions
Harboe et al.	1970	Optimal policy for reservoir operation
Aron and Scott	1971	Dynamic programming for conjunctive use
Burt and Stauber	1971	Economic analysis of irrigation in sub-humid climate
Dudley et al.	1971	Optimal intra-seasonal irrigation water allocation
Heidari et al.	1971	Discrete differential dynamic programming approach to water resources systems optimization
Biere and Lee	1972	A model for managing reservoir water releases
Trott and Yeh	1973	Optimization of multiple reservoir system
Argaman et al.	1973	Design of optimal sewerage systems
Aguado and Remsen	1974	Groundwater hydraulics in aquifer management
Aguado et al.	1974	Optimal pumping for aquifer
Becker and Yeh	1974	Optimization of real-time operation of a multiple reservoir system
Cohon and Marks	1975	A review and evaluation of multiobjective programming techniques
Mishra	1975	Optimization of conjunctive use of ground water and surface water
Becker et al.	1976	Operations models for central valley project

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Author(s)	Date	Title
Alley et al.	1976	Aquifer management under transient and steady-state conditions
Dudley et al.	1976	Reliability, trade-offs and water resources development modeling with multiple crops
Fults et al.	1976	A practical monthly optimum operations model
Futagami et al.	1976	FEM coupled with LP for water pollution control
Willis	1976	Optimal groundwater quality management: well injection of waste water
Alperovits and Shamir	1977	Design of optimal water distribution systems
Gorelick et al.	1979	Management model of a groundwater system with a transient pollutant source
Willis	1979	A planning model for the management of groundwater quality
Croley and Rao	1979	Multi-objective risks in reservoir operation
Tauxe et al.	1979	Multi-objective dynamic programming with application to reservoir
Murray and Yakowitz	1979	Constrained differential dynamic programming and its application to multireservoir control
Maji and Heady	1980	Optimal reservoir management and crop planning under deterministic and stochastic inflows
Bras and Cordova	1981	Interseasonal water allocation in deficit irrigation
Becker and Yeh	1981	Improved hourly reservoir operation model
Giles and Wunderlich	1981	Weekly multipurpose planning model for TVA reservoir system
Rydzewski and Rashid	1981	Optimization of water resources for irrigation in east Jordan
Vedula and Roger	1981	Multiobjective analysis of irrigation planning in river basin development
Quindry et al.	1981	Optimization of looped water distribution systems
Khepar and Chaturvedi	1982	Optimum cropping and groundwater management
Becker et al.	1982	Central Arizona project operation
Hashimoto et al.	1982	Reliability, resiliency and vulnerability criteria for water resource system performance evaluation
Yaron and Dinar	1982b	Optimal allocation of water on a farm during feed season
Khanjani and Busch	1983	Optimal irrigation distribution systems with internal storage
Wasimi and Kitanidis	1983	Real-time forecasting and daily operation of a multireservoir system during floods by linear quadratic Gaussian control
Can and Houck	1984	Real-time reservoir operations by goal programming
Shamir et al.	1984	Optimal annual operation of a coastal aquifer
Stedinger et al.	1984	Stochastic dynamic programming models for reservoir optimization
Tsakiris and Kiountouzis	1984	Optimal intraseasonal irrigation water distribution

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Author(s)	Date	Title
Ozden	1984	A binary state DP algorithm for operation problems of multireservoir systems
Grygier and Stedinger	1985	Algorithms for optimizing hydropower system operation
Pereira and Pinto	1985	Stochastic optimization of a multireservoir hydroelectric system: a decomposition approach
Yeh	1985	Reservoir management and operation models: a state-of-the-art review
Mariño and Loaiciga	1985	Dynamic model for multireservoir operation
Casola et al.	1986	Optimal control model for groundwater management
Lefkoff and Gorelick	1986	Design and cost analysis of rapid aquifer restoration systems using flow simulation and quadratic programming
Wanakule et al.	1986	Optimal management of large-scale aquifers: methodology and applications
Tung	1986	Groundwater-management by chance-constrained model
Bosch et al.	1987	A review of methods in evaluating the economic efficiency of irrigation
Chávez-Morales et al.	1987	Planning model of irrigation district
Jonesm et al.	1987	Optimal control of nonlinear groundwater hydraulics using differential dynamic programming
Karamouz and Houck	1987	Comparison of stochastic and deterministic dynamic-programming for reservoir operating rule generation
Yaron et al.	1987	Irrigation scheduling—theoretical approach and application problems
Georgakakos, and Marks	1987	New method for the realtime operation of reservoir systems
Kitanidis	1987	A first-order approximation to stochastic optimal control of reservoirs
Trezos and Yeh	1987	Use of stochastic dynamic programming for reservoir management
Ahlfeld et al.	1988	Applications of optimal hydraulic control to groundwater systems
Benedini	1988	Developments and possibilities of optimization models
Dudley	1988	A single decision maker approach to irrigation reservoir and farm management decision making
Galeati and Gambolati	1988	Optimal dewatering schemes in the foundation design of an electronuclear plant
Foufoula-Georgiou and Kitanidis	1988	Gradient dynamic programming for stochastic optimal control of multidimensional water resources systems
Nix and Heaney	1988	Optimization of storm water storage-release strategies
Georgakakos	1989a	Extended linear quadratic Gaussian control for the real time operation of reservoir systems
Georgakakos	1989b	Extended linear quadratic Gaussian control: further extensions

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Author(s)	Date	Title
Kitanidis and Andricevic	1989	Accuracy of the first-order approximation to the stochastic optimal control of reservoirs
Ponnambalam et al.	1989	An application of Karmarkar's interior-point linear programming algorithm for multireservoir operations optimization
Yaron and Dinar	1982a, b	Optimal allocation of farm irrigation water during peak seasons
Sobel	1989	A multi-reservoir model with a myopic optimum
Andricevic	1990	A real-time approach to management and monitoring of groundwater hydraulics
Andricevic and Kitanidis	1990	Optimization of the pumping schedule in aquifer remediation under uncertainty
Bardossy et al.	1990	Fuzzy regression in hydrology
Kelman et al.	1990	Sampling stochastic dynamic-programming applied to reservoir operation
McLaughlin and Velasco	1990	Real-time control of a system of large hydropower reservoirs
Paudyal and Gupta	1990	Irrigation planning by multilevel optimization
Paudyal et al.	1990	Optimal hydropower system configuration based on operational analysis
Protopapas and Georgakakos	1990	An optimal control method for real-time irrigation scheduling
Vedula and Mohan	1990	Real-time multipurpose reservoir operation:a case study
Dariane and Hughes	1991	Application of crop yield functions in reservoir operation
Dougherty and Marryott	1991	Optimal groundwater management (Simulated annealing)
Lee and Kitanidis	1991	Optimal estimation and scheduling in aquifer remediation with incomplete information
Onta et al.	1991	Multistep planning model for conjunctive use of surface and groundwater resources
Tao and Lennox	1991	Reservoir operations by successive linear programming
Yuan et al.	1991	A study on the optimal allocation model of limited irrigation water
Pereira and Pinto	1991	Multi-stage stochastic optimization applied to energy planning
Braga et al.	1991	Stochastic optimization of multiple-reservoir-system operation
Afzal et al.	1992	Optimization model for alternative use of different quality irrigation water
Chang et al.	1992	Optimal time-varying pumping rates for groundwater remediation: application of a constrained optimal control algorithm
Culver and Shoemaker	1992	Dynamic optimal control for groundwater remediation with flexible management periods
Finney et al.	1992	Quasi-three dimensional optimization model of Jakarta basin

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Author(s)	Date	Title
Ko et al.	1992	Multiobjective optimization of reservoir systems operation
Ratzlaff et al.	1992	Optimal design of ground-water capture systems using segmental velocity-direction constraints
Vedula and Mujumdar	1992	Optimal reservoir operation for irrigation of multiple crops
Karamouz and Vasiliadis	1992	Bayesian stochastic optimization of reservoir operation using uncertain forecast
Mohan and Raipure	1992	Multiobjective analysis of multireservoir system
Crawley and Dandy	1993	Optimal operation of multiple-reservoir system
Culver and Shoemaker	1993	Optimal control for groundwater remediation by differential dynamic programming with quasi-Newton approximations
Datta	1993	Operation models for single and multipurpose reservoirs—a review
Karatzas and Pinder	1993	Groundwater management using numerical simulation and the outer approximation method for global optimisation
Kuczera	1993	Network linear programming codes for water-supply headworks modeling
Marryott et al.	1993	Optimal groundwater management
Tejada-Guibert et al.	1993	Comparison of 2 approaches for implementing multireservoir operating policies derived using stochastic dynamic programming
Wurbs	1993	Reservoir-system simulation and optimization models
Cardwell and Ellis	1993	Stochastic dynamic programming models for water quality management
Ahlfeld and Heidari	1994	Groundwater management with fixed charges
Simpson et al.	1994	Genetic algorithms compared to other techniques for pipe optimization
Mannocchi and Mecarelli	1994	Optimization analysis of deficit irrigation systems
Shyam et al.	1994	Optimal operation scheduling model for a canal system
Shih and ReVelle	1994	Water-supply operation during drought: continues hedging rule
Cieniawski et al.	1995	Using genetic algorithms to solve multiobjective groundwater monitoring problem
Peralta et al.	1995	Optimal large scale conjunctive water use planning: a case study
Sun et al.	1995	Generalized network algorithm for water-supply-system optimization
Shih and ReVelle	1995	Water supply operation during drought: A discrete hedging rule
Lamond and Boukhtouta	1995	Optimizing future hydropower production using Markov decision processes
Lund and Israel	1995	Optimization of transfers in urban water supply planning

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Agarwal and Roest	1996	Towards improved water management in Haryana State: final report of the Indo-Dutch operational research Project on Hydrological Studies
Datta and Dhiman	1996	Chance constrained optimal monitoring network design for pollutants in groundwater
Hallaji and Yazicigil	1996	Optimal management of coastal aquifer in Southern Turkey
Keshari and Datta	1996	Multiobjective management of a contaminated aquifer for agricultural use
Raman and Chandramouli	1996a, b	Deriving a general operating policy for reservoirs using neural network
Rizzo and Dougherty	1996	Design optimisation for multiple management period groundwater remediation
Vedula and Kumar	1996	An integrated model for optimal reservoir operation for irrigation of multiple crops
Nitivattananon et al.	1996	Optimization of water supply system operation
Wurbs	1996	Modelling and analysis of reservoir system operation
Liang et al.	1996	A comparison of two methods for multiobjective optimization for reservoir operation
Raman and Chandramouli	1996a, b	Deriving a general operating policy for reservoirs using neural network
Dandy et al.	1996	An improved genetic algorithm for pipe network optimization
Yang et al.	1996a	Water distribution network reliability: connectivity analysis
Garcia and Rivera	1996	Optimal estimation of storage-release alternatives for stormwater detention systems
Chang and Wang	1997	A fuzzy goal programming approach for the optimal planning of metropolitan solid waste management systems
Fontane et al.	1997	Planning reservoir operations with imprecise objectives
Huang and Mayer	1997	Pump-and-treat optimization using well locations and pumping rates as a decision variable
Khaliquzzaman and Chander	1997	Network flow programming model for multireservoir sizing
Mainuddin et al.	1997	Optimal crop planning model for an existing groundwater irrigation project in Thailand
Misirli and Yazicigil	1997	Optimal ground-water pollution containment with fixed charges
Mujumdar and Ramesh	1997	Real-time reservoir operation for irrigation
Sepaskhah and Kamgar-Haghighi	1997	Water use and yield of sugarbeet grown under every-other-furrow irrigation with different irrigation intervals
Oliveira and Loucks	1997	Operating rules for multireservoir systems
Archibald et al.	1997	An aggregate stochastic dynamic programming model of multireservoir systems

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Author(s)	Date	Title
Savic and Walters	1997	Genetic algorithms for the least-cost design of water distribution networks
Papa and Adams	1997	Application of derived probability and dynamic programming techniques to planning regional stormwater management systems
Alidi	1998	A goal programming model for an integrated solid waste management system
Carvallo et al.	1998	Irrigated cropping optimization
Emch and Yeh	1998	Management model for conjunctive use of coastal surface water and groundwater
Garg and Ali	1998	Two-level optimization model for lower Indus Basin
Haastrup et al.	1998	A decision support system for urban waste management
Wang and Zheng	1998	Ground water management optimization using genetic algorithms and simulated annealing: formulation and comparison
Evers et al.	1998	Integrated decision making for reservoir, irrigation, and crop management
Jain et al.	1998	Reservoir operation studies of Sabarmati system, India
Galan and Grossmann	1998	Optimal design of distributed wastewater treatment networks
Chang and Chen	1998	Real-coded genetic algoritsed flood control reservoir management
Hajilal et al.	1998	Real-time operation of reservoir based canal irrigation systems
Bear et al.	1999	Seawater Intrusion in Coastal Aquifers—Concepts, Methods and Practices
Philbrick and Kitanidis	1999	Limitations of deterministic optimization applied to reservoir operations
Wardlaw and Barnes	1999	Optimal allocation of irrigation water supplies in real time
Wardlaw and Sharif	1999	Evaluation of genetic algorithms for optimal reservoir system operation
Das and Datta	1999a	Development of multiobjective management models for coastal aquifers
Das and Datta	1999b	Development of management models for sustainable use of coastal aquifers
Agarwal	2000	Optimal design of on-farm reservoir for paddy-mustard cropping pattern using water balance approach
Cheng et al.	2000	Pumping optimization in saltwater-intruded coastal aquifers
Das and Datta	2000	Optimisation based solution of density dependent seawater intrusion in coastal aquifers
Mousavi and Ramamurthy	2000	Optimal design of multi-reservoir systems for water supply
Needham et al.	2000	Linear programming for flood control in the Iowa and Des Moines rivers
Paul et al.	2000	Optimal irrigation allocation: a multilevel approach

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Sharif and Wardlaw	2000	Multireservoir systems optimization using genetic algorithms: case study
Simonovic	2000	Tools for water management: one view of future
Acreman	2000	Managed flood releases from reservoirs: issues and guidance. Center for Ecology and Hydrology, UK
Yang et al.	2000	Optimization of regional water distribution system with blending requirements
Diogo et al.	2000	Three dimensional optimization of urban drainage system
Jairaj and Vedula	2000	Multireservoir system optimization using fuzzy mathematical programming
World Commission on Dams	2000	Dams and development: a new framework for decision-making
Anwar and Clarke	2001	Irrigation scheduling using mixed integer linear programming
Archibald et al.	2001	Controlling multi-reservoir systems
Chandramouli and Raman	2001	Multireservoir modeling with dynamic programming and neural networks
Das and Datta	2001	Application of optimization techniques in groundwater quantity and quality management
Eschenbach et al.	2001	Goal programming decision support system for multiobjective operation of reservoir systems
Faber and Stedinger	2001	Reservoir optimization using sampling SDP with ensemble streamflow prediction (ESP) forecasts
Haouari and Azaiez	2001	Optimal cropping patterns under water deficits
Singh et al.	2001	Optimal cropping pattern in a canal command area
Reca et al.	2001a	Optimisation model for water allocation in deficit irrigation systems. I. Description of the model
Reca et al.	2001b	Optimisation model for water allocation in deficit irrigation systems. II. Application to the Bembezar irrigation system
Deb	2001	Multi-Objective Optimization using evolutionary Algorithms
Savic and Walters	2001	Evolutionary computing in water distribution and wastewater systems
Dahe and Srivastava	2002	Multireservoir multiyield model with allowable deficit in annual yield
Ghahraman and Sepaskhah	2002	Optimal allocation of water from a single purpose reservoir to an irrigation project with pre-determined multiple cropping patterns
Hsiao and Chang	2002	Dynamic optimal groundwater management with inclusion of fixed costs
Hsu and Cheng	2002	Network flow optimization model for basin-scale water supply planning
Palmer et al.	2002	An application of water conflict resolution in the Kum river basin, Korea

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Author(s)	Date	Title
Sethi et al.	2002	Optimal crop planning and conjunctive use of water resources in a coastal river basin
Shangguan et al.	2002	A model for regional optimal allocation of irrigation water resources under deficit irrigation and its applications
Shim et al.	2002	Spatial decision support system for integrated river basin flood control
Teixeira and Mariño	2002	Coupled reservoir operation–irrigation scheduling by dynamic programming
Shangguan et al.	2002	A model for regional optimal allocation of irrigation water resources under deficit irrigation and its applications
Tilmant et al.	2002	Optimal operation of multipurpose reservoir stochastic dynamic programming
Alaya et al.	2003	Optimization of Nebhana reservoir water allocation by stochastic dynamic programming
Barlow et al.	2003a	Conjunctive management model for sustained yield of stream–aquifer systems
Barros et al.	2003b	Optimization of large-scale hydropower system operations
Benli and Kodal	2003	A non-linear model for farm optimization with adequate and limited water supplies application to the south-east Anatolian project (GAP) region
Kuo et al.	2003	Comparative study of optimization techniques for irrigation project planning
Nema and Gupta	2003	Multi-objective risk analysis and optimization of regional hazardous waste management system
Cheung et al.	2003	Multiobjective evolutionary algorithms applied to the rehabilitation of a water distribution system: a comparative study
Keedwell and Khu	2003	More choices in water distribution system optimization
Chen	2003	Real coded genetic algorithm optimization of long term reservoir operation
Karamouz et al.	2003	Water resources systems analysis
Rao et al.	2003	Optimal groundwater management in deltaic regions using simulated annealing and neural networks
Tu et al.	2003	Optimization of reservoir management and operation with hedging rules
Chang et al.	2003	Optimization of operation rule curves and flushing schedule in reservoir
Barros et al.	2003a, b	Optimization of large-scale hydropower system operations
Cunha and Ribeiro	2004	Tabu search algorithms for water network optimization
Karamouz et al.	2004a, b	Monthly water resources and irrigation planning: case study of conjunctive use of surface and groundwater resources
Labadie	2004	Optimal operation of multireservoir systems: state-of-the-art review
Mantoglou et al.	2004	Management of coastal aquifers based on nonlinear optimization and evolutionary algorithms

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Author(s)	Date	Title
McPhee and Yeh	2004	Multiobjective optimization for sustainable groundwater management in semiarid regions
Mujumdar et al.	2004	Irrigation water allocation in canal command areas
Park and Aral	2004	Multi-objective optimization of pumping rates and well placement in coastal aquifers
Draper and Lund	2004	Optimal hedging and carryover storage value
Ghahraman and Sepaskhah	2004	Linear and non-linear optimization models for allocation of a limited water supply
Karamouz et al.	2004a, b	Analysis of hydrologic and agricultural droughts in central part of Iran
Van Zyl et al.	2004	Operational optimization of water distribution systems using a hybrid genetic algorithm
Mousavi et al.	2004a	A stochastic dynamic programming model with fuzzy storage states for reservoir operations
Wu	2004	A benchmark study on maximizing energy cost saving for pump operation
Cembrano et al.	2004	Optimal control of urban drainage systems. a case study
Mousavi et al.	2004b	Application of an interior-point algorithm for optimization of a large-scale reservoir system
Azaiez et al.	2005	A chance-constrained multi-period model for a special multi-reservoir system
Bhattacharjya and Datta	2005	Optimal management of coastal aquifer using linked simulation optimization approach
Kapelan et al.	2005	Optimal sampling design methodologies for water distribution model calibration
Gorantiwar and Smout	2005	Multilevel approach for optimizing land and water resources and irrigation deliveries for tertiary units in large irrigation schemes
Mousavi et al.	2005	Reservoir operation using a dynamic programming fuzzy rule-based approach
Qahman et al.	2005	Optimal and sustainable extraction of groundwater in coastal aquifers
Reis et al.	2005	Multi-reservoir operation planning using hybrid genetic algorithm and linear programming (GA–LP): an alternative stochastic approach
Smout and Gorantiwar	2005	Multilevel approach for optimizing land and water resources and irrigation deliveries for tertiary units in large irrigation schemes. I: Method
Tospornsampan et al.	2005	Optimization of a multiple reservoir system using a simulated annealing—a case study in the Mae Klong system, Thailand
Wang et al.	2005	Stochastic multiobjective optimization of reservoirs in parallel
Yurtal et al.	2005	Hydropower optimization for the lower Seyhan system in Turkey using dynamic programming

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Author(s)	Date	Title
Sepaskhah and Akbari	2005	Deficit irrigation planning under variable seasonal rainfall
Loucks and van Beek	2005	Water resource systems planning and management: an introduction to methods, models and applications
Tu et al.	2005	Optimization of water distribution and water quality by hybrid genetic algorithm
Abrishamchi et al.	2005	Case study: application of multicriteria decision making to urban water supply
Maqsood et al.	2005	ITOM: an interval-parameter two-stage optimization model for stochastic planning of water resources systems
Simonovic and Nirupama	2005	A spatial multi-objective decision-making under uncertainty for water resources management
Farmani et al.	2005	Trade-off between total cost and reliability for Anytown water distribution network
Karamouz	2005	Qualitative and quantitative planning and management of water allocation with emphasis on conflict resolution
Keedwell and Khu	2005	A hybrid genetic algorithm for the design of water distribution networks
Abarca et al.	2006	Optimal design measures to correct seawater intrusion
Archibald et al.	2006	Modeling the operation of multireservoir systems using decomposition and stochastic dynamic programming
Booker and O'Neill	2006	Can reservoir storage be uneconomically large?
Haddad et al.	2006	Honey-bees mating optimization (HBMO) algorithm: a new heuristic approach for water resources optimization
Karmakar and Mujumdar	2006	Grey fuzzy optimization model for water quality management of a river system
Katsifarakis and Petala	2006	Combining genetic algorithms and boundary elements to optimize coastal aquifers' management
Kumar et al.	2006	Optimal reservoir operation for irrigation of multiple crops using genetic algorithms
Karamouz et al.	2006	System dynamics-based conflict resolution model for river water quality management
Lee et al.	2006	Two-stage stochastic linear programming model for coordinated multi-reservoir operation
Lund	2006	Drought storage allocation rules for surface reservoir systems
Prasad et al.	2006	Optimal irrigation planning under water scarcity
Reddy and Kumar	2006	Optimal reservoir operation using multi-objective evolutionary algorithm
Sethi et al.	2006	Optimal crop planning and water resources allocation in a coastal groundwater basin, Orissa, India
Gill et al.	2006	Multiobjective particle swarm optimization for parameter estimation in hydrology
Mannocchi and Todisco	2006	Optimal reservoir operations for irrigation using a three spatial scales approach

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Author(s)	Date	Title
Purves	2006	2006 annual review of planning variables for water supply and demand assessment: A review of the changes in demand assumptions for future water options for the ACT
Cervellera et al.	2006	Optimization of a large scale water reservoir network by stochastic dynamic programming with efficient state space discretization
Smout et al.	2006	Performance-based optimization of land and water resources within irrigation schemes. II: application
Gorantiwar et al.	2006	Performance-based optimization of land and water resources within irrigation schemes. I: Method
Suen and Eheart	2006	Reservoir management to balance ecosystem and human needs: Incorporating the paradigm of the ecological flow regime
Dandy et al.	2006	Sustainability objectives for the optimization of water distribution networks
Chang et al.	2007	Multi-step-ahead neural networks for flood forecasting
Ferreia da Silva and Haie	2007	Optimal locations of groundwater extractions in coastal aquifers
Grunow et al.	2007	Supply optimization for the production of raw sugar
Hsu and Wei	2007	A multipurpose reservoir real-time operation model for flood control during typhoon invasion
Joubert et al.	2007	An optimization model for the management of a South African game ranch
Karterakis et al.	2007	Application of linear programming and differential evolutionary optimization methodologies for the solution of coastal subsurface water management problems subject to environmental criteria
Moradi-Jalal et al.	2007	Reservoir operation in assigning optimal multi-crop irrigation areas
Nandalal and Bogardi	2007	Dynamic programming based operation of reservoirs: applicability and limits
Smith et al.	2007	Optimization of hydropower resources: current practices and opportunities for improvement
Reddy and Kumar	2007	Multi-objective particle swarm optimization for generating optimal trade-offs in reservoir operation
Yang et al.	2007	Multiobjective planning of surface water resources by multiobjective genetic algorithm with constrained differential dynamic programming
Abolpour and Javan	2007	Optimization model for allocating water in a river basin during a drought
Hajkowicz and Collins	2007	A review of multiple criteria analysis for water resource planning and management

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Author(s)	Date	Title
Ganji et al.	2007	Development of stochastic dynamic Nash game model for reservoir operation. I: The symmetric stochastic model with perfect information
Fedra et al.	2007	Water resources management: Economic valuation and participatory multi-criteria optimization
Ayvaz and Karahan	2008	A simulation/optimization model for the identification of unknown groundwater well locations and pumping rates
Baltar and Fontane	2008	Use of multiobjective particle swarm optimization in water resource management
Hsu et al.	2008	Optimization and capacity expansion of a water distribution system
Kale et al.	2008	Optimal design of pressurized irrigation subunit
Azamathulla et al.	2008	Comparison between genetic algorithm and linear programming approach for real time operation
Minciardi et al.	2008	Multiobjective optimization of solid waste flows: environmentally sustainable strategies for municipalities
Karamouz and Araghinejad	2008	Drought mitigation through long-term operation of reservoirs: Case study
Karamouz et al.	2008	Dealing with conflict over water quality and quantity allocation: A case study
Cetinkaya et al.	2008	Optimization methods applied for sustainable management of water-scarce basins
Castelletti et al.	2008	Water reservoir control under economic, social and environmental constraints
Li et al.	2008	Modeling of the flow changes due to reservoir operations and the impacts on aquatic ecosystem downstream
Baltar and Fontane	2008	Use of multiobjective particle swarm optimization in water resources management
Barros et al.	2008	Planning and operation of large-scale water distribution systems with preemptive priorities
Dandy et al.	2008	Optimizing the sustainability of water distribution systems
Ayvaz	2009	Application of harmony search algorithm to the solution of groundwater management models
Bar et al.	2009	Use of a dynamic programming model to estimate the value of clinical mastitis treatment and prevention options utilized by dairy producers
Karamouz et al.	2009	Probabilistic reservoir operation using bayesian stochastic model and support vector machine
Rejani et al.	2009	Simulation–optimization modelling for sustainable groundwater management in a coastal basin of Orissa, India.
Sarker and Ray	2009	An improved evolutionary algorithm for solving multiobjective crop planning models
Yang et al.	2009	Multi-objective planning for conjunctive use of surface and subsurface water using genetic algorithm and dynamics programming

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Yoo	2009	Maximization of hydropower generation through the application of a linear programming model
Madsen et al.	2009	Energy optimization of well fields
Han et al.	2009	A multi-objective linear programming model with Interval parameters for water resources allocation in Dalian City
Rosenberg and Lund	2009	Modeling integrated decisions for a municipal water system with recourse and uncertainties: Amman, Jordan
Herstein et al.	2009a	Evaluating environmental impact in water distribution system design
Herstein et al.	2009b	Environmental input–output (EIO)-based multi-objective design of water distribution systems
Afshar et al.	2010	Large-scale nonlinear conjunctive use optimization problem: decomposition algorithm
Chiu et al.	2010	Optimal pump and recharge management model for nitrate removal in the Warren Groundwater Basin, California
Moghaddasi et al.	2010	Long-term operation of irrigation dams considering variable demands: case study of Zayandeh-rud Reservoir, Iran
Rani and Moreira	2010	Simulation–optimization modeling: a survey and potential application in reservoir systems operation
Singh	2010	Decision support for on-farm water management and long-term agricultural sustainability in a semi-arid region of India
Hakimi-Asiabar et al.	2010	Deriving operating policies for multi-objective reservoir systems:Application of Self-Learning Genetic Algorithm
Reichold et al.	2010	A simulation–optimization framework to support sustainable watershed development by mimicking the predevelopment flow regime
Huang and Liu	2010	Multiobjective water quality model calibration using a hybrid genetic algorithm and neural network-based approach
Safavi et al.	2010	Simulation–optimization modeling of conjunctive use of surface water and groundwater.
Arena et al.	2010	A simulation/optimization model for selecting infrastructure alternatives in complex water resource systems
Alemu et al.	2011	Decision support system for optimizing reservoir operations using ensemble streamflow predictions
Gaur et al.	2011	Analytic elements method and particle swarm optimization based simulation–optimization model for groundwater management
Divakar et al.	2011	Optimal allocation of bulk water supplies to competing use sectors based on economic criterion – An application to the Chao Phraya River Basin, Thailand
Moeini et al.	2011	Fuzzy rule-based model for hydropower reservoirs operation

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Caxaria et al.	2011	Small scale hydropower: generator analysis and optimization for water supply systems
Hejazi and Cai	2011	Building more realistic reservoir optimization models using data mining—a case study of Shelbyville reservoir
Afshar et al.	2011	Particle swarm optimization for automatic calibration of large scale water quality model (CE-QUAL-W2): application to Karkheh reservoir, Iran
Mortazavi et al.	2012	Multiobjective optimization of urban water resources : Moving toward more practical solutions
Deschaine et al.	2011	Optimised planning for management of integrated surface water and groundwater systems
Li et al.	2011	Modelling the impacts of reservoir operations on the downstream riparian vegetation and fish habitats in the Lijiang River
Karamouz et al.	2011	Development of a demand driven hydro-climatic model for contingency planning during drought: a case study
George et al.	2011	An integrated hydro-economic modelling framework to evaluate water allocation strategies I: Model development
Zhang et al.	2011	Water quantity and quality optimization modeling of dams operation based on SWAT in Wenyu River Catchment, China
Sun et al.	2011	Optimization of yield and water-use of different cropping systems for sustainable groundwater use in North China Plain
Afshar	2012	Large scale reservoir operation by constrained particle swarm optimization algorithms
Broitman et al.	2012	One size fits all? An assessment tool for solid waste management at local and national levels
Huang et al.	2012	Optimization of the irrigation water resources for agricultural sustainability in Tarim River Basin, China
Mouatasim	2012	Boolean integer nonlinear programming for water multireservoir operation
Ziaei et al.	2012	Optimization and simulation modeling for operation of the Zayandehrud reservoir
Eum et al.	2012	Integrated reservoir management system for flood risk assessment under climate change
Lee et al.	2012	A watershed-scale design optimization model for stormwater best management practices
Dehdari et al.	2012	Comparison of optimization algorithms for reservoir management with constraints—a case study
Singh	2012b	Optimal allocation of resources for the maximization of net agricultural return
Chen et al.	2012	Adapting the operation of two cascaded reservoirs for ecological flow requirement of a de-watered river channel due to diversion-type hydropower stations

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Author(s)	Date	Title
Goodarzi et al.	2013	Reservoir operation management by optimization and stochastic simulation
Ramos et al.	2013	Optimization of retention ponds to improve the drainage system elasticity for water-energy nexus
Geng and Wardlaw	2013	Application of multi-criterion decision making analysis to integrated water resources management
Damodaram and Zechman	2013	Simulation–optimization approach to design low impact development for managing peak flow alterations in urbanizing watersheds
Price and Ostfeld	2013	Iterative linearization scheme for convex nonlinear equations: application to optimal operation of water distribution systems
Kang and Lansey	2013	Scenario-based robust optimization of regional water and wastewater infrastructure
Zhang et al.	2013	Optimal operation of multi-reservoir system by multi-elite guide particle swarm optimization
Afshar et al.	2013	Multiobjective calibration of reservoir water quality modeling using multiobjective particle swarm optimization (MOPSO)
Kurek and Ostfeld	2013	Multi-objective optimization of water quality, pumps operation, and storage sizing of water distribution systems
Shammout et al.	2013	Participatory optimization scenario for water resources management: a case from jordan
Guo et al.	2013	Multi-objective optimization of the proposed multi-reservoir operating policy using improved NSPSO
Zagonari	2013	An optimization model for integrated coastal management: development and a case study using Italy's Comacchio municipality
Giustolisi et al.	2013	Operational optimization: water losses versus energy costs
Kourakos and Mantoglou	2013	Development of a multi-objective optimization algorithm using surrogate models for coastal aquifer management
Huang et al.	2013	A stochastic optimization approach for integrated urban water resource planning
Damodaram and Zechman	2013	Simulation–Optimization approach to design low impact development for managing peak flow alterations in urbanizing watersheds
Pianosi et al.	2013	Tree-based fitted Q-iteration for multi-objective Markov decision processes in water resource management
Shokri et al.	2013	Reservoir operation for simultaneously meeting water demand and sediment flushing: stochastic dynamic programming approach with two uncertainties

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